Price vs Quantity
Martin Weitzman

Thomas Bourany
THE UNIVERSITY OF CHICAGO

Macro reading group

September 2023

Introduction – Choice of planning instrument

- If regulators want to regulate a market, is it more efficient to regulate with *prices* or *quantity* ?
  - Example of pollution (clean air) : is it better to use price instruments or caps/quotas ?
  - Conventional economists wisdom : vague preference for price/tax (example : carbon tax)
  - Non-economists : direct controls on quantity more efficient
- What friction implies a difference between the two ?
  - Here : Uncertainty and information frictions
  - Simple static cost-benefit analysis
- Main result :
  - Result ambiguous depending on cost and benefit elasticity and cost uncertainty
  - Does **not** depends on benefit uncertainty
  - Quantity regulation preferred in most cases / largest share of the parameter space

Planning problem

- Planning problem :
  - Choice of a quantity of a good q
  - Produced a private cost $C(q)$, increasing and convex $C''(q) > 0$
  - Yielding (public) benefit $B(q)$, increasing and concave $B''(q) < 0$
- Quantity regulation : $q^\star$

$$\max_q B(q) - C(q)$$

$$\Rightarrow \qquad B'(q^\star) = C'(q^\star) =: p^\star$$

- Equivalence :
  Choice by the planner of price : $p^\star$ and let producers maximize :

$$\max_q \; p^\star q - C(q)$$

Uncertainty and information frictions

- Two sources of incomplete information :
- – Stochastic private cost $C(q, \theta)$, with slope $C'' = C_{qq}(q, \theta) > 0$ [***known to firms ex-post***]
- – Stochastic benefit $B(q, \eta)$, increasing and concave $B'' = C_{qq}(q, \eta) < 0$

- Ideal instrument : ex-ante quantity schedule $q^\star(\theta, \eta)$ or (equivalently) and price schedule $p^\star(\theta, \eta)$ satisfying :

$$B_q(q^\star(\theta, \eta), \eta) = C_q(q^\star(\theta, \eta), \theta) = p^\star(\theta, \eta)$$

  - Can reach the first-best and *eliminate* ex-post uncertainty
  - But infeasible : requires information about all states of the world (θ, η)

Quantity or price regulation under uncertainty

- Choice of target quantity $\hat{q}$ to maximize $\mathbb{E}[B(q,\eta) - C(q,\theta)]$

$$\mathbb{E}[B_q(\hat{q},\eta)] = \mathbb{E}[C_q(\hat{q},\theta)]$$

- Choice of target price $\tilde{p}$
  - Ex-post firm policy $q = h(p,\theta)$ implicitly defined by FOC

$$p = C_q\big(h(p,\theta),\theta\big)$$

  - price instrument $\tilde{p}$ chosen ex-ante to maximize $\mathbb{E}\Big[B\big(h(\tilde{p},\theta),\eta\big) - C\big(h(\tilde{p},\theta),\theta\big)\Big]$
  - Ex-ante FOC :

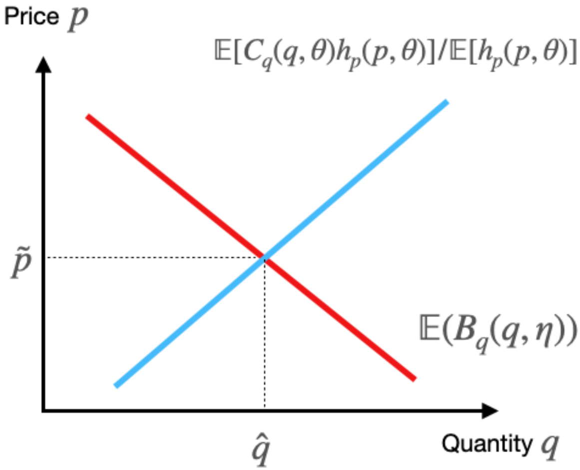
$$\mathbb{E}\Big[B_q\big(h(\tilde{p},\theta),\eta\big)\ h_p(\tilde{p},\theta)\Big] = \mathbb{E}[C_q\big(h(\tilde{p},\theta),\theta\big)h_p(\tilde{p},\theta)]$$

  - Yielding optimal price :

$$\tilde{p} = \frac{\mathbb{E}\Big[B_q\big(h(\tilde{p},\theta),\eta\big)\ h_p(\tilde{p},\theta)\Big]}{\mathbb{E}\Big[\ h_p(\tilde{p},\theta)\Big]}$$

  - And ex-post quantity : $\tilde{q}(\theta) = h(\tilde{p},\theta)$

Quantity or price regulation under uncertainty



Model approximation

- Approximate $C(q,\theta)$ and $B(q,\eta)$ to 2nd order around the optimal quantity choice $\hat{q}$
  - Cost curve :
$$C_q(q,\theta) = (C' + \alpha(\theta)) + C''(q - \hat{q})$$
  - Benefit curve :
$$B_q(q,\theta) = (B' + \beta(\eta)) + B''(q - \hat{q})$$
  - Assumptions :
$$\mathbb{E}[\alpha(\theta)] = \mathbb{E}[\beta(\eta)] = 0 \qquad \mathbb{V}\mathrm{ar}(\alpha(\theta)) = \sigma^2 \qquad \text{and} \qquad \mathbb{V}\mathrm{ar}(\beta(\eta)) = \gamma^2$$
  - Optimal price and producer decision :
$$\tilde{p} = \mathbb{E}[B_q(h(\tilde{p},\theta),\eta)] = B' \qquad \tilde{q} = h(\tilde{p},\theta) = \hat{q} - \frac{\alpha(\theta)}{C''}$$

Model approximation

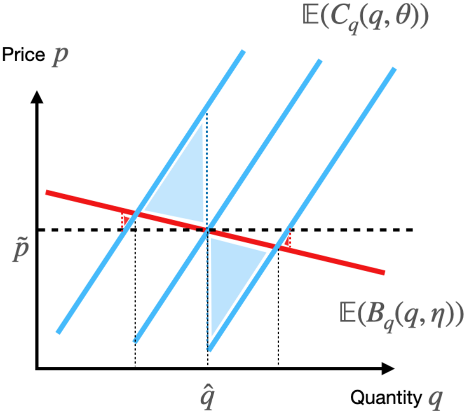
- Δ Expected surplus (welfare) advantage of prices over quantity

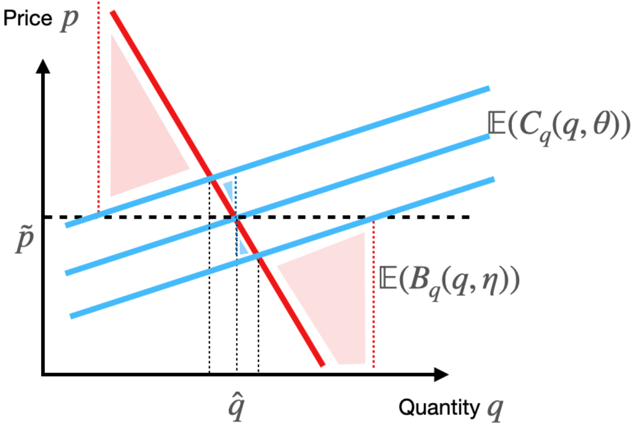
$$\Delta = \frac{\sigma^2}{2C''}(B'' + C'')$$

  - Recall $B'' < 0 < C''$
  - If $|B''| < C''$ (cost very elastic), price $\tilde{p}$ preferred over $\hat{q}$
  - If $|B''| > C''$ (cost less elastic), quantity $\hat{q}$ preferred over $\tilde{p}$

  - Benefit uncertainty $\gamma = \mathbb{V}\mathrm{ar}(\beta(\eta))$ does *not* appear !

Gain from prices (blue)





Extensions :

- Risk in slope

$$C_q(q,\theta) = (C' + \alpha(\theta)) + \frac{C''}{f(\theta)}(q - \hat{q})$$

$$B_q(q,\theta) = (B' + \beta(\eta)) + \frac{B''}{g(\eta)}((q - \hat{q})$$

$$\text{with} \qquad \delta^2 = \mathbb{V}\text{ar}(f(\theta))$$

$$\Rightarrow \qquad \Delta = \frac{\sigma^2}{2C''}[B''(1+\delta^2) + C'']$$

- Many producers

$$\max_{q/p} \mathbb{E}\left[B(q,\eta) - \sum_i c^i(q_i,\theta_i)\right]$$

$$\Delta = \sum_i \sum_j \frac{B_{ij}\sigma_{ij}^2}{2c_{qq}^i c_{qq}^j} - \sum_k \frac{\sigma_i i^2}{2c_{qq}^k}$$