Positive Long-Run Capital Taxation : Chamley-Judd Revisited

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Macro Reading Group

March 2019

Capital taxation

- Classic question : **Should we tax capital income ?**
  - Two common rationales :
    1. reduce distortionary labor taxes
    2. redistribution
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- **Both :** *zero* tax on capital is optimal on the long-run (steady-state)
  - Framework and assumptions questioned...
  - Still, Chamley-Judd remains an important benchmark...
- This paper – Straub-Werning – revisits their result

  ... using their own model

Chamley-Judd

- **Chamley (1986)**
  - **Trade-off** : lower labor taxes vs efficiency
  - Representative agent
  - Intertemporal government budget
- **Judd (1985)**
  - **Trade-off** : redistribution vs efficiency
  - Workers and Capitalists
  - Balanced budget

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- Straub-Werning revisits their result ... *using their own model*
  - Show results / proofs incomplete
  - Preferences : overturn conclusions when **IES** $< \mathbf{1}$!
- Main issues :
  - Related to the convergence (or not) to interior steady-state for quantities and multipliers.

Judd (1985) – Capitalist and workers

- Two class economy without government debt :
  - Capitalist save and consume C_t, utility $U(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma}$
  - Workers work and consume (hand-to-mouth) c_t, utility $u(c)$)

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- **Capitalists' problem**

$$\max_{\{C_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$C_t + a_{t+1} = R_t a_t \qquad a_{t+1} \geq 0$$

$R_t =$ *after-tax* interest on capital, $a_t =$ wealth of capitalists

- First order optimality :

$$U'(C_t) = \beta R_{t+1} U'(C_{t+1})$$

$$\beta^t U'(C_t) k_{t+1} \to 0$$

- **Resource constraint**

$$c_t + C_t + g + k_{t+1} \leq f(k_t) + (1-\delta)k_t$$

- Neoclassical technology :
  - *Before-tax* interest : $R_t^* = f'(k_t) + 1 - \delta$
  - Wage : $w_t = f(k_t) - f'(k_t)k_t$

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- **Government :** balanced-budget

$$\underbrace{g}_{\text{gov. expenses}} + \underbrace{T_t}_{\text{transfers to workers}} = \underbrace{(R_t^* - R_t)k_t}_{\text{taxed capital}}$$

- Workers : $c_t = w_t + T_t$
- **Market clearing :** $a_t = k_t$

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⇒ First order optimality + market clearing + (capitalist) budget constraint = **Implementability constraint**

Social planner's problem

- Primal approach : Maximize weighted sum of utilities :
  - ***Aim :*** redistribution from capitalist to workers : low γ $(= 0)$.

$$\max \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + \gamma U(C_t) \right\}$$

$$c_t + C_t + g + k_{t+1} = f(k_t) + (1-\delta)k_t \quad (1)$$

$$\beta\, U'(C_t)\,(C_t + k_{t+1}) = U'(C_{t-1})k_t \quad (2)$$

$$\beta^t U'(C_t)k_{t+1} \to 0 \quad (3)$$

  - λ_t Lagrange multipliers on ressource constraint : eq. (1)
  - μ_t Lagrange multipliers on Implementability : eq. (2)

Planner's First-Order conditions

μ_t on Implementability, λ_t on ressource, $\kappa_t = k_t / C_{t-1}$, $\upsilon_t = U'(C_t)/u'(c_t)$

$$\mu_0 = 0$$

$$\mu_{t+1} - \mu_t = \left(\frac{\sigma - 1}{\sigma \kappa_{t+1}} \right) \mu_t + \frac{1}{\beta \sigma \kappa_{t+1} \upsilon_t} (1 - \gamma \upsilon_t)$$

$$\frac{u'(c_{t+1})}{u'(c_t)} (f'(k_{t+1}) + 1 - \delta) = \frac{1}{\beta} + \upsilon_t (\mu_{t+1} - \mu_t)$$

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- Judd (1985) studies interior steady state
  - for allocation + multipliers
  - $c_t = c > 0$, $C_t = C > 0$, $k_t = k > 0$, $\mu_t = \mu$
  - Last FOC $\Rightarrow R^* = 1/\beta$
  - Capitalists' Euler $\Rightarrow R = 1/\beta$
  - Hence : **Zero capital tax !**

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... or not ?

Taxation results : Judd, 1985 and Lansing, 1999

- ***Thm 1 (Judd, 1985)*** **If** quantities and multipliers converge to an *interior* steady state – i.e. c_t, C_t, k_t converge to positive values and μ_t converge, **then** the tax on capital is *zero in the limit* : $\mathcal{T}_t = 1 - \frac{R_t}{R_t^*} \to 0$

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- ***Logarithm Utility*** : Assume $\sigma = 1 : U(C) = \log(C)$
  $\Rightarrow$ Constant saving rate β.

$$C_t = (1-\beta)R_t k_t \qquad k_{t+1} = \beta R_t k_t = \frac{\beta}{1-\beta} C_t$$

  - Substitute out C_t, and redistribute $\gamma = 0$, you obtain :

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \qquad s.t. \quad c_t + \frac{1}{\beta} k_{t+1} + g = f(k_t) + (1-\delta)k_t$$

  $\Rightarrow$ *Neoclassical growth model!* (with higher cost $\frac{1}{\beta}$ of capital).
  - ***Prop 1*** : Planner FOC : $R^* = 1/\beta^2$; Euler $R = 1/\beta$, and capital tax $\mathcal{T} = 1 - \beta$

Overturning the results – Straub - Werning

- Why positive tax ?
  - Multipliers do not converge (Reinhorn, 2002)
  - Extend to $\gamma \neq 0$ (Lansing, 1999),
  - Specific to log ?

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- ***Prop 3*** If $\sigma \geq 1$ and $\gamma = 0$, then ***any*** solution of the planning problem converge to the non-interior steady state : $\mathcal{T}_t \rightarrow \mathcal{T}_g > 0$

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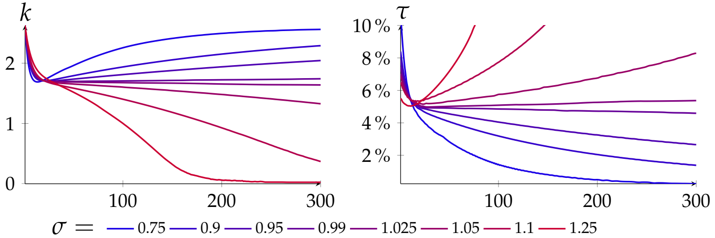
- Extension : generalize to
  1. Ad-hoc saving function $S(R_t k_t, R_{t+1}, ...)$
  2. $\gamma \neq 0$ but redistribution toward workers

Intuition and numerical example

- Intuition for *increasing slope* of capital tax :
  - Incentivize capitalists to save
  - Announce forever increasing tax
  - If IES < 1 : capitalist increase saving (income effect $>$ substitution)
- Left graph : capital stock k_t, right graph : wealth tax $\mathcal{T}_t$



- Red : IES < 1 : non-interior steady state, 85% long run tax.
- Blue : IES > 1 : interior steady state, 0% tax (Judd result)

Chamley (1986)

- Model overview
  - Representative agent
  - Optimal tax policy (labor taxes/capital taxes) and government debt
  - Koopmans' recursive utility $\mathcal{V}(U_t, U_{t+1}, \dots) = W(U_t, \mathcal{V}_{t+1})$.
  - Bound on capital tax : non-confiscatory constraint $R_t \geq 1$
- Main results :
  - Chamley : zero capital tax
  - Revisiting : add assumption : ***if*** "steady state is interior" and "constraint is asymptotically slack"
    - If non separable, then $\tau_k \to 0$ and either first best or tax base is zero
    - Separable utility : If IES < 1 then constraint may bind forever $\tau_k = \bar{\tau}$
  - Moreover : need to look at the transition path !
- Judd (1999) : Straub-Werning also revisit consumption tax analogy

Conclusion – takeaways

- Revisit Judd-Chamley models
  - If IES > 1 : zero long-run capital taxation
  - If IES < 1 : can have **positive** long-run capital taxation
- Methodological : Think twice before making assumptions on *endogenous* multipliers