Networks and the Macroeconomy
Theory and applications

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June 2020

Introduction – Motivation

- Network economies is another step to breakdown the standard neoclassical macro framework
  - Firms operates in a input-output structure
  - Exposure and strategic interactions are not-trivial
    $\Rightarrow$ far from the island model !

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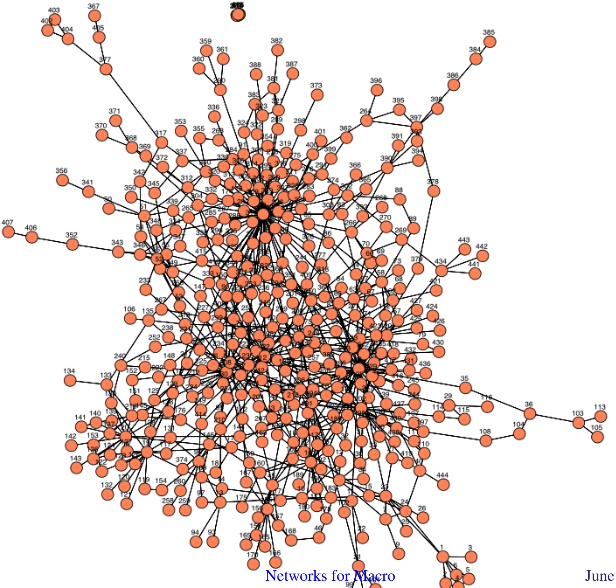
  - Exposure and strategic interactions are not-trivial
    $\Rightarrow$ far from the island model !
- Shocks propagate (and potentially amplify) :
  $\Rightarrow$ link with macroeconomic fluctuation
- Empirical approach
  - Exercise made available thanks to input-ouput matrices and more disaggregated firms data
  - Sadly, I won't have too much time to talk about it

Motivatio



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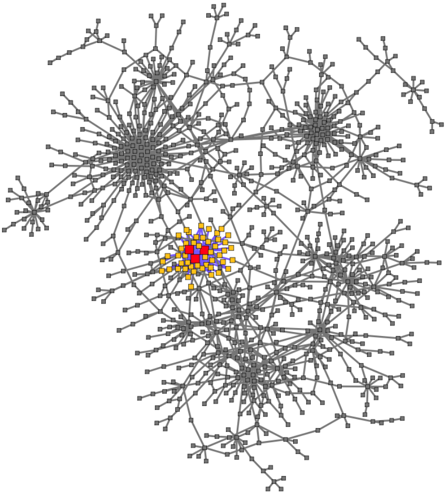

Fig. 2. Buyer–supplier network in 2006. GM, Ford, and Chrysler are colored red. Their suppliers are colored orange. All other firms are gray.

Articles

- Today I'll try to talk about 4 articles :
  1. Basic network framework and notations :
     - *Production Networks : A Primer*, 2017, Annual Reviews, by V. Carvalho and A. Tahbaz-Salehi
     - *From Micro to Macro via Production Networks*, 2014, JEP, by V. Carvalho
  2. *Networks and the Macroeconomy : An Empirical Exploration*, 2016, Macro-Annuals, by D. Acemoglu, U. Akcigit and W. Kerr
     - ▷ Most simple frameworks with empirical counterparts
  3. *The Network Origins of Aggregate Fluctuations*, 2012, Ecma, by D. Acemoglu, V. Carvalho, A. Ozdaglar, and A. Tahbaz- Saleh
     - Idiosyncratic shocks can propagate if the network has high variability in sectoral "degree"
  4. *The Macroeconomic Impact of Microeconomic Shocks : Beyond Hulten's Theorem*, 2019, Ecma, by D. Baqaee and E. Farhi
     - Non-linearities critical for understanding source and welfare of business cycle

Other Articles – 1

- Endogenous network structure
  - A Theory of Input-Output Architecture, 2018, Econometrica, by E. Oberfield
    - Theory of network structure formation : firms choose exactly one intermediate input (question : which one ?)
    - If elasticity of output to input os high, star suppliers emerge endogenously
  - The Origins of Firm Heterogeneity : A Production Network Approach, 2019, by A. Bernard, E. Dhyne, G. Magerman, K. Manova, and A. Moxnes,
    - Data : all buyer-supplier relationships in Belgium
    - 2 new facts : more connected firms have lower sales per costumer + negative assortativity
    - Heterogeneous firms model, with 2-dims : productivity and relationship capability

Other Articles – 2

- Trade and network :
  - All the articles by F. Tintelnot, for example :
  - The margins of global sourcing : Theory and evidence from US firms, 2017, by P. Antras, T. Fort, F. Tintelnot
    - Interdependencies/complementarities in firms' sourcing decisions across markets
  - Putting the Parts Together : Trade, Vertical Linkages, and Business Cycle Comovement, 2010, by Di Giovanni, and Levchenko
    - International business cycles driven by vertical linkages and bilateral trade between sectors
- Plently of more Macro/theory articles (Winberry and vom Lehn, Rubbo's JMP, Hak's JMP)

Notations - basic framework - 1

- Most important objects :
  - $\mathcal{N}$ units : sectors/industries/goods/firms
  - Input-Output matrix :

$$\omega_{ij} := \frac{p_j x_{ij}}{p_i y_i} = \frac{\text{sales from } j \text{ to } i}{\text{sales of } i} \qquad \Omega := \{\omega_{ij}\}_{ij}$$

    - Sales shares, measuring direct exposure of i to sector j

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  - Out-degrees and weighted outdegree :

$$d_i^{out} = \#\{j \text{ s.t. } \omega_{ji} > 0\}$$
$$d_i^{out,w} = \sum_j \omega_{ji} \qquad d^{out} = \Omega' \mathbb{1}$$

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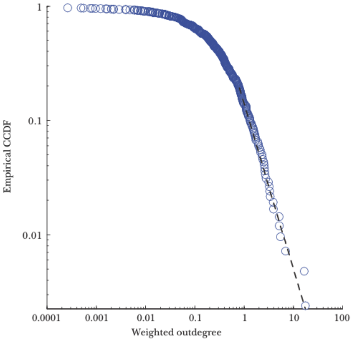
    – Measure connectedness/centrality as a supplier
  - In-degrees and weighted indegree :

$$d_i^{in} = \#\{j \text{s.t. } \omega_{ij} > 0\}$$
$$d_i^{in,w} = \sum_j \omega_{ij} = 1 \quad \text{by definition of } \Omega$$

    – Measure centrality as a costumer

Notations - basic framework - 1 - Outdegree

Figure 3
The Weighted Outdegree Distribution Associated with 2002 US Input-Output Data



Source: Bureau of Economic Analysis, detailed input-output table for 2002. For more details, see Data Appendix available with this paper at http://e-jep.org.
Notes: The x-axis gives the weighted outdegree for each sector, presented on a log scale. The y-axis, also in log scale, gives the probability of finding a sector with weighted outdegree larger than or equal to *x*, that is the empirical counter-cumulative distribution (CCDF).

Notations, basic framework - 2

- Most important objects :
  - Leontieff Inverse Matrix :

$$\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \cdots = \sum_{k=0}^{\infty} \Omega^k$$

    – Measure total exposure (direct and indirect) for i
    – Linked to another measure of centrality : Katz-Bonacich centrality

$$\boldsymbol{c} = \eta(I - \delta\Omega')^{-1}\mathbb{1}$$

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  - Sales shares / Domar weights :

$$\lambda_i = \frac{p_i y_i}{Y} = \frac{p_i y_i}{\sum_{i \in \mathcal{N}} p_i c_i} = \frac{\text{sales of } i}{GDP}$$

    - Share in final consumption / GDP

Notations, basic framework - 3 - Cobb Douglas structure

- Production function for i- output :

$$y_i = z_i \zeta_i \prod_{j=1}^{n} x_{ij}^{\omega_{ij}}$$

- Utility of final consumer :

$$u(c_1, \ldots, c_n) = \xi \prod_{i=1}^{n} c_i^{\beta_i}$$

- We obtain a link btw Domar weights and the Leontieff matrix :

$$\lambda_i = \frac{p_i y_i}{Y} = \beta_i + \sum_k \omega_{ki} \lambda_k$$
$$= \sum_k \beta_k \psi_{ki}$$

$$\Rightarrow \qquad \Lambda = \Psi' \boldsymbol{\beta} = (I - \Omega')^{-1} \boldsymbol{\beta}$$

Cobb Douglas structure : Hulten's theorem

- Hulten's theorem (1978)
  - Holds with efficient economy and production function with homogeneity of degree one
    $$\frac{dY}{Y} = \sum_{i=1}^{\mathcal{N}} \lambda_i \frac{dz_i}{z_i} = \Lambda \cdot \frac{dz}{z}$$
  - Domar weights as sufficient statistics for the transmission of shocks from sector i to the whole economy, up to a first order approximation
- For Cobb Douglas, Hulten's theorem holds globally :
  $$\ln Y = \sum_i \lambda_i \varepsilon_i = \Lambda \cdot \boldsymbol{\varepsilon}$$

Model – Propagation of shocks

- *Networks and the Macroeconomy : An Empirical Exploration*, 2016, Macro-Annuals, by D. Acemoglu, U. Akcigit and W. Kerr
  - Cobb Douglas economy
  - Preferences : $\beta = 1/n$ and disutil of labor $\gamma(\ell) = (1-\ell)^\lambda$
- Comparison supply shocks vs. Demand shocks :
  - industry imports from China (demand side),
  - changes in federal spending (demand side),
  - TFP shocks (supply side),
  - knowledge/ideas coming from foreign patenting (supply side)
- Result :
  - Demand propagate upstream while supply propagate downstream

  $\Rightarrow$ Cobb Douglas assumpt. : income & substitution effects cancel out

Model – Propagation of shocks

(1) Productivity shocks : dz_i

$$d\ln y_i = \underbrace{dz_i}_{\text{own effect}} + \underbrace{\sum_{j=1}^{n} \left(\psi_{ij} - \mathbb{1}_{\{j=i\}}\right) \times dz_j}_{\text{network effect}}$$

$$d\ln \mathbf{y} = \Psi'\beta \cdot d\mathbf{z}$$

- Supply shocks only propagates downstream
- Indirect effects much larger than direct effects

Model – Propagation of shocks

(2) Government spending shocks :

$$d \ln y_i = \underbrace{\frac{d\tilde{G}_i}{p_i y_i}}_{\text{own effect}} + \underbrace{\sum_{j=1}^{n} \left(\hat{\psi}_{ji} - \mathbf{1}_{j=i} \right) \times \frac{d\tilde{G}_j}{p_j y_j}}_{\text{network effect}} - \underbrace{\sum_{j=1}^{n} \hat{\psi}_{ji} \times \frac{1}{p_j y_j} \times \frac{\beta_j}{1+\lambda} \times \sum_{k=1}^{n} d\tilde{G}_k}_{\text{resource constraint effect}}$$

$$d \ln \mathbf{y} = \widehat{\Psi} \mathbf{D} \big(I - \tfrac{\mathbb{1}'}{(1+\lambda)n} \big) \mathbf{d}\widetilde{\mathbf{G}}$$

- With dG_i and $d\widetilde{G}_i = p_i dG_i$, and re-weighting : $\widehat{\omega}_{ij} = \frac{\omega_{ij}}{y_j}$ and $\widehat{\Psi} = (I - \widehat{\Omega})^{-1}$ and $D = \text{diag}(p_i y_i)^{-1}$

- Demand shocks only propagates upstream
- Indirect effects much larger than direct effects

Model – Propagation of shocks

(3) Empirical counterpart to the equations above :

$$\Delta \ln Y_{i,t} = \delta_t + \psi \Delta \ln Y_{i,t-1} + \beta^{\text{own}}\, Shock_{i,t-1} + \beta^{\text{upst}} Upstream_{i,t-1} + \beta^{\text{downst}} Downstream_{i,t-1} + \varepsilon_{i,t}$$

with
$$\text{Downstream}_{i,t} = \sum_j \left(\text{Input}\%_{j\to i}^{1991} - \mathbf{1}_{j=i}\right) \cdot\ \text{Shock}_{j,t}$$
$$\text{Upstream}_{i,t} = \sum_j \left(\text{Output}\%_{i\to j}^{1991} - \mathbf{1}_{j=i}\right) \cdot\ \text{Shock}_{\,j,t}$$

- Results in estimations consistent with the theory

Origins of Aggregate Fluctuations

- *The Network Origins of Aggregate Fluctuations*, 2012, Ecma, by D. Acemoglu, V. Carvalho, A. Ozdaglar, and A. Tahbaz- Saleh
- Foundation of business cycle by microeconomic shocks, similar as Gabaix (2011)
    - Failure of Lucas (1977) argument, where idiosyncratic shocks should diversify in the aggregate, as weight $1/\sqrt{n} \to 0$.
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  - Due to Fat-tail (power law) distribution in Gabaix (2011)
  - Basic idea from Hulten's thm + i.i.d. assumption $\sigma_i = \sigma$:

$$d\ln Y = \sum_i \lambda_i \, d\ln z_i$$

$$\Rightarrow \qquad \sigma_Y^2 = \mathbb{V}\text{ar}(\ln Y) = \overbrace{\left(\sum_i \lambda_i^2\right)}^{=h^2,\text{ Herfindahl}} \sigma^2 \qquad \Rightarrow \qquad \sigma_Y = h\sigma$$

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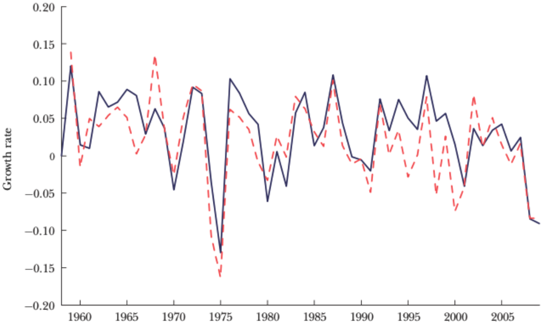
  - Show weak versions of Central Limit Theorem
  - Here, Input-output linkages instead of Fat-tail firm distrib : Conditions on the network structure for the idiosyncratic shocks to remains, even as size of units $\to 0$

Origins of Aggregate Fluctuations

Figure 6
Comovement of Productivity Growth in Central Sectors and Aggregate Output Growth



Source: NBER-CES Manufacturing Industry Database and Bureau of Economic Analysis detailed input-output tables for 1987.
Notes: The solid line gives manufacturing real value added growth for the period 1959–2009. The dashed line gives the simple average of total factor productivity growth across the ten most central sectors in the production network.

Network origins of Macro Fluctuations

- Almost same idea as in Gabaix :
  - Hulten's theorem with "influence vector" $\boldsymbol{v}$

$$\ln y_i = \Lambda \cdot \boldsymbol{\varepsilon} = \sum_i v_i \varepsilon_i = \psi' \beta \cdot \boldsymbol{\varepsilon} = \frac{1}{n} (\boldsymbol{I} - \Omega')^{-1} \cdot \boldsymbol{\varepsilon}$$

  - Influence vector $v_i := [\psi' \beta]_i = \frac{p_i x_i}{\sum_j p_j x_j}$
    (red : result of CD assumption) + $v_i = c_i$ (for specific δ and η)

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  - Question : does risk average out when level of disaggr°, $n \to \infty$?
    - Same answer as Gabaix (2011)

$$\sigma_Y^2 = \mathbb{V}\mathrm{ar}(\ln Y) = \sum_i v_{n,i}^2 \sigma_i^2$$
$$= \mathcal{O}(||v_n||_2)$$

  - Central Limit Thm : (Thm 1),
    - If $\sigma_i = \sigma$, and if ε Normal, *or* more concentred law + $\frac{||v_n||_\infty}{||v_n||_2} \to 0$, then

$$\frac{1}{||v_n||_2} y_n \xrightarrow[n \to \infty]{\mathcal{D}} \mathcal{N}(0, \sigma^2)$$

Network origins of Macro Fluctuations

- Characterization of the decay of agg. volatility as fct of network interconnection :
  - Thm 2 :

$$\sigma_Y^2 = \mathbb{V}\text{ar}(\ln Y) = \mathcal{O}^{-1}\left(\frac{1}{n}\sqrt{\sum_i (d_i^n)^2}\right) = \mathcal{O}^{-1}\left(\frac{1}{\sqrt{n}}\left(1 + \frac{\sqrt{\mathbb{V}\text{ar}(d_i^n)}}{\mathbb{E}(d_i^n)}\right)\right)$$

    – Key concept here : variability of weighted outdegree d_i^n
    – Notations : $y_n = \mathcal{O}(x_n)$ if $\lim_n \frac{y_n}{x_n} < M$ and $y_n = \mathcal{O}^1(x_n)$ if $\lim_n \frac{x_n}{y_n} < M$ (or $\lim_n \frac{y_n}{x_n} > m$)
  - Using Hulten thm, one can also show that :

$$\sigma_Y = \frac{\sigma/\alpha}{\sqrt{n}}\sqrt{1 + n^2\alpha^2\mathbb{V}\text{ar}(\lambda_i^n)}$$

Network origins of Macro Fluctuations : Extensions -1

- Distribution of interconnection (power law) :
  - Larger aggregate volatility is degrees d_i, and second-order interconnectivity coeff, have a fat-tail distribution (Thm 3)

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  - Less interconnected sectors are more volatile
- Great diversification : Carvalho and Gabaix (2013), AER
  - Great moderation due to decline in manufacturing
  - Recent volatility due to rise in financial sector

Network origins of Macro Fluctuations : Extensions - 2

- Systemic risk due to financial contagion : D. Acemoglu, A. Ozdaglar, and A. Tahbaz-Salehi (2015), AER
  - Threshold effect : if shock below, connectedness improve stability
  - If shock large enough : propagation and source of fragility and systemic risk

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- Origins of macro tail risks, D. Acemoglu D, Ozdaglar A, Tahbaz-Salehi :
  - Tail-risk related to largest Domar-weights $||\lambda_i||_\infty$
  - Slightly different from case where σ_Y related to $\mathbb{V}$ar and $||\lambda_i||_2$

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$\Rightarrow$ All these analyses stay in the Cobb-Douglas production world

From Hulten's theorem to Non-linear world

- Linear economy :
  - Cobb-Douglas assumption or first order approximation yields the Hulten's thm
  - Details of network structure irrelevant as sales shares λ_i are sufficient stats.
  - Is this reasonable ?
- Farhi-Baqaee research agenda
  - Extension to second order approximation to capture the non-linearity
  - 9 articles based on this framework (in 4 years, 3 Top 5)
  - General Production function (e.g. Nested CES) and homothetic preferences

Baqaee-Farhi 2019, Beyond Hulten's thm

- General framework : N goods, M factors :

$$y_i = A_i F_i(\ell_{i1}, ..., \ell_{iM}, x_{i1}, ..., x_{iN})$$
$$Y = \mathcal{D}(c_1, \ldots, c_N)$$
$$s.t. \qquad \sum_{i \in \mathcal{N}} p_i c_i = \sum_{i=1}^{M} w_f L_f + \sum_{i=1}^{N} \pi_i$$

- Hulten's theorem (Thm 1) and input-output multiplier :

$$\frac{d \ln Y}{d \ln A_i} = \lambda_i \qquad \sum_{i=1}^{N} \frac{d \ln Y}{d \ln A_i} = \sum_{i=1}^{N} \lambda_i = \xi$$

  - ξ is te macro-return to scale
  - ξ constant if and only if $C(\cdot)$ homogenous of degree ξ

Baqaee-Farhi 2019, Beyond Hulten's thm - More notations

- For output fct. $Y(A_1, \ldots, A_N)$, GE-elasticity of substitution is :

$$\frac{1}{\rho_{ij}} = -\frac{\mathrm{d}\log(MRS_{ij})}{\mathrm{d}\log(A_i)} = -\frac{\mathrm{d}\ln(\partial_i Y/\partial_j Y)}{\mathrm{d}\log(A_i)}$$

$$\frac{d\log(\lambda_i/\lambda_j)}{d\log A_i} = 1 - \frac{1}{\rho_{ij}}$$

- First result (Thm 2)

$$\frac{d^2\log Y}{d\log A_i^2} = \frac{d\lambda_i}{d\log A_i} = \frac{\lambda_i}{\xi}\sum_{\substack{1\le j\le N\\ j\neq i}}\lambda_j\left(1-\frac{1}{\rho_{ji}}\right) + \lambda_i\frac{d\log\xi}{d\log A_i}$$

  - Change in share of sales to others industries :
    GE-elasticity of substitution ρ_{ji}
  - Change in aggregate sales of all other industries :
    Elasticity of input-ouput ξ

Baqaee-Farhi 2019, Beyond Hulten's thm

- Second-order approximation

$$\log Y \approx \log \bar{Y} + \frac{\mathrm{d}\log Y}{\mathrm{d}\log A_i}\log A_i + \frac{1}{2}\frac{d^2\log Y}{\mathrm{d}\log A_i^2}(\log A_i)^2$$

$$\approx \log \bar{Y} + \lambda_i \log A_i + \frac{1}{2}\frac{\lambda_i}{\xi}\sum_{1\leq j\leq N, j\neq i}\lambda_j\left(1-\frac{1}{\rho_{ji}}\right)(\log A_i)^2 + \frac{1}{2}\lambda_i\frac{d\log\xi}{d\log A_i}(\log A_i)^2$$

  - Cobb-Douglas as a knife edge case where $\rho_{ij} = 1$ and ξ is constant

More results - Origins of aggregate fluctuations

- Macro moment :
  - Mean of output, if $\mathbb{V}\text{ar}(\log A_i) = \sigma_i^2$

$$\mu_Y = \mathbb{E}(\log(Y/\bar{Y})) \approx \frac{1}{\xi} \sum_i \frac{\sigma_i^2}{2\xi} \lambda_i \sum_{j \neq i} \lambda_j \left(1 - \frac{1}{\rho_{ij}}\right) + \sum_i \frac{\sigma_i^2}{2} \lambda_i \frac{d \log \xi}{d \log A_i}$$

  - Variance of macro fluctuation, if $\mathbb{V}\text{ar}(\log A_i) = \sigma^2$

$$\sigma_Y^2 = \mathbb{V}\text{ar}(\log(Y/\bar{Y})) \approx \left(\sum_i \lambda_i^2 + 2\left(\frac{\mu_Y}{\sigma}\right)^2\right) \sigma^2 \geq \left(\sum_i \lambda_i^2\right) \sigma^2$$

    - Potentially larger than the benchmark in Acemoglu et al. (2012)
  - Others formula for skewness and kurtosis

More results - Welfare costs of business cycles in a non-linear world

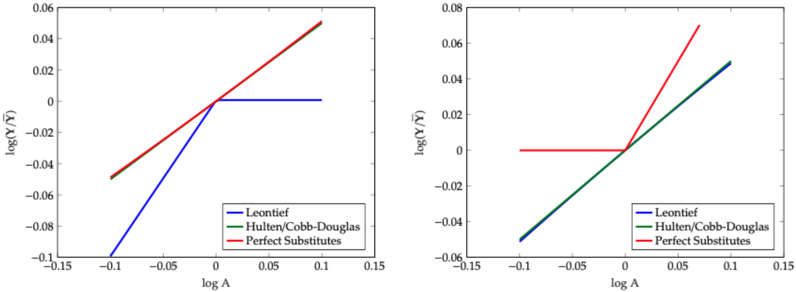
- Welfare costs of business cycle :
  - Let $u(\cdot)$ be CRRA (param. γ), $\mathbb{V}\text{ar}(\log A_k) = \sigma_k^2$ and $\bar{Y} = Y(\bar{A})$, we have :

$$\frac{[\mathbb{E}(u(Y)) - u(\bar{Y})]}{u'(\bar{Y})\bar{Y}} \approx \underbrace{-\frac{1}{2}\gamma \sum_k^N \lambda_k^2 \sigma_k^2}_{\text{Consumption nonlinearities}} + \underbrace{\frac{1}{2}\bar{Y} \sum_k^N \frac{\partial^2 Y}{\partial A_k^2} \sigma_k^2}_{\text{Production nonlinearities}}$$

    - Consumption nonlinearities : small as in Lucas (1987)
    - Production nonlinearities : potentially larger in magnitude

Illustrative example

- Industry output with reallocation (or not) of labor + CES utility
  - Factor allocation matters !



(a) log aggregate output with no realloca-tion/extreme decreasing returns. Perfect substitutes and Hulten's approximation overlap almost perfectly.

(b) log aggregate output with full realloca-tion/constant returns. Leontief and Hulten's approximation overlap almost perfectly.

General Nested CES

- Nested CES production : (pooling factors and inputs)

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i} \left(\sum_{j=1}^{N+M} \omega_{ij} \left(\frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}$$

- To understand these models, two sets of equations are key : Forward and Backward equations.
  - Forward equations describe downstream influences of A_i on price p_i
  - Backward equations describe the choices of allocation λ_i of upstream suppliers (given price i)

General Nested CES

- Forward equation : effects of A_i on p_i downstream
  - Let α denote the factor shares.

$$d\log p_i = -d\log A_i + \sum_j \omega_{ij} d\log p_j + \sum_f \alpha_{ij} d\log \Lambda_f$$

$$d\log \boldsymbol{P} = \Psi(\alpha d\log\Lambda - d\log A)$$

- This implies an extension of Hulten's theorem

$$d\log Y = -b' d\log \boldsymbol{P} = \lambda' d\log A + \Lambda' d\log \Lambda$$

General Nested CES – Mooore notations

- Backward equations : effects of p_i on λ_i (given the exposure ψ_k)
  - Account for substitution effects for allocations
  - Input-output covariance operators : measure correlation price/exposure

$$\mathbb{C}\text{ov}_{\Omega^{(j)}}\left(\Psi_{(k)}, d\log P\right) = \sum_i \omega_{ji}\Psi_{ik} d\log p_i - \left(\sum_i \omega_{ji}\Psi_{ik}\right)\left(\sum_i \omega_{ij} d\log p_i\right)$$

$$d\log\lambda_i = \sum_{k=0}^{N}(1-\theta_k)\lambda_k \mathbb{C}\text{ov}_{\Omega^{(k)}}\left(d\log P, \Psi_{(i)}\right)$$

  - Network irrelevance result : $\mathbb{C}$ov and Λ as sufficient statistics
    - With $\theta_i = \theta$, the terms $\frac{d^2\log Y}{d\log A_i d\log A_j} = (\theta-1)\lambda_i(\mathbb{1}_{\{i=j\}} - \lambda_j)$ don't depend on the network structure
  - We can solve for p_i and λ by plugging in the forward equation
  - Recover all the remaining terms and the 2nd order approx of Y

Baqaee-Farhi 2019, Extensions

- This analysis holds in economies with :
  - Wedges and markets frictions : *Productivity and misallocation in general equilibrium*, QJE 2020
  - Multi countries and trade barriers : *Networks, Barriers, and Trade*, R&R Ecma
  - Heterogeneous agents : *Macroeconomics with heterogeneous agents and input-output networks*
  - Capacity constraints and changes in tastes : *Nonlinear Production Networks with an Application to the Covid-19 crisis*
  - Capacity constraints and nominal rigidities : *Supply and Demand in Disaggregated Keynesian Economies with an Application to the Covid-19 crisis*
  - Entry and market frictions : *Entry vs. Rents*

Productivity and Misallocation in GE

- Can extend the above analysis with markups, taxes and wedges

$$d\log Y = \underbrace{\frac{\partial \log \mathscr{Y}}{\partial \log A} d\log A}_{\Delta \text{ Technology}} + \underbrace{\frac{\partial \log \mathscr{Y}}{\partial X} dX}_{\Delta \text{ Allocative Efficiency}}$$

- More precisely

$$\frac{d\log Y}{d\log A_k} = \tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d\log \Lambda_f}{d\log A_k}$$

$$\frac{d\log Y}{d\log \mu_k} = -\tilde{\lambda}_k - \sum_f \tilde{\Lambda}_f \frac{d\log \Lambda_f}{d\log \mu_k}$$

- Decomposition of output changes :

$$\mathrm{d}\log Y = \underbrace{\tilde{\lambda}' d\log A}_{\Delta \text{ Technology}} \underbrace{-\tilde{\lambda}' d\log \mu - \tilde{\Lambda}' d\log \Lambda}_{\Delta\text{Allocative Efficiency}}$$

Conclusion

- Plenty of different avenues for future research
  - Possibilities for structural/empirical approach
  - Different bridges to build with macro and IO
  - Structural changes and entry-exit of firms ?
  - Endogeneous networks : not-trivial at all

Conclusion

- Plenty of different avenues for future research
  - Possibilities for structural/empirical approach
  - Different bridges to build with macro and IO
  - Structural changes and entry-exit of firms ?
  - Endogeneous networks : not-trivial at all
- *Thank you for attending !*

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