A THEORY OF INPUT-OUTPUT ARCHITECTURE

Ezra Oberfield, Econometrica (2018)

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Network reading group

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Introduction – Motivation

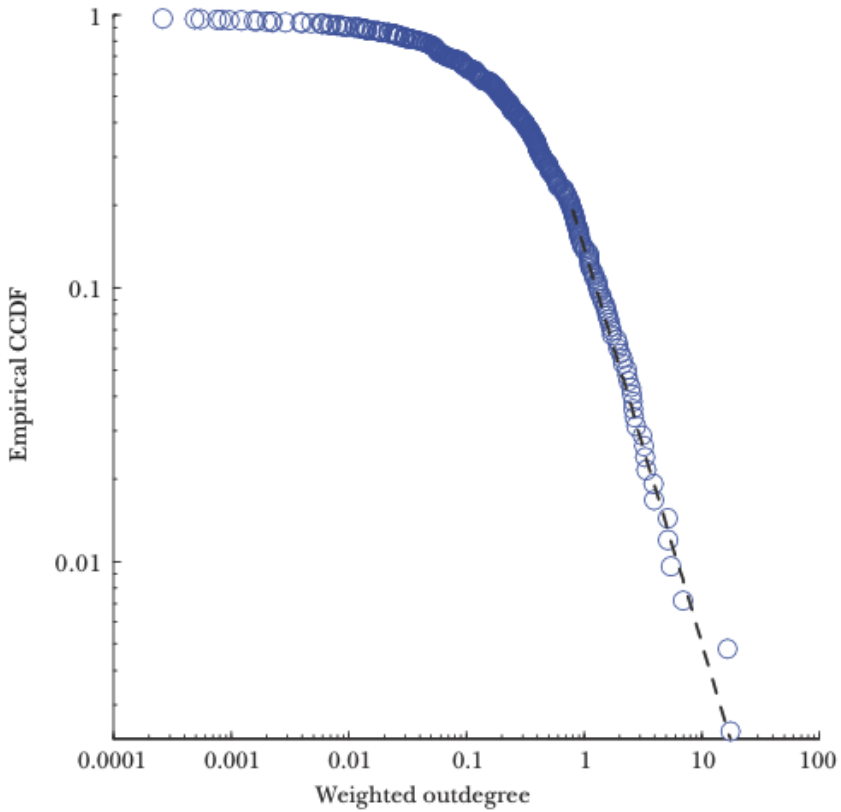
- What explains the heterogeneity in firms size and the existence of superstar firms ?
- Endogenous network formation :
  - Supply chain and choice of supplier / input technology
  - ... instead of stochastic productivity growth/demand residual

Motivation

- Size and degree distribution of U.S. firms :
  - Pareto/Zipf tail of the firm distribution

Figure 3

The Weighted Outdegree Distribution Associated with 2002 US Input-Output Data



Source: Bureau of Economic Analysis, detailed input-output table for 2002. For more details, see Data Appendix available with this paper at http://e-jep.org.

Novel properties of the model - 1 : Allocation

- Setting :
  - A production process in firm j is not firm specific but technique/match specific $\widehat{\phi}$ and $\widetilde{\phi}$
    - Depends on *all* suppliers $\widehat{\phi} \in U_j$, through all $z(\widehat{\phi})$ and $l(\widehat{\phi})$
    - Depends on demand from *all* costumer techniques $\widetilde{\phi}$, through all $x(\widetilde{\phi})$
- Outcome :
  - Endogenous choice of one unique supplier : $q_j = \max_{\widehat{\phi} \in U_j} z(\widehat{\phi}) q^{\alpha}_{s(\widehat{\phi})}$
  - Supply chain has a simple mathematical structure :
    - Feasible sequence $\omega = \{\phi\}_k$ such that $j = b(\phi_0)$ and $s(\phi_k) = b(\phi_{k+1})$
    - Firm production : $q_j = \sup_{\Omega_j} \prod_{k=0}^{\infty} \left[z_k(\omega)\right]^{\alpha^n}$ (prop 1 + c.f. vertical economy in Farhi-Baqaee (2020, QJE)
  - Payoff : surplus transfer from $b(\phi)$ to $s(\phi)$ is bounded from below by zero, and from above by total surplus $\mathcal{S}(\phi)$ of a technique (prop. 2)

Novel properties - 2 : Probabilistic structure and distribution

- Same logic as problstic. trade models : determine equilb. using LLN
  - Nb of techniques available to j : $|U_j| \sim \mathscr{P}(M)$
  - Probability of different pro
  - Efficiency of one technique : $q_j \sim G(q) = \int_{z_0}^{\infty} F\left(\left(\frac{q}{z}\right)^{1/\alpha}\right)$ d
  - Fixed point of the distribution (Kolmogorov Forward eq.)

$$F(q) = e^{-M \int_{z_0}^{\infty} \left[1 - F\left((q/z)^{1/\alpha}\right)\right] dH(z)}$$

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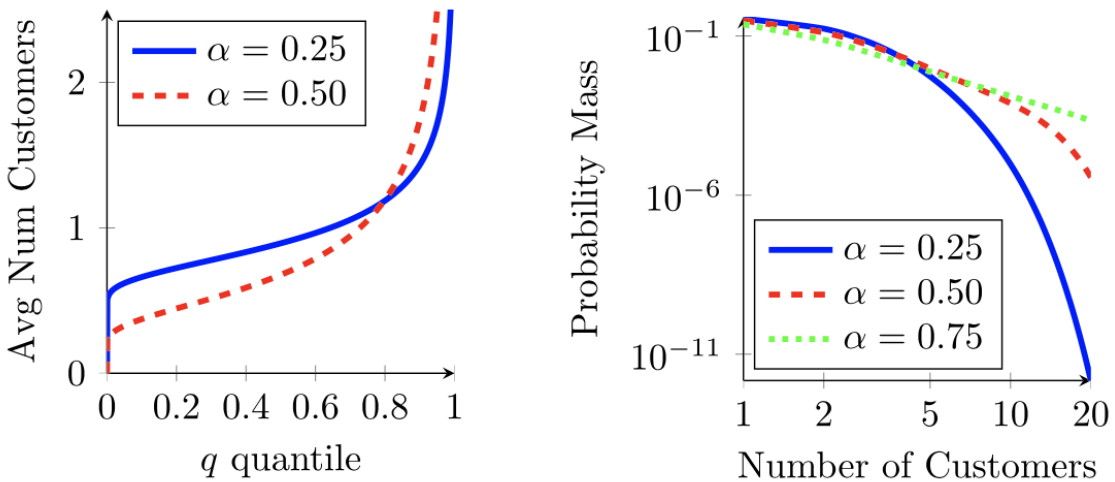
- Functional forms
  - Assumption $z \sim H \sim Pareto(z_0, \zeta)$ and $M = mz_0^{-\zeta}$ s.t. $M(1 - H(z)) = mz^{-\zeta}$
  - Outcome : $q \sim Fréchet(\zeta, \theta^{-\zeta})$ where θ Gamma fct involving α
  - Prop 4. Outdegree $\sim \mathscr{P}(\frac{m}{\theta} q^{\alpha\zeta})$ and asymptotically $Pareto(1/\alpha)$
  - Prop 5. Consequence : asymptotically $Pareto(\min\{1/\alpha, \zeta/(1-\varepsilon)\})$

Endogeneous distribution of sales/degree

- Cross sectional distribution
  - Depends on α

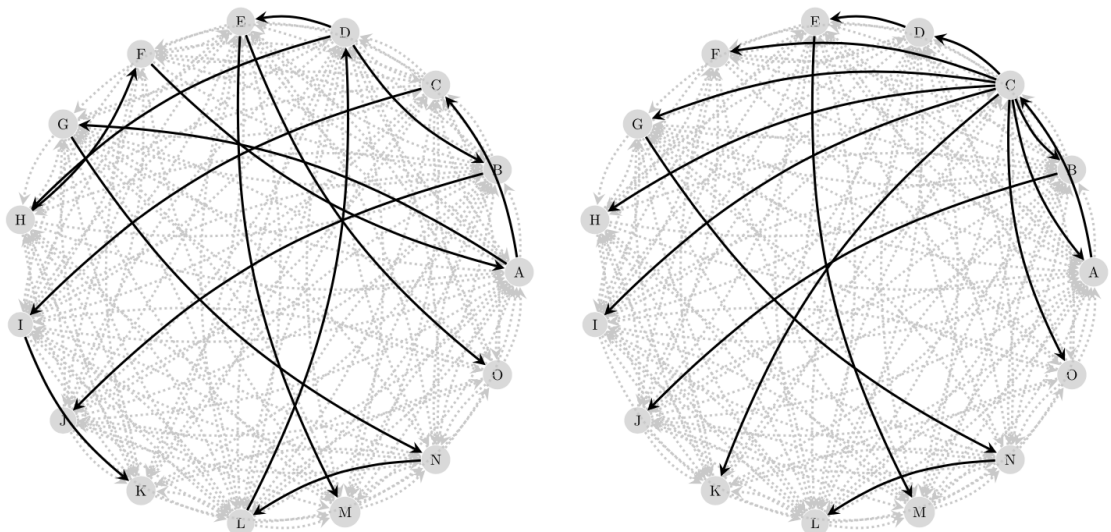


(a) Conditional Means (b) Unconditional Distribution

FIGURE 2.—Distribution of customers. Panel (a) shows the mean number of actual customers for each quantile in the efficiency distribution. Panel (b) gives the mass of entrepreneurs with n customers on a log-log plot. Under Assumption 2, the curves in each plot depend only on α.

Endogeneous distribution of sales/degree

- Cross sectional distribution
  - Rise of superstar firms :



(a) $\alpha = 0.1$ (b) $\alpha = 0.9$

FIGURE 3.—Equilibrium supply chains and α. This figure shows entrepreneurs' choices of techniques. The set of techniques, Φ, is held fixed; the only difference is the value of α. The dark edges represent techniques that are used. $M = 15$ and $H(z) = 1 - z^{-2}$ for $z \geq 1$.

Comparison distribution of sales/degree

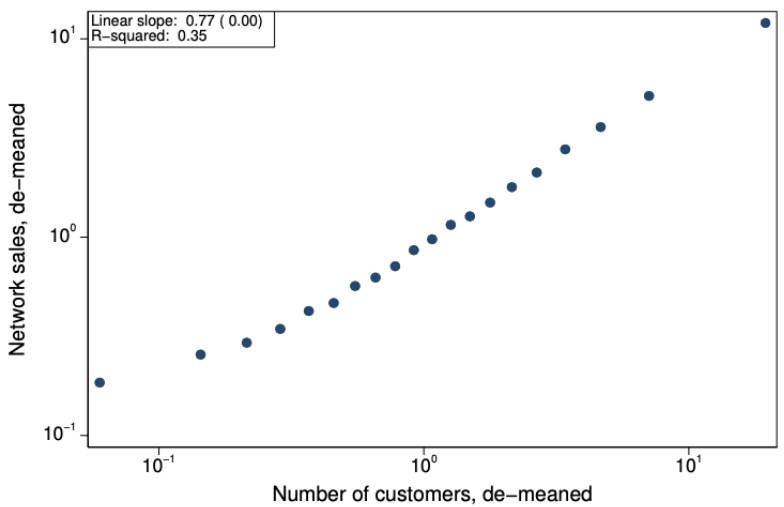
- Bernard, Dhyne, Magerman, Manova, Moxne (wp, 2019) : "Origins of firm heterogeneity : A production network approach"
  - Empirical facts (Belgian firm network data :
    1. High dispersion and skewness in distributions of firms' total sales, buyer-supplier connections, and buyer-supplier bilateral sales
    2. Firms with more customers : higher sales but lower sales per customer
    3. Negative degree assortativity among sellers and buyers
  - Propose theoretical model + SMM estimation with heterogeneous firms, endogenous network :
    - Two dimensions, sales ability (quality/productivity) and matching ability (relationship capability)
    - These two should be negatively correlated !
    - Similar to Bernard and Ulltveit-Moe (2018) : Two-sided Heterogeneity and Trade

Comparison distribution of sales/degree

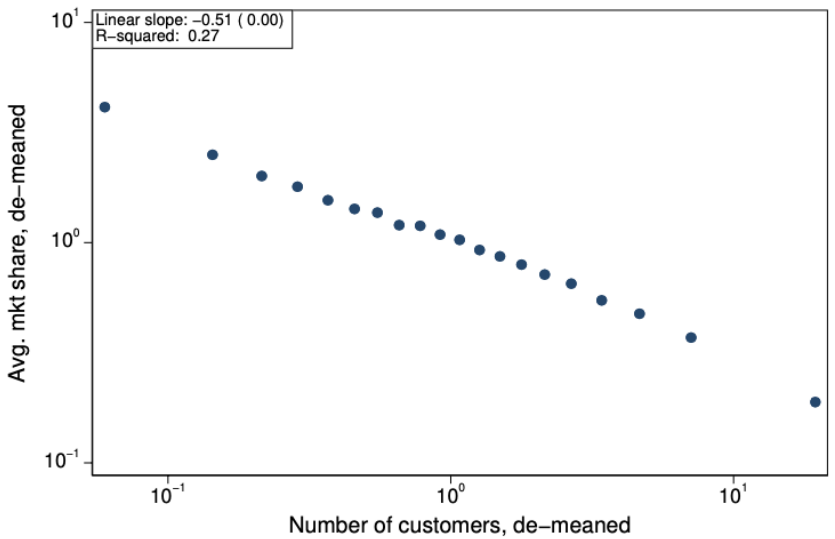
- Bernard, Dhyne, Magerman, Manova, Moxne (2019) :

Figure 2: Total Network Sales, Average Sales and Number of Customers

(a) Total Sales



Total sales vs. nb of costumers



(b) Market Share

Avg. market share per costumer

Comparison distribution of sales/degree

- Barabasi, Albert (1999) : Preferential Attachment :

$$\frac{d}{dt}k_i(t) = \frac{k_i(t)}{2t} \qquad \text{sol :} \quad k_i(t) = m\left(\frac{t}{i}\right)^{\frac{1}{2}} \quad \Rightarrow \quad \ln(1-F(k)) = \alpha - \beta \ln k + \varepsilon$$

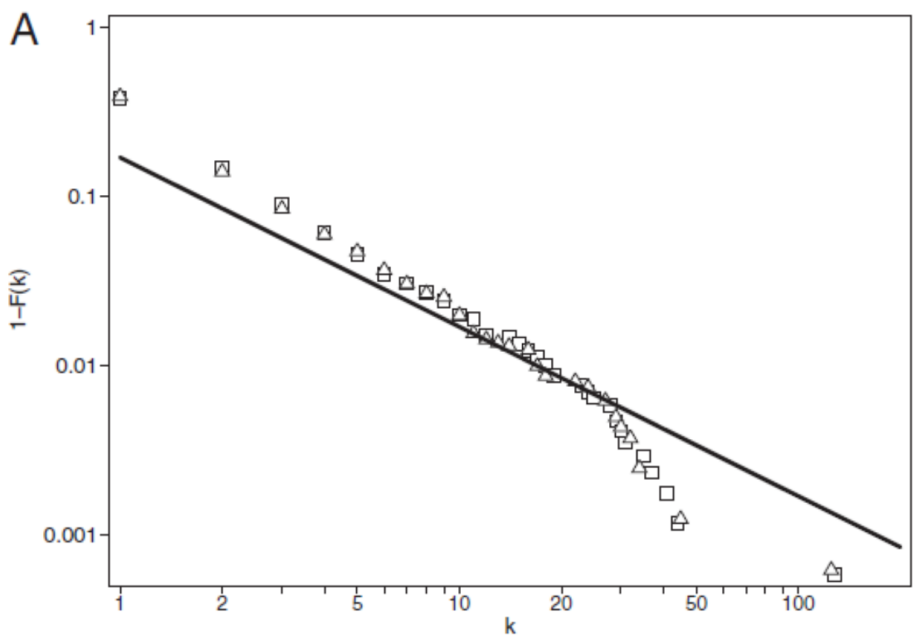
- Uniform probability :

$$\frac{d}{dt}k_i(t) = \frac{m}{t} \qquad \text{sol :} \quad k_i(t) = m + m \ln\left(\frac{t}{i}\right) \qquad \Rightarrow \quad F(d) = 1 - e^{-\frac{k-m}{m}}$$

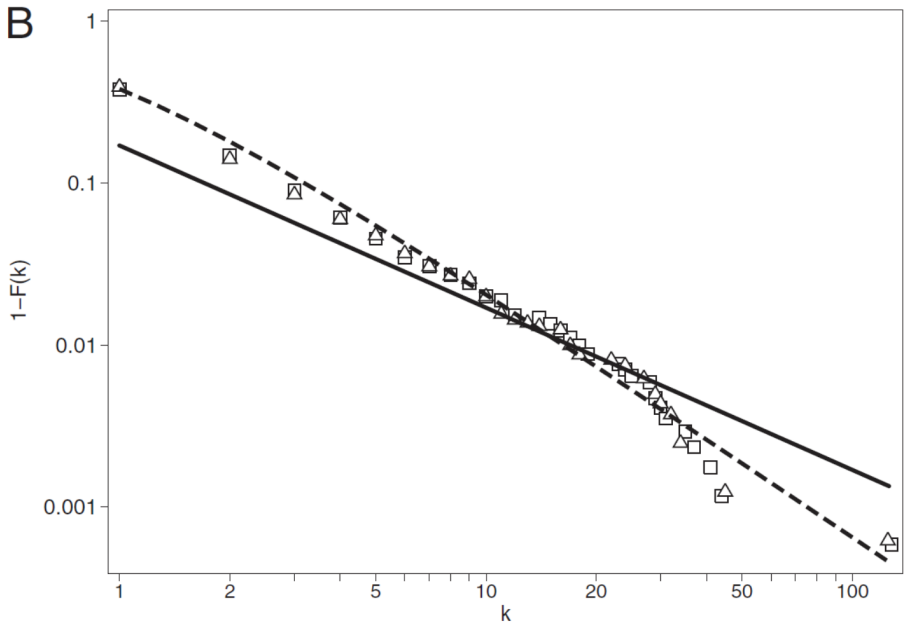
- Atalay, Hortaçsu, Roberts, Syverson (2011 PNAS), modified model
  1. Firms (nodes) die
  2. Surviving nodes that lose a link “rewire” themselves through a combination of UP and PA links
  3. New firms incorporate using a combination of UP and PA links

Endogeneous distribution of sales/degree

- Bernard, Dhyne, Magerman, Manova, Moxne (2019) :



Preferential Attachment – Barabasi, Albert



Modified model AHRS

Other things I found interesting and extensions/research ideas

- Cost share affected by final demand
  - Surplus split share to supplier β, with $\tau(\phi) = \beta S(\phi)$
  - Can a change in consumer preference or some sectoral productivity shift be a force behind large change in the production structure ... and potentially structural change, job destruction (and skill-mismatch)
  - Also question related to the fall in the labor share and the rise of superstar firms.
- Entry and exit determined in equilibrium.
  - Time varying default ? Recessions ?