

Supply and Demand in Disaggregated Keynesian Economies  
with an Application to the Covid-19 Crisis  
David Baqaee and Emmanuel Farhi

*Thomas Bourany*

*Macro Reading Group*

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## Plans

- ▶ Initial plan :
- ▶ Sectoral effects of social distancing  
Jean-Noël Barrot, Basile Grassi and Julien Sauvagnat
- ▶ Other more interesting paper with an empirical approach :
- ▶ Barrot, J.-N. and J. Sauvagnat (2016).  
Input Specificity and the Propagation of Idiosyncratic Shocks in  
Production Networks.  
The Quarterly Journal of Economics 131(3), 1543–1592.

## Introduction & Motivation

- ▶ Study the effects of supply and demand shocks in a model with multiple sectors/factors/input-output linkages
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    - Changing shape of the AS curve : due to complementarity & binding rigidities constraints
  - ▶ Plenty of results and extensions :
    - (i) Quantitative model : need both supply shocks and agg. demand,
    - (ii) Demand spillovers, (iii) Policy responses



## Model : two periods

► Consumers :

$$\max_{c_i} \quad (1 - \beta) \frac{Y^{1-1/\rho} - 1}{1 - 1/\rho} + \beta \frac{Y_*^{1-1/\rho} - 1}{1 - 1/\rho} \quad Y = \mathcal{D}(x_{01}, \dots, x_{0N}; \omega_{0j})$$

$$\sum_{i \in \mathcal{N}} p_i x_{0i} + \frac{\bar{p}^Y Y_*}{1 + i} = \sum_{f \in \mathcal{G}} w_f L_f + \sum_{i \in \mathcal{N}} \pi_i + \frac{\bar{I}_*}{1 + i'} \quad E = p^Y Y$$

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► Production :

$$\pi_i = \max_{y_i, x_{ij}, L_{if}} p_i y_i - \sum_{j \in \mathcal{N}} p_j x_{ij} - \sum_{f \in \mathcal{G}} w_f L_{if}$$

$$\frac{y_i}{\bar{y}_i} = \frac{A_i}{\bar{A}_i} \left( \sum_{j \in \mathcal{N} + \mathcal{G}} \bar{\omega}_{ij} \left( \frac{x_{ij}}{\bar{x}_{ij}} \right)^{\frac{\theta_i - 1}{\theta_i}} \right)^{\frac{\theta_i}{\theta_i - 1}}$$

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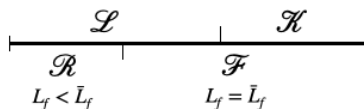
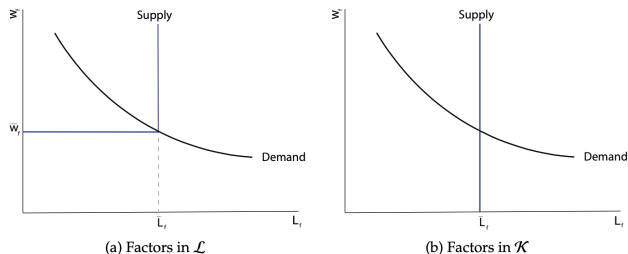
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- Market clearing :
- $$x_{0i} + \sum_{j \in \mathcal{N}} x_{ji} = y_i$$

## Model – Nominal rigidities for factors

- Slackness constraints :

$$(w_f - \bar{w}_f) (L_f - \bar{L}_f) = 0, \quad \bar{w}_f \leq w_f, \quad L_f \leq \bar{L}_f \quad L_f = \sum_{i \in \mathcal{N}} L_{if}$$



## Model – Shocks and Input Output definition

► Shocks :

- Supply : Productivity  $A_i$ , Factor supply shocks :  $\bar{L}_f$
- Demand : Composition :  $\omega_{0j}$  and aggregate shocks (i) Patience :  $\frac{\beta}{1-\beta}$ ,  
(ii) Monetary shocks :  $i$ ,  $\bar{p}_*^Y$  and  $\bar{Y}_*$

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### ► Network definitions :

- Input-Output matrix :  $\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i} = \frac{p_j x_{ij}}{\sum_{k \in \mathcal{N} + \mathcal{G}} p_k x_{ik}}$
- Leontieff Inverse Matrix :  $\Psi = (I - \Omega)^{-1} = I + \Omega + \Omega^2 + \dots$
- Domar weights : goods :  $\lambda_i = \frac{p_i y_i}{E} = \frac{p_i y_i}{\sum_{i \in \mathcal{N}} p_i x_{0i}}$        $\lambda_f = \frac{w_f L_f}{\sum_{f \in \mathcal{G}} w_f L_f}$

## Result – Local comparative statics :

- ▶ Euler equation for output :

$$d\log Y = -\rho d\log p^Y + d\log \zeta$$

$$d\log \zeta = -\rho \left( d\log(1+i) + \frac{\beta}{1-\beta} d\log \beta - d\log \bar{p}_*^Y \right) + d\log \bar{Y}_*$$

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- Stagflationary effect of negative supply shocks,

- ▶ Prop. 1 :

$$\begin{aligned} d\log Y &= \sum_{i \in \mathcal{N}} \lambda_i d\log A_i + \sum_{f \in \mathcal{G}} \lambda_f d\log L_f \\ &= \underbrace{\sum_{i \in \mathcal{N}} \lambda_i d\log A_i + \sum_{f \in \mathcal{G}} \lambda_f d\log \bar{L}_f}_{\text{neoclassical effect}} + \underbrace{\sum_{f \in \mathcal{L}} \lambda_f \min \{ d\log \lambda_f + d\log E - d\log \bar{L}_f, 0 \}}_{\text{Keynesian effect}} \end{aligned}$$



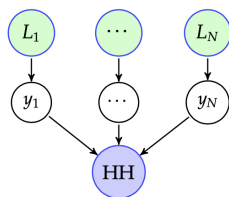
## Result – Local comparative statics : factor share and price

- Prop 2 : Forward propagation vs. Backward propagation :

$$d \log p_k = - \sum_{i \in \mathcal{N}} \Psi_{ki} d \log A_i + \sum_{f \in \mathcal{G}} \Psi_{kf} \underbrace{(d \log \lambda_f + d \log E - d \log L_f)}_{=d \log w_f}$$

$$d \log \lambda_k = \theta_0 \text{Cov}_{\Omega^{(0)}} \left( d \log \omega_0, \frac{\Psi^{(k)}}{\lambda_k} \right) + \sum_{j \in 1+\mathcal{N}} \lambda_j (\theta_j - 1) \text{Cov}_{\Omega^{(j)}} \left( -d \log p, \frac{\Psi^{(k)}}{\lambda_k} \right)$$

## A simpler example : Horizontal economy



$$Y/\bar{Y} = \left( \sum_i \bar{\lambda}_i (y_i/\bar{y}_i)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}$$

$$y_i/\bar{y}_i = L_i/\bar{L}_i,$$

$$L_i = \min\{\bar{L}_i, \lambda_i E/\bar{w}_i\}$$

$$w_i = \max\{\lambda_i E/\bar{L}_i, \bar{w}_i\}.$$

- Change in labor in flexible and rigid sectors :

$$d \log L_{\mathcal{F}} = d \log \bar{L}_{\mathcal{F}} := \sum_{f \in \mathcal{F}} \frac{\lambda_f}{\lambda_{\mathcal{F}}} d \log \bar{L}_f \quad d \log L_{\mathcal{R}} < d \log \bar{L}_{\mathcal{R}}$$

- By Proposition 2 , the change in the share of a factor  $f$  is given by

$$\lambda_{\mathcal{F}} d \log \lambda_{\mathcal{F}} = - \frac{(1-\theta)(1-\lambda_{\mathcal{F}})}{1-(1-\theta)(1-\lambda_{\mathcal{F}})} \lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}$$

## A simpler example : Horizontal economy

- ▶ Use the above relation : reallocation from rigid to flexible sectors :

$$\lambda_{\mathcal{R}} d \log L_{\mathcal{R}} = \sum_{f \in \mathcal{R}} \lambda_f d \log \lambda_f = - \sum_{f \in \mathcal{F}} \lambda_f d \log \lambda_f = - \lambda_{\mathcal{F}} d \log \lambda_{\mathcal{F}}$$

- ▶ Aggregate output change : Amplification through Keynesian channels :

$$\begin{aligned} d \log Y &= \lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d \log L_{\mathcal{R}} = \frac{\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}}{1 - (1 - \theta)(1 - \lambda_{\mathcal{F}})} \\ &= \underbrace{\lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}} + \lambda_{\mathcal{R}} d \log \bar{L}_{\mathcal{R}}}_{\text{neoclassical effect}} + \underbrace{\sum_{f \in \mathcal{R}} \lambda_f \left( \frac{(1 - \theta) \lambda_{\mathcal{F}} d \log \bar{L}_{\mathcal{F}}}{1 - (1 - \theta)(1 - \lambda_{\mathcal{F}})} - d \log \bar{L}_f \right)}_{\text{Keynesian effect}} \end{aligned}$$

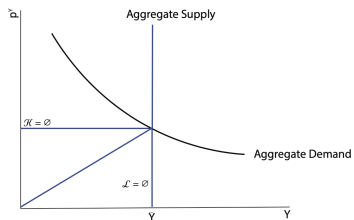
## A simpler example : Horizontal economy

- ▶ Prop 3 : (Global Sufficient Statistics).
  - With log util  $\rho = 1$ , uniform elasticities of substitution  $\theta_j = \theta, \forall j \in 1 + \mathcal{N}$ , and if shocks affect only factor supply shocks  $\Delta \log \bar{L}$  and agg. demand  $\Delta \log \zeta$  (not productivity  $A$  or demand composition  $\omega$ ), Then :

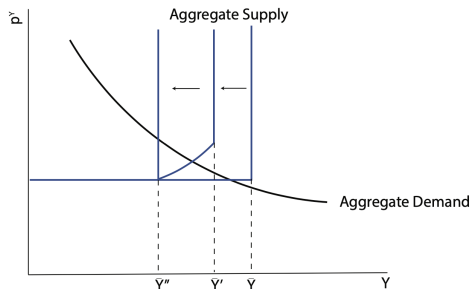
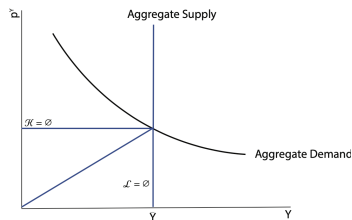
$$\Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}) = \Delta \log Y(\Delta \log \bar{L}, \Delta \log \zeta, \bar{\Omega}')$$

$\forall \bar{\Omega}$  and  $\bar{\Omega}'$  such that  $\bar{\lambda}_f = \bar{\Psi}_{0f} = \bar{\Psi}'_{0f} = \bar{\lambda}'_f$  for every  $f \in \mathcal{G}$

## AS-AD representation :

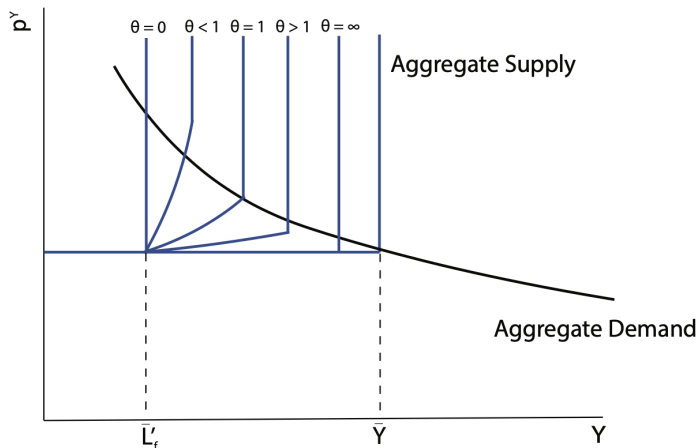


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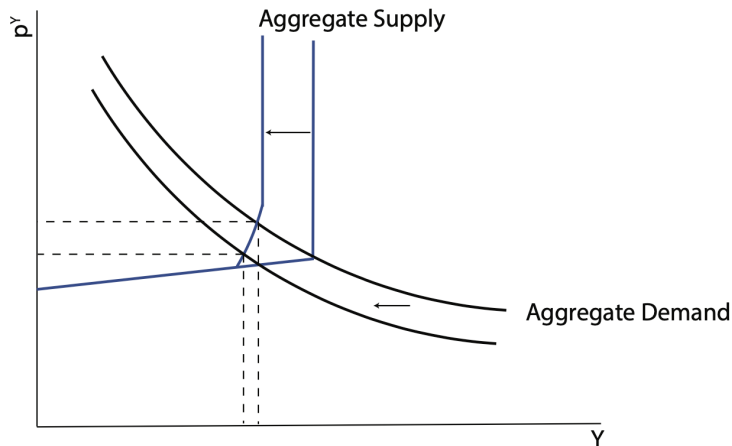


- ▶ Shock to one sector with Keynesian spillovers on aggregate supply

## AS-AD representation : shocks



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## Extensions :

- ▶ Quantitative model :
  - Between 4 and 12 % Keynesian unemployment
  - Around 10% drop in real GDP (50%-100% more than in neoclassical model alone)
  
- ▶ Extensions : endogenous demand spillovers and policy
  - Constrained costumers
  - Firm failures
  - Policy responses : (less effective with complementarities)
    - Monetary policy
    - Payroll tax cut
    - Targeted fiscal policy