A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management

P. Bolton, H. Chen, N. Wang, Journal of Finance (2011)

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Macro reading group

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Introduction – Motivation

- Under financial frictions, decisions on funding (via cash holding, equity or debt), firm investment and risk management (hedging systemic risk) are intertwined

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- Under financial frictions, decisions on funding (via cash holding, equity or debt), firm investment and risk management (hedging systemic risk) are intertwined
- *This paper* : a "tractable" dynamic model with :
  1. A neoclassical q theory of investment, c.f. Hayashi 82
  2. External financing cost (of equity)
     $\Rightarrow$ cash inventory problem, c.f. Miller Orr 66
  3. Four financial instruments : cash, equity, credit line and derivatives
- Rich set of prescriptions for a simple model

Introduction – Motivation

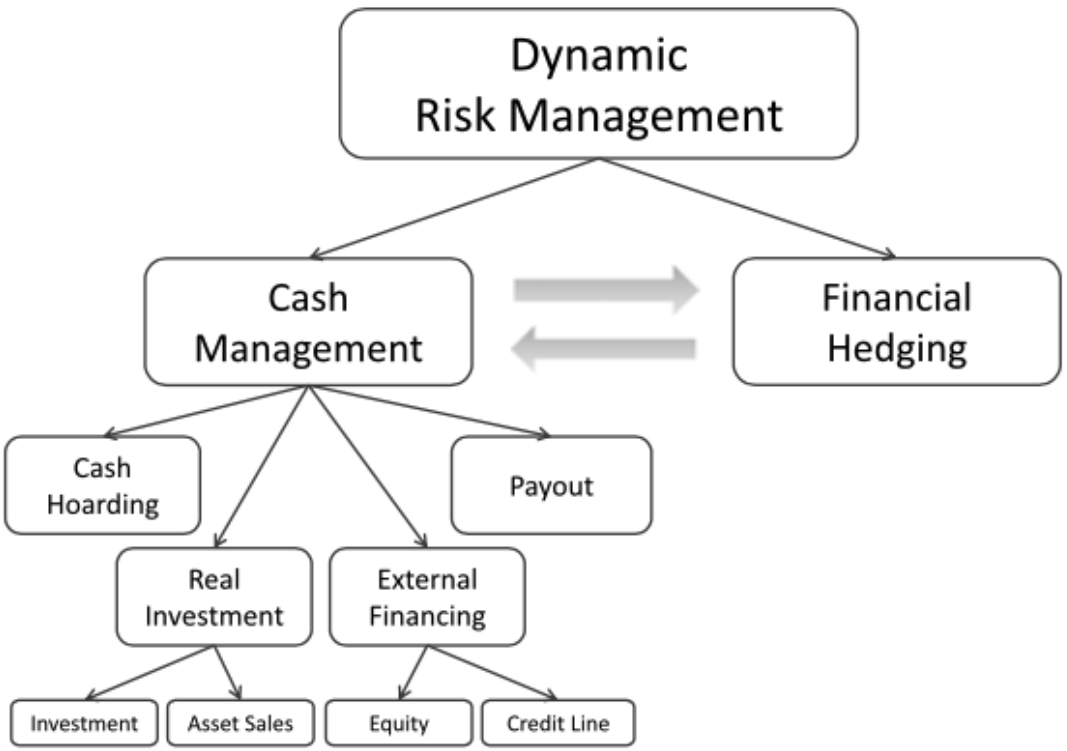

Figure 1. A unified framework for risk management.

Introduction – Preview of the results

0. With financing constraints :
   - no longer the simple $MC\ invest^{nt} = q$, instead :

$$\text{marg. cost. of investing} = \frac{\overbrace{\text{marg. } q}^{\text{marg. val. of inside capital}}}{\underbrace{\text{marg. cost. of financing}}_{\text{marg. val. of liquidity}}} = \frac{P_K(K,W)}{P_W(K,W)}$$

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1. Relation btw q and $invest^{nt}$
   - If funding is cash : positively related, but with credit line : negatively correlated because of leverage constraint
2. Cash inventory policy : (endogenous) double barrier policy
   - Pecking order btw internal/external funds and interplay with investment (or hedging) decisions
3. Different complementary channels for risk management :
   - State-noncontingent instruments (cash) for idiosyncratic risk versus derivative (future) for hedging systemic risk

Literature

- Dynamic literature beyond Hayashi and Modigliani-Miller :

1. Investment & hedging : Mello, Parsons, and Triantis (1995) and Morellec and Smith (2007)
2. Cash manag^{nt} & hedging : Mello and Parsons (2000)
3. Risk & funding structure : Fischer, Heinkel, and Zechner (1989) and Leland (1994)
4. Investment, liquidity & financing constraints : Fazzari, Hubbard, and Petersen (1988), Kaplan and Zingales (1997), Gomes (2001) and Hennessy and Whited (2005, 2007), Hennessy, Levy, and Whited (2007), Riddick and Whited (2009)
5. Loads of macro models on investment and adjustment costs. + financing costs : Decamps, Mariotti, Rochet, Villeneuve (2008)
6. Investment & dynamic agency frictions : DeMarzo, Fishman, He, Wang (2012)

AK Model of investment

- Two states : Capital K_t and Cash (liquidity/networth) W_t and reduced to one : cash-wealth $w = \frac{W}{K}$
- AK technology : $dA_t = \mu dt + \sigma dZ_t$
- Capital accumulation $dK_t = (I_t - \delta K_t)dt$ and liquidation : $L_t = lK_t$
- Investment $i_t = \frac{I_t}{K_t}$ and quadratic adjustment $g(i_t) = \frac{\theta i^2}{2}$

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- Investment $i_t = \frac{I_t}{K_t}$ and quadratic adjustment $g(i_t) = \frac{\theta i^2}{2}$
- Operating profit Y and $y = \frac{Y}{K}$ of the firm :

$$dy_t = dA_t - \big(i_t + g(i_t)\big)dt - y_t(i_t - \delta)dt$$

Model : Financial frictions

- Cost of external finance : issuing equity dH_t bear a cost dX_t :
  - Fixed cost : $\Phi = \phi K_t$ and marginal cost γH
- Firm will hoard cash W_t as liquidity to avoid raising equity.
  - Cash earns risk-free interest r and agency implies a carry cost λ
- Equity payout (i.e. share repurchase or dividend) : dU_t

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- Value function, up to liquidation τ

$$P(K_{t_0}, W_{t_0}) = \mathbb{E}_{t_0}\left[\int_{t_0}^{\tau} e^{-rt}(dU_t - dH_t - dX_t) + e^{-r\tau}(\ell K_\tau + W_\tau)\right]$$

- Homogeneity of the problem : $w = \frac{W}{K}$, and $p(w) = \frac{P(K,W)}{K}$

Model : Tobin's q

- Tobin's q related to value fct $p(w)$
  - Marginal $q := q_m = P_K(K, W) = p(w) - wp'(w)$
  - Marginal value of cash : $P_W(K, W) = p'(w)$

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- Optimal investment
  - FOC of the HJB

$$1 + \theta i^{\star}(w) = \frac{P_K}{P_W} = \frac{p(w) - wp'(w)}{p'(w)} = \frac{q_m}{p'(w)}$$

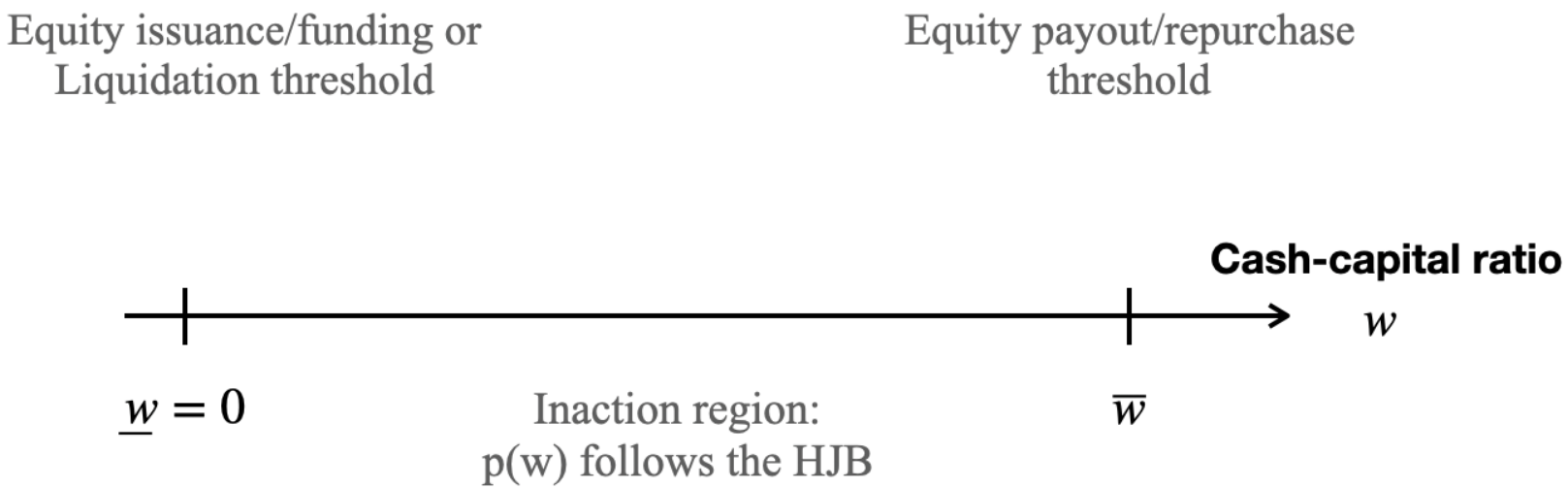
  - First best (without fin. frictions) : $P_W = p'(w) = 1$

$$1 + \theta i^{FB} = q^{FB}$$

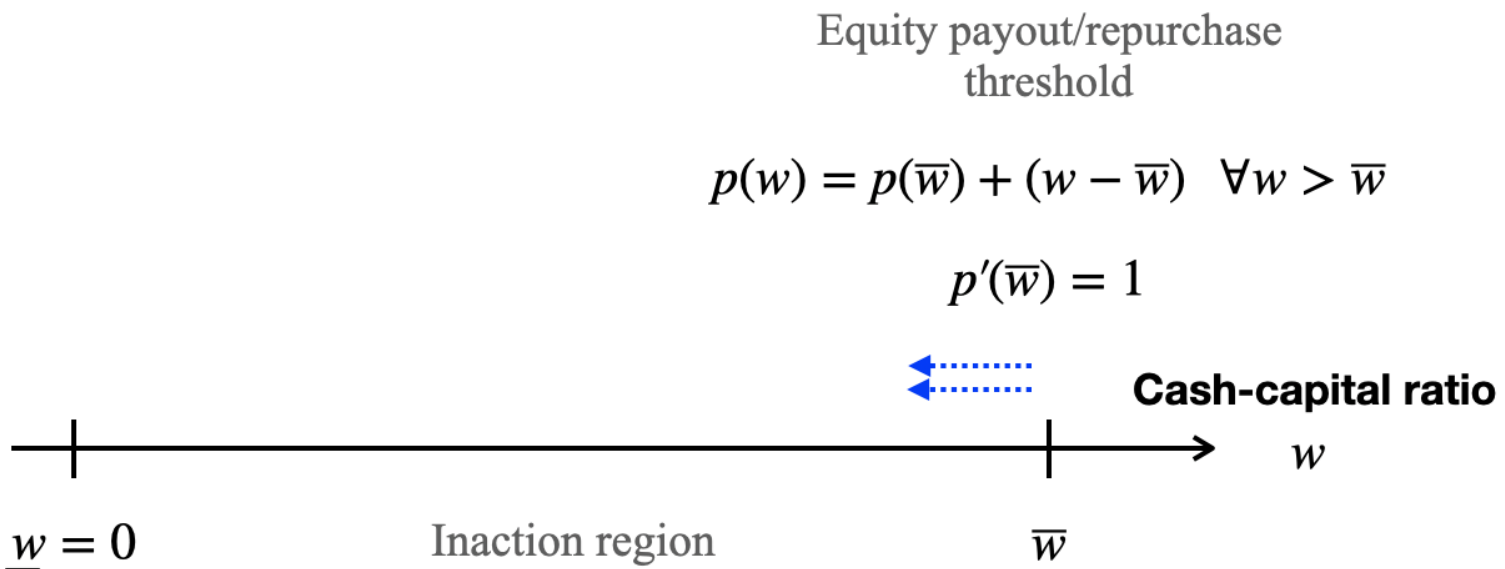
  - HJB equation :

$$rp(w) = (i(w)-\delta)\,(p(w) - wp'(w)) + ((r-\lambda)w + \mu - i^{\star}(w) - g(i^{\star}(w)))p'(w) + \frac{\sigma^2}{2}p''(w)$$

Value function - 1



Value function - 2



Value function - case I : Liquidation

1) Liquidation threshold

Equity payout/repurchase threshold

$p(0) = l$

If low μ high ϕ

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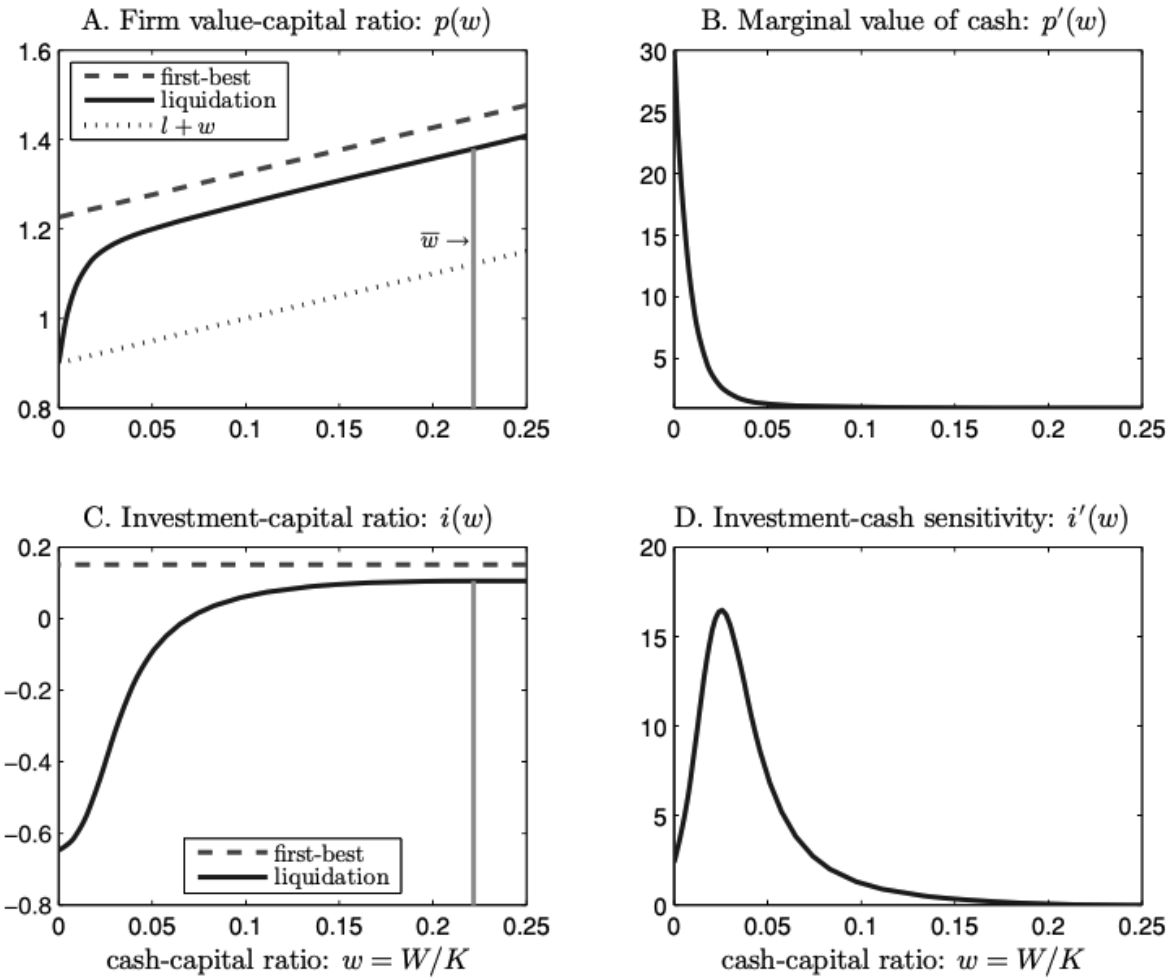

Figure 2. Case I—liquidation. This figure plots the solution for the case in which the firm has to liquidate when it runs out of cash ($w = 0$).

Value function - case II : Refinancing (equity issuance)

2) Equity issuance

Equity payout/repurchase threshold

$$p(0) = p(m) - \phi - (1+\gamma)m$$

If high μ low ϕ

Value function - case II : Refinancing (equity issuance)

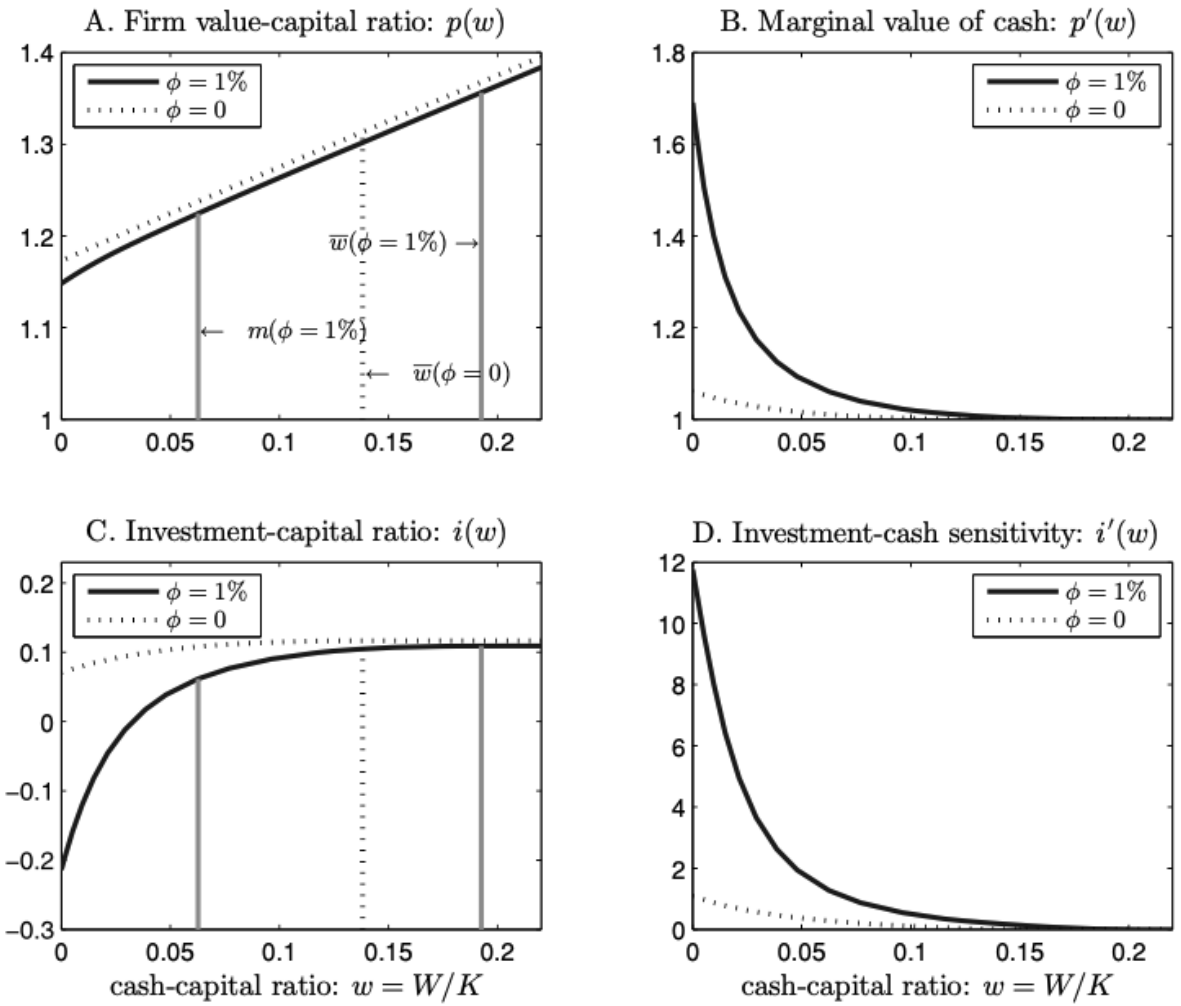

Figure 3. Case II—optimal refinancing. This figure plots the solution for the case of refinancing.

Hedging systemic risk

- Firm productivity dZ_t is correlated (corr. ρ) with agg. risk dB_t
- Can take (short-)position $\phi_t w_t$ on futures replicate the market index $dF_t = \sigma_m F_t dB_t$
- Needs cash in a "margin account" $\kappa_t w_t$, that incurs a cost εw_t. The future position (short-selling) is constrained

$$\psi_t w_t \geq -\pi \kappa_t w_t$$

- Optimal Hedging :
  - Without friction : $\pi \to \infty$

$$\psi^*(w) = \frac{-\rho\sigma}{w\sigma_m}$$

  - With margin requirement $\pi = 5$

$$\psi^*(w) = \begin{cases} -\pi \\ \frac{1}{w}\left(\frac{-\rho\sigma}{\sigma_m} - \frac{\epsilon}{\pi}\frac{p'(w)}{p''(w)}\frac{1}{\sigma_m^2}\right) \end{cases}$$

Hedging svstemic risk

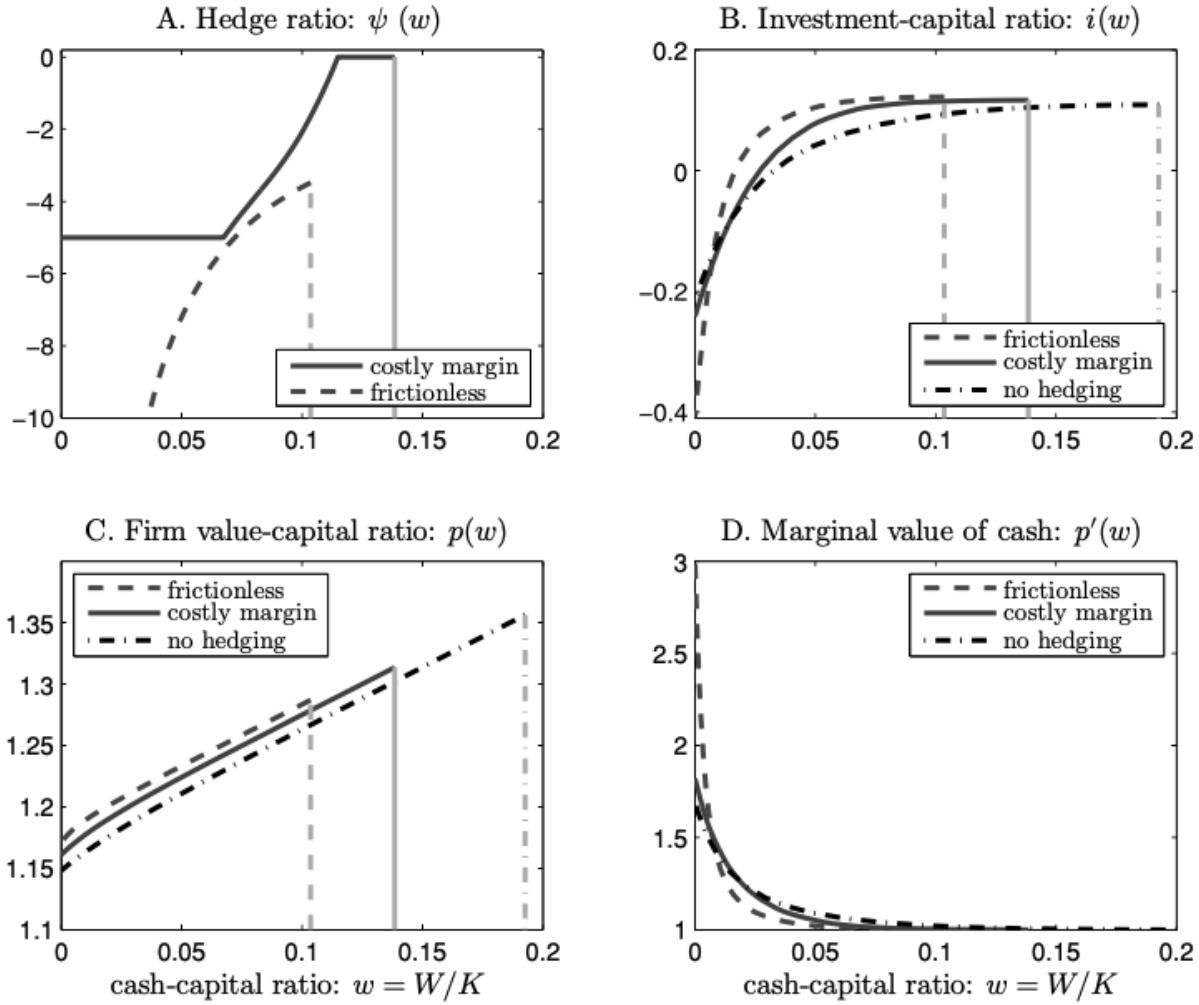

Figure 6. Optimal hedging. This figure plots the optimal hedging and investment policies, the firm value-capital ratio, and the marginal value of cash for Case II with hedging (with or without margin requirements). In Panel A, the hedge ratio for the frictionless case is cut off at -10 for

Credit line as additional funding

- Firm can borrow – i.e. negative cash $w < 0$ – additional funds
  - At a cost of a spread α
  - Up to a borrowing limit $-c$
  - The boundary for external funding/liquidation and payout shift

Credit line as additional funding

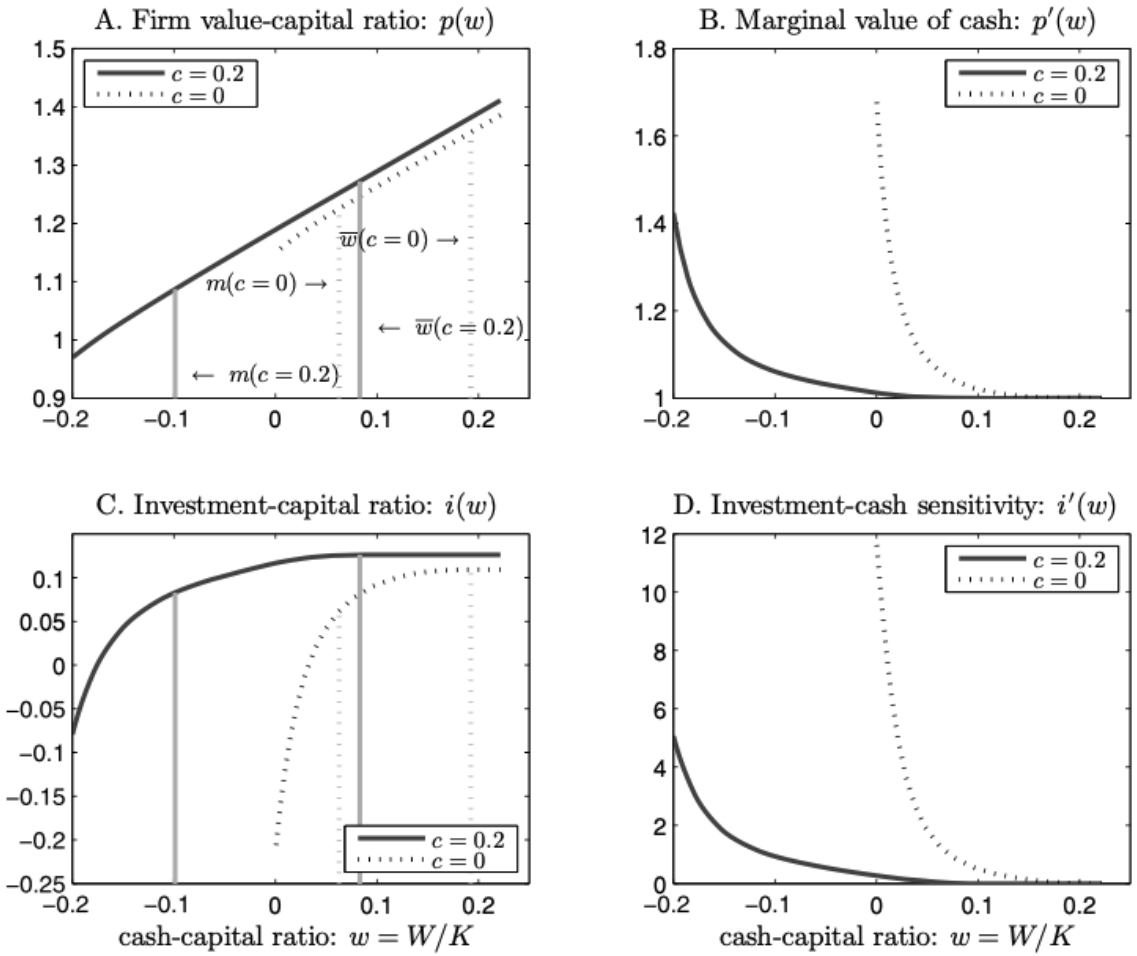

Figure 7. Credit line. This figure plots the model solution with credit lines and external equity. Each panel plots two scenarios: one without a credit line ($c = 0$) and the other with a credit line ($c = 20\%$). The spread on the credit line is $\alpha = 1.5\%$ over the risk-free rate r.

Conclusion

- An interesting article to include different new elements in a standard framework
- Interaction of Investment, cash hoarding and hedging in the life cycle of the firm
- However, in my sense, not a good model for :
  - investment (non-convex adj. cost ?)
  - equity/debt tradeoff (equity/leverage not a state)
  - firm cash distribution (due to positive drift, issuance happens with proba ≈ 0)