The New Keynesian Model

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Advanced Macroeconomics – Part 2 – Lecture 2 and 3

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Roadmap

- Introduction: from the Old Keynesian (IS–LM, AS–AD) to the NK model
- Building the New Keynesian model
  - households, monopolistic competition, Calvo pricing
  - non-linear model first, then log-linearization
- Determinacy and Taylor principle
- Monopolistic competition and aggregate demand externality (Blanchard–Kiyotaki)
- Transmission of shocks in the 3-equation model
- Empirics of the building blocks: Euler equation, price rigidity, Phillips curve
- Evidence on monetary non-neutrality
- Policy: divine coincidence, optimal policy, monetary vs. fiscal policy, forward guidance

Introduction

Why a New Keynesian model?

- In a nutshell: NK model = RBC model + monopolistic competition + nominal rigidities
- RBC / neoclassical model: classical dichotomy
  - real allocation determined independently of nominal variables ⇒ money is neutral
  - fluctuations are efficient ⇒ no role for stabilization policy
- Yet, in the data:
  - prices and wages are sticky: median duration of prices 8–11 months (Nakamura & Steinsson, 2008), wages ≈ 1 year (Taylor, 1999)
  - monetary policy has real effects (Christiano, Eichenbaum & Evans, 1999; Romer & Romer, 2004; Nakamura & Steinsson, 2018)
- Sticky prices break monetary neutrality *and* the first welfare theorem
  ⇒ rationale for stabilization policy
- ***"New"*** = methodological: microfounded, rational expectations
  "Keynesian" = substantive: aggregate demand matters in the short run
- Workhorse at central banks: Fed (FRB/US, EDO), ECB (NAWM), Smets-Wouters (2007)

The Old Keynesian model: IS–LM

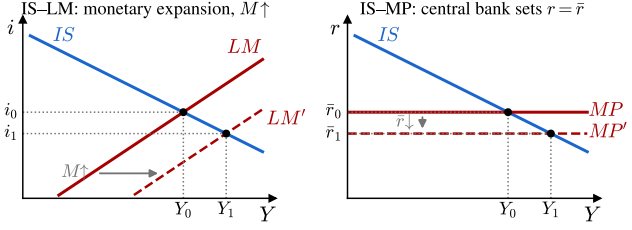
- Hicks (1937) formalization of Keynes (1936). Prices are fixed in the short run
- ***IS*** (goods market):

$$Y = C(Y - T) + I(r) + G, \qquad 0 < C' < 1, \quad I' < 0, \qquad r = i - \pi^e$$

- ***LM*** (money market):

$$\frac{M}{P} = L(i, Y), \qquad L_i < 0, \quad L_Y > 0$$

- Output is demand-determined: Keynesian cross, multiplier $\frac{1}{1-C'} > 1$
- Modern variant (Romer, 2000): replace LM by central bank interest rate rule (MP curve)

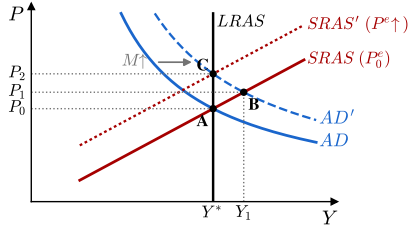


The Old Keynesian model: AS–AD and the Phillips curve

- ***AD***: combine IS and LM, eliminate $i \Rightarrow\ Y = Y^D(M/P, G, T)$, decreasing in P
- ***AS***: expectations-augmented Phillips curve (Friedman, 1968; Phelps, 1968)

$$\pi_t = \pi_t^e + \kappa\left(Y_t - Y_t^*\right) + u_t$$

  with adaptive expectations $\pi_t^e = \pi_{t-1}$: accelerationist Phillips curve
- Short run: upward-sloping AS; long run: vertical at potential Y^*
- Problems:
  - behavioral equations not derived from optimization $\Rightarrow$ not policy-invariant (Lucas, 1976)
  - backward-looking expectations $\Rightarrow$ systematically exploitable trade-off; 1970s' stagflation
  - no welfare criterion



Target of today: the three-equation NK model

$$
\begin{aligned}
&\text{(DIS)} && x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) \\
&\text{(NKPC)} && \pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, x_t \\
&\text{(MP)} && i_t = \rho + \phi_\pi\, \pi_t + \phi_y\, x_t + \nu_t
\end{aligned}
$$

- $x_t = y_t - y_t^n$: output gap w.r.t. the *natural* (flexible-price) level of output
- Modern version of IS–LM, AS–AD:

Old Keynesian	**New Keynesian**	**Microfoundation**
IS curve	Dynamic IS (DIS)	household Euler equation
LM curve	interest rate rule (MP)	central bank (Taylor) rule
AS / Phillips curve	NK Phillips curve (NKPC)	Calvo price setting

- DIS & NKPC are forward-looking: what matters is the *entire path* of expected policy

Building the New Keynesian Model

Ingredients

1. ***Representative household*** (key! – relaxed in HANK lectures)
   - consumes a CES basket of varieties, supplies labor, trades a nominal bond
2. ***Production*** with nominal price rigidities
   - continuum of monopolistically competitive firms (Dixit–Stiglitz)
   - Calvo (1983) pricing: random fraction $1-\theta$ of firms can reset their price
   - alternative: Rotemberg (1982) quadratic adjustment costs: same Phillips curve to first order
3. ***Monetary policy***: central bank sets the nominal rate i_t through a rule (Taylor rule)

- Simplifications: no capital, no explicit money: cashless limit (Woodford, 2003)
  $\Rightarrow$ a "neo-Wicksellian" model: the price *level* is a nominal anchor problem
- Strategy: write non-linear model, then log-linearize around zero-inflation steady state

Households: demand for varieties

- Continuum of varieties $i \in [0,1]$, Dixit–Stiglitz aggregator with elasticity $\varepsilon > 1$:

$$C_t = \Big(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \Big)^{\frac{\varepsilon}{\varepsilon-1}} \qquad s.t. \int_0^1 P_t(i) C_t(i) \, di = I_t$$

- Lagrangian $\mathcal{L}\big(\{C_t(i)\}, \lambda\big) = \Big(\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \Big)^{\frac{\varepsilon}{\varepsilon-1}} - \lambda \Big(\int_0^1 P_t(i) C_t(i) \, di - I_t \Big)$
- Solution: isoelastic demand curves and price index

$$C_t(i) = \Big(\frac{P_t(i)}{P_t} \Big)^{-\varepsilon} C_t, \qquad P_t = \Big(\int_0^1 P_t(i)^{1-\varepsilon} \, di \Big)^{\frac{1}{1-\varepsilon}}, \qquad \int_0^1 P_t(i) C_t(i) \, di = P_t C_t = I_t$$

- ε = elasticity of substitution btw varieties = price elasticity of demand faced by each firm

Households: intertemporal problem

- Representative household:

$$\max_{\{C_t,N_t,B_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \Big]$$

$$\text{s.t.} \qquad P_t C_t + B_t \leq (1+i_{t-1}) B_{t-1} + W_t N_t + D_t - T_t$$

  B_t: one-period nominal bond, D_t: dividends from firms, T_t: lump-sum taxes
  σ: inverse EIS; φ: inverse Frisch elasticity

- Two first-order conditions (+ no-Ponzi / transversality):

$$\text{Euler:} \qquad C_t^{-\sigma} = \beta \, \mathbb{E}_t \Big[(1+i_t) \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \Big]$$

$$\text{Labor supply:} \qquad \frac{W_t}{P_t} = N_t^{\varphi} \, C_t^{\sigma}$$

- Inflation $\Pi_t \equiv P_t / P_{t-1}$; nominal SDF $\Lambda_{t,t+k} = \beta^k \big(\frac{C_{t+k}}{C_t} \big)^{-\sigma} \frac{P_t}{P_{t+k}}$

Firms: technology and the flexible-price benchmark

- Firm i: $Y_t(i) = Z_t N_t(i)$, faces demand $Y_t(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$
- Nominal marginal cost W_t/Z_t, common to all firms; real marginal cost $mc_t \equiv \frac{W_t}{P_t Z_t}$
- ***Flexible prices***: $\max_{P(i)} \left(P(i) - W_t/Z_t\right)\left(P(i)/P_t\right)^{-\varepsilon} Y_t$

$$\Rightarrow \quad P_t(i) = \mu \frac{W_t}{Z_t}, \qquad \mu \equiv \frac{\varepsilon}{\varepsilon - 1} > 1 \qquad \Rightarrow \qquad mc_t = \frac{1}{\mu}$$

- Equilibrium real block: labor supply $W_t/P_t = N_t^{\varphi} C_t^{\sigma}$; $mc_t = 1/\mu$; $Y_t = Z_t N_t$; $C_t = Y_t$

$$\Rightarrow \qquad \left(Y_t^n\right)^{\sigma+\varphi} = \frac{Z_t^{1+\varphi}}{\mu} \qquad \text{(natural output)}$$

  and Euler equation gives natural real rate r_t^n

- Real allocation independent of i_t, M_t, P_t: classical dichotomy
  - monetary policy only determines nominal variables (and with a peg, not even P_t)
  - still inefficient: markup $\mu > 1 \Rightarrow Y_t^n$ below the efficient level

Why sticky prices matter: a two-period illustration

- Two periods, $Y_1 = A_1 K_1$, $Y_2 = A_2 K_2$, $K_2 = I_1$, money quantity $M_t = P_t C_t$, policy (i_1, M_2)
  Here $\eta = 1/\sigma$ = EIS: Euler equation: $C_1 = C_2[\beta(1+r_1)]^{-\eta}$
- ***Flexible*** P_1: $Y_1 = A_1 K_1$, $1 + r_1 = A_2$: monetary policy only moves P_1, P_2
- ***Sticky*** P_1 (set in advance): output determined by the short side, $Y_1 = \min\{Y_1^D, Y_1^S\}$

$$Y_1 = \left[1 + \left(\frac{1}{\beta A_2}\right)^{\eta} A_2\right] \frac{M_2}{(1+i_1)P_1}$$

  - $i_1 \downarrow \Rightarrow C_1, I_1, Y_1 \uparrow$: monetary non-neutrality
  - demand shocks (news $A_2 \downarrow$, patience $\beta \uparrow$) $\Rightarrow$ recession (with $\eta < 1$)
- Key mechanism: household income depends on aggregate demand, $\Omega = P_1 Y_1 = P_1 Y_1^D$
  "one person's spending is someone else's income" $\Rightarrow$ Keynesian-cross feedback
- Paradox of thrift: $\beta \uparrow \Rightarrow C_1 \downarrow, Y_1 \downarrow$ but I_1 unchanged: saving does not rise

Calvo pricing: the reset price problem

- Each period, a firm can reset its price with probability $1-\theta$, independently of history
  $\Rightarrow$ expected price duration $1/(1-\theta)$
- A resetting firm chooses P_t^* to maximize the value of profits while the price is in place:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \, \mathbb{E}_t \left[\Lambda_{t,t+k} \left(P_t^* \, Y_{t+k|t} - \frac{W_{t+k}}{Z_{t+k}} Y_{t+k|t} \right) \right]$$

$$\text{s.t.} \qquad Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k}$$

- First-order condition:

$$\sum_{k=0}^{\infty} \theta^k \, \mathbb{E}_t \left[\Lambda_{t,t+k} \, Y_{t+k|t} \left(P_t^* - \mu \, \frac{W_{t+k}}{Z_{t+k}} \right) \right] = 0 \qquad\qquad P_t^* = \mu \frac{\sum_k \mathbb{E}_t \left[\lambda_{t,t+k} mc_{t+k} \right]}{\sum_k \mathbb{E}_t \left[\lambda_{t,t+k} \right]}$$

with $\lambda_{t,t+k} = \theta^k \underbrace{\beta^k (P_t/P_{t+k})(C_{t+k}/C_t)^{-\sigma}}_{\Lambda_{t,t+k}} P_{t+k}^{\varepsilon} Y_{t+k}$

- Reset price = markup μ over a weighted average of expected future marginal costs
- $\theta = 0$: back to static markup over marginal cost: $P_t^* = \mu \, mc_t = \mu W_t / Z_t$

The non-linear Phillips curve

- Rearranging the FOC, with $\Pi_{t,t+k} \equiv P_{t+k}/P_t$:

$$p_t^* \equiv \frac{P_t^*}{P_t} = \frac{\mu\, \mathbb{E}_t \sum_{k\geq 0} (\beta\theta)^k C_{t+k}^{-\sigma} Y_{t+k}\, \Pi_{t,t+k}^{\varepsilon}\, mc_{t+k}}{\mathbb{E}_t \sum_{k\geq 0} (\beta\theta)^k C_{t+k}^{-\sigma} Y_{t+k}\, \Pi_{t,t+k}^{\varepsilon-1}} = \frac{F_t}{H_t}$$

- where F_t and H_t can be defined recursively (forward looking):

$$F_t = C_t^{-\sigma} Y_t\, \mu\, mc_t + \beta\theta\, \mathbb{E}_t\left[\Pi_{t+1}^{\varepsilon} F_{t+1}\right]$$
$$H_t = C_t^{-\sigma} Y_t + \beta\theta\, \mathbb{E}_t\left[\Pi_{t+1}^{\varepsilon-1} H_{t+1}\right]$$

  as reset price today depends on future path of inflation and marg. cost
- Price index: resetters are drawn at random and all choose P_t^*

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon} \qquad \Rightarrow \qquad \frac{1-\theta\,\Pi_t^{\varepsilon-1}}{1-\theta} = \left(\frac{P_t^*}{P_t}\right)^{1-\varepsilon} = \left(\frac{F_t}{H_t}\right)^{1-\varepsilon}$$

- Highest inflation possible ($P_t^* \to \infty$): $\Pi_t < \theta^{-1/(\varepsilon-1)}$
  standard calibration $\theta = 0.85$, $\varepsilon = 6$: $\Pi < 1.03$ per quarter!

Price dispersion and aggregate production

- Aggregate labor demand:

$$N_t = \int_0^1 N_t(i)\,di = \int_0^1 \frac{Y_t(i)}{Z_t}\,di = \frac{Y_t}{Z_t}\underbrace{\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di}_{\equiv \Delta_t} \qquad \Rightarrow \qquad Y_t = \frac{Z_t N_t}{\Delta_t}$$

- $\Delta_t \geq 1$ (Jensen): price dispersion $\Rightarrow$ misallocation of labor across varieties $\Rightarrow$ TFP loss

$\Rightarrow$ Price stickiness provide a microfundation for an efficiency wedge

- Law of motion:

$$\Delta_t = \theta \underbrace{\Pi_t^{\varepsilon}\,\Delta_{t-1}}_{\substack{\text{price dispersion} \\ \text{non-reset firms}}} + (1-\theta)\underbrace{\left(\frac{1-\theta\,\Pi_t^{\varepsilon-1}}{1-\theta}\right)^{\frac{\varepsilon}{\varepsilon-1}}}_{=(P_t^*/P_t)^{\varepsilon}\text{ reset}}$$

- Around $\Pi = 1$: $\Delta_t - 1 = \mathcal{O}(\|\pi\|^2)$
  - second order: disappears in the log-linear model
  - but key for welfare: this is *the* cost of inflation in NK models

Taking stock: the non-linear NK model

1. Household optimality:

$$C_t^{-\sigma} = \beta\, \mathbb{E}_t\Big[\frac{1+i_t}{\Pi_{t+1}} C_{t+1}^{-\sigma}\Big], \qquad w_t \equiv \frac{W_t}{P_t} = N_t^{\varphi} C_t^{\sigma},$$

2. Non-linear Phillips curve, with (F_t, H_t) as above (fct of $\{\Pi_{t+k}, \}_k, mc_{t+k}$) w/ $mc_t = w_t/Z_t$

$$\Big(\frac{1-\theta\Pi_t^{\varepsilon-1}}{1-\theta}\Big)^{\frac{1}{1-\varepsilon}} = \frac{F_t}{H_t}$$

3. Output and price dispersion: $C_t = Y_t = Z_t N_t/\Delta_t$, Δ_t law of motion above
4. Monetary policy rule $i_t = f(\Pi_t, Y_t, v_t)$, e.g. $1 + i_t = \frac{1}{\beta}\,\Pi_t^{\phi_\pi}\big(Y_t/Y_t^n\big)^{\phi_y} e^{v_t}$

- ***Steady state*** with zero inflation $\Pi = 1$:
  $p^* = 1, \qquad \Delta = 1, \qquad w/z = mc = P/\mu = 1/\mu, \qquad 1+i = 1/\beta = 1+\rho, \qquad Y = Y^n$
- Can be solved non-linearly (Dynare, perfect foresight / sequence-space methods)
  $\Rightarrow$ useful for large shocks, high inflation, ZLB

Log-linearization I: households and marginal cost

- Lowercase = log-deviations from steady state; i_t, π_t = net rates; $\rho \equiv -\log \beta$
- Euler equation:
$$c_t = \mathbb{E}_t\, c_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - \rho\right)$$
- Labor supply: $w_t = \sigma c_t + \varphi n_t$
- Production and goods market: $y_t = c_t,\ y_t = z_t + n_t$ (Δ_t drops out at first order)
- Real marginal cost:
$$\widehat{mc}_t = w_t - z_t = \sigma y_t + \varphi(y_t - z_t) - z_t = (\sigma + \varphi)\, y_t - (1 + \varphi)\, z_t$$
- Firms' marginal cost rises with output: higher labor $\Rightarrow$ higher wage (labor supply) and lower marginal utility of consumption (wealth effect)

Log-linearization II: the New Keynesian Phillips curve

- Log-linearize the reset-price FOC around the zero-inflation steady state:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k\,\mathbb{E}_t\left[\widehat{mc}_{t+k} + p_{t+k}\right]$$

  i.e. recursively: $p_t^* - p_{t-1} = \beta\theta\,\mathbb{E}_t\left[p_{t+1}^* - p_t\right] + (1-\beta\theta)\widehat{mc}_t + \pi_t$

- Price index: $\pi_t = (1-\theta)(p_t^* - p_{t-1})$
- Combining:

$$\boxed{\pi_t = \beta\,\mathbb{E}_t\pi_{t+1} + \lambda\,\widehat{mc}_t}, \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

- Solving forward: inflation = present value of future real marginal costs

$$\pi_t = \lambda\sum_{k=0}^{\infty}\beta^k\,\mathbb{E}_t\,\widehat{mc}_{t+k}$$

- Purely forward-looking: firms raise prices today if they expect high future costs

Log-linearization III: output gap and natural rate

- Natural output (flexible prices $\Leftrightarrow \widehat{mc}_t = 0$):

$$y_t^n = \frac{1+\varphi}{\sigma+\varphi} z_t \qquad \Rightarrow \qquad \widehat{mc}_t = (\sigma+\varphi)\left(y_t - y_t^n\right) =: (\sigma+\varphi)\, x_t$$

- NKPC in terms of the output gap:

$$\pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, x_t, \qquad \kappa \equiv \lambda(\sigma+\varphi)$$

- Natural real rate (Euler equation in the flex-price economy):

$$r_t^n = \rho + \sigma\, \mathbb{E}_t \Delta y_{t+1}^n = \rho + \sigma \frac{1+\varphi}{\sigma+\varphi}\, \mathbb{E}_t \Delta z_{t+1}$$

(+ demand / preference shocks shift r_t^n)

- Subtracting the two Euler equations: the Dynamic IS curve

$$x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right)$$

The three-equation NK model

$$
\begin{aligned}
x_t &= \mathbb{E}_t x_{t+1} - \tfrac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\right) && \text{(DIS)} \\
\pi_t &= \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, x_t \left[+ \; u_t ? \right] && \text{(NKPC)} \\
i_t &= \rho + \phi_\pi\, \pi_t + \phi_y\, x_t + v_t && \text{(MP)}
\end{aligned}
$$

- Two jump variables (x_t, π_t), no endogenous state (no backward-looking variable!)
- Shocks: r_t^n (technology, demand), v_t (monetary), u_t (cost-push / markup)
- Solving the DIS forward:

$$
x_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} \mathbb{E}_t\left(r_{t+k} - r_{t+k}^n\right) + \lim_{T \to \infty} \mathbb{E}_t x_{t+T}, \qquad r_t \equiv i_t - \mathbb{E}_t \pi_{t+1}
$$

$\Rightarrow$ the output gap depends on the whole path of future real-rate gaps

Old vs. New Keynesian: what changes?

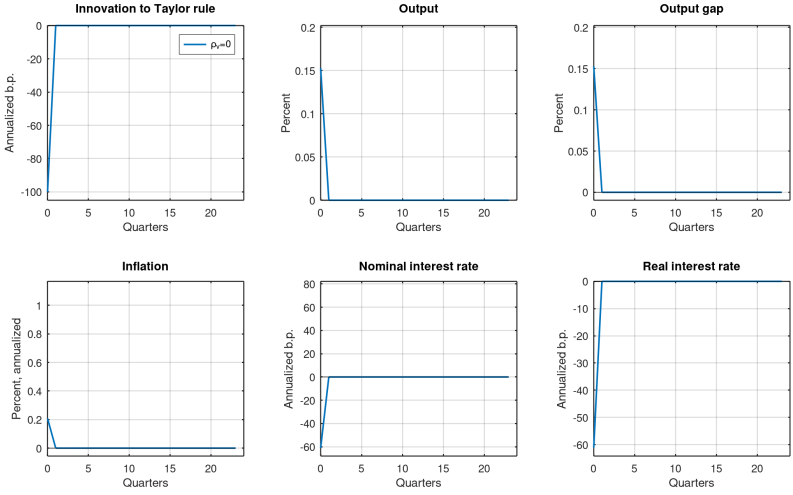
- Output gap measured w.r.t. the natural level of output, which fluctuates
  $\Rightarrow$ not all fluctuations are bad (e.g. efficient response to TFP z_t)
- Aggregate demand depends on expectations of future interest rates and inflation
  $\Rightarrow$ central banking = managing expectations: communication, inflation targets, credibility
- Microfounded: deep parameters $(\beta, \sigma, \varphi, \varepsilon, \theta)$, welfare analysis
  (Lucas critique: is the Calvo θ structural? $\rightarrow$ menu costs model)
- But: the NKPC is purely forward-looking
  - no intrinsic inflation persistence: credible disinflations are costless, Ball, (1994)
  - fix: indexation of non-resetting prices to past inflation, Christiano, Eichenbaum, Evans (2005)

$$\pi_t = \frac{\beta}{1+\beta}\mathbb{E}_t\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} + \frac{\kappa}{1+\beta}x_t$$

  - Galí & Gertler (1999): forward-looking behavior dominant, backward-looking component significant but small

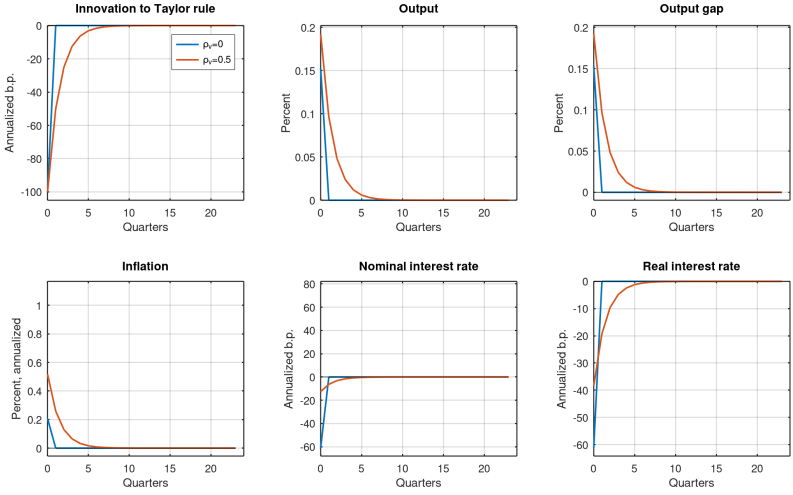
Transmission of Shocks

Monetary policy shock: impulse responses



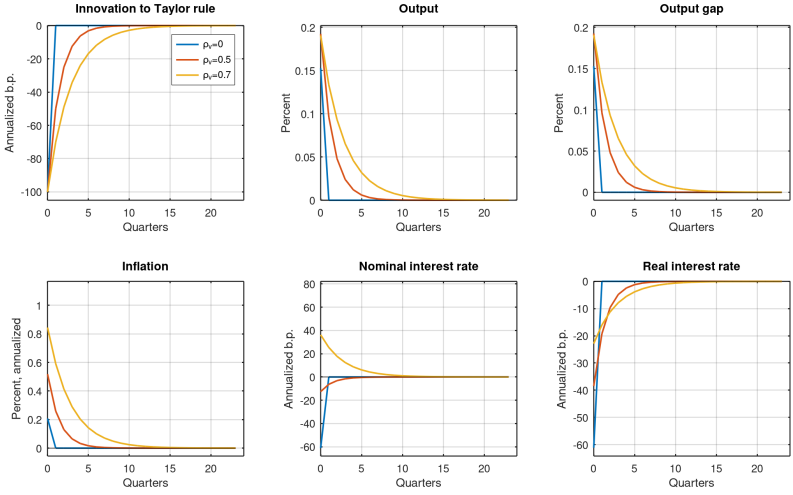
- Qualitatively reasonable for such a simple model: $r \downarrow, x \uparrow, \pi \uparrow$

Monetary policy shock: impulse responses



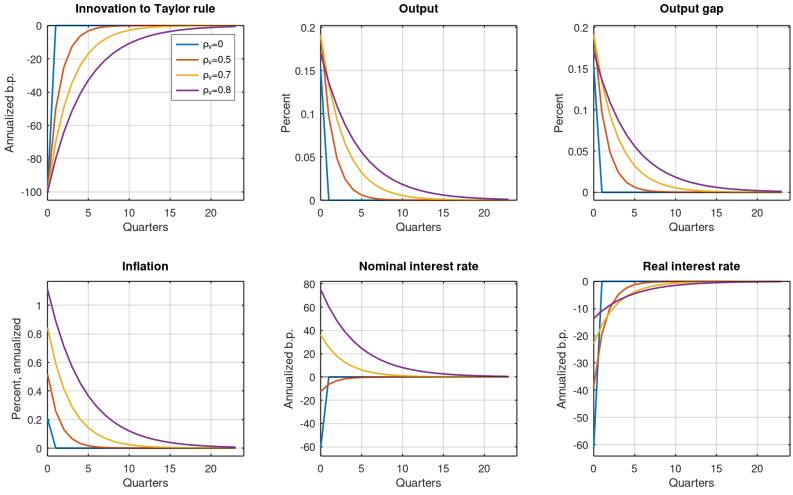
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Monetary policy shock: impulse responses



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Monetary policy shock: impulse responses



- Qualitatively reasonable for such a simple model: $r \downarrow, x \uparrow, \pi \uparrow$

Monetary policy shock: analytical solution

- Monetary rule $i_t = \rho + \phi_\pi \pi_t + v_t$, with $v_t = \eta\, v_{t-1} + \varepsilon_t^v$, and $r_t^n = \rho$
- Guess

$$x_t = -A v_t \qquad\qquad \pi_t = -B v_t$$

so we get $\left(\mathbb{E}_t \pi_{t+1} = \eta \pi_t \;\; \& \;\; \mathbb{E}_t y_{t+1} = \eta y_t\right)$

Guess & verify using NKPC $\Rightarrow \pi_t = \frac{\kappa}{1-\beta\eta} x_t$ and DIS $\Rightarrow \sigma(1-\eta)x_t = -(\phi_\pi - \eta)\pi_t - v_t$

$$\Rightarrow \qquad x_t = -(1-\beta\eta)\,\Lambda\, v_t, \qquad \pi_t = -\kappa\Lambda\, v_t$$

with $\Lambda \equiv \left[\sigma(1-\eta)(1-\beta\eta) + \kappa(\phi_\pi - \eta)\right]^{-1} > 0$

- Real rate: $r_t = i_t - \mathbb{E}_t \pi_{t+1} = \sigma(1-\eta)(1-\beta\eta)\Lambda\, v_t > 0 \Rightarrow$ contraction
- Nominal rate: $i_t - \rho = (1 - \phi_\pi \kappa \Lambda)\, v_t$
  - there is always a persistence η such that $\phi_\pi \kappa \Lambda = 1$: policy works *without* moving i_t!
  - for very persistent shocks, i_t *falls* after a tightening: the inflation (Fisher) effect dominates
  - related to the Neo-Fisherian debate (Cochrane, 2017; Uribe, 2022)

Monetary policy shock: impulse responses

- Cleaner to think in terms of real rates:

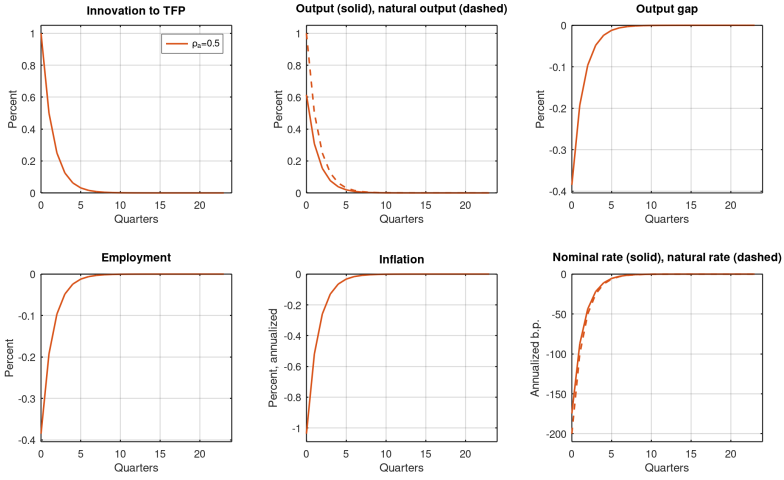
$$x_t = -\frac{1}{\sigma} \sum_k \mathbb{E}_t(r_{t+k} - r^n_{t+k})$$

$$\pi_t = \kappa \sum_k \beta^k \mathbb{E}_t x_{t+k}$$

- Qualitative lessons in line with the effect of monetary shocks – see empirics later
- But:
  - output and inflation respond on impact and decay monotonically. Data: delayed, hump-shaped (CEE, 1999, Romer & Romer, 2004)
  - markups and profits countercyclical, real wages very procyclical

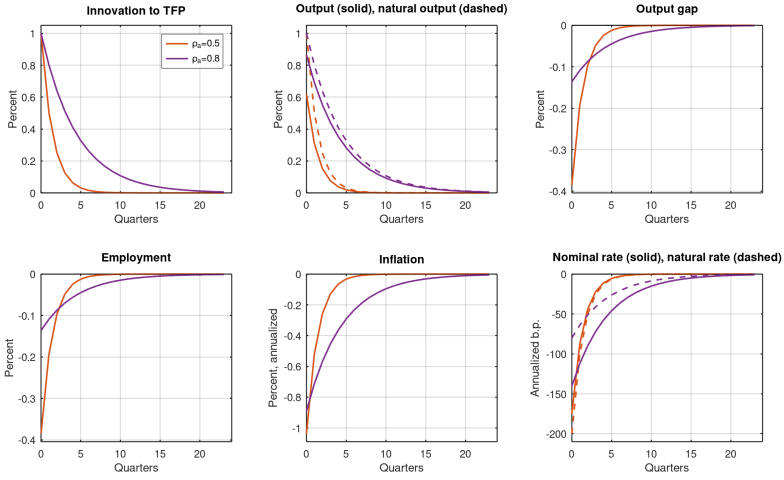
  $\Rightarrow$ medium-scale models add habits, investment adjustment costs, sticky wages, indexation (Christiano, Eichenbaum & Evans, 2005; Smets & Wouters, 2007)

Technology shock



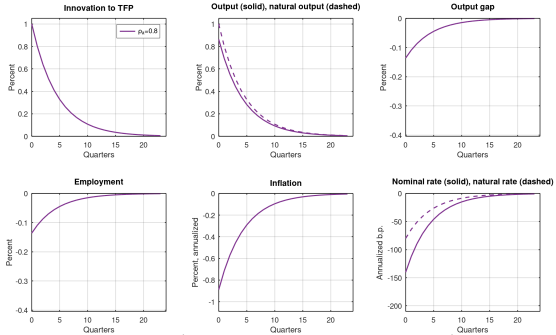
- Positive TFP shocks increase output, but nominal rigidities lower output gap: price stickiness create an *efficiency wedge*

Technology shock



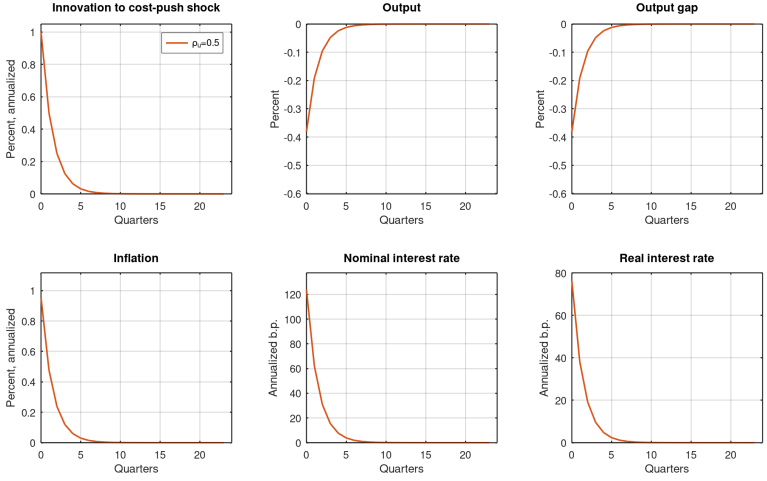
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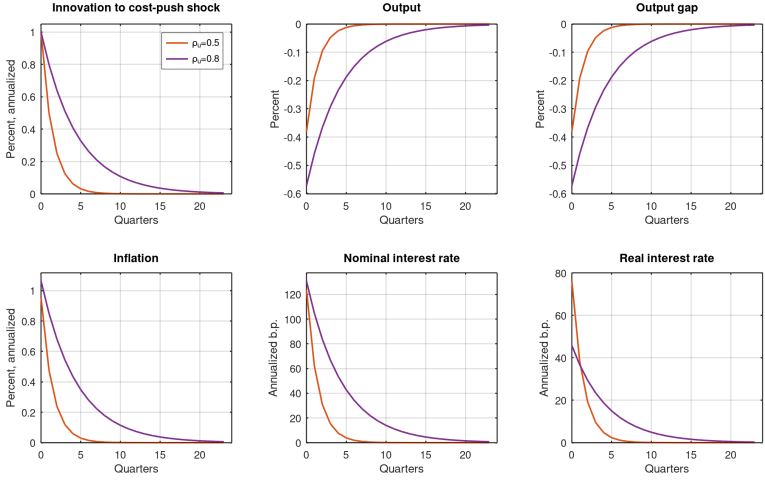
- Recall $z_t = \rho_z z_{t-1} + \varepsilon_t^z \;\Rightarrow\; y_t^n = \frac{1+\varphi}{\sigma+\varphi} z_t, \quad r_t^n - \rho = -\sigma \frac{1+\varphi}{\sigma+\varphi}(1-\rho_z)\, z_t$
- A positive TFP shock lowers natural rate: output must rise today, but fall back $\mathbb{E}_t \Delta z_{t+1} < 0$
- If the central bank follows a Taylor rule (does not track r_t^n): the real rate falls *too little*
  - $x_t < 0$, $\pi_t < 0$: output rises, but less than natural output
  - employment falls on impact ($n_t = y_t - z_t$) – consistent with Galí (1999) VAR evidence

Cost push shock



- Cost shocks increase inflation, triggering central bank monetary reaction, increasing interest rates and lowering output

Cost push shock



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Demand shocks and the Keynesian cross

- Preference (patience) shock $\beta_t \uparrow$ or pessimistic news about the future $\Rightarrow r_t^n \downarrow$
- ***Flexible prices***: the real rate falls, output unaffected (y_t^n depends only on z_t)
- ***Sticky prices***: if i_t does not track r_t^n, the real rate gap $r_t - r_t^n > 0 \Rightarrow x_t < 0$
  - households want to consume less $\Rightarrow$ firms sell and produce less
  - $\Rightarrow$ labor income and dividends fall $\Rightarrow$ consumption falls further
    "*one person's spending is someone else's income*" (Keynesian cross in GE)
- Recessions driven by insufficient aggregate demand: output below its efficient level
- The central bank can fully offset it by setting $i_t = r_t^n \rightarrow$ *divine coincidence*, later

Determinacy and the Taylor Principle

Determinacy: the problem(s)

- Two types of multiplicity:
  1. *nominal*: price level P_t indeterminacy (also with flexible prices)
  2. *real*: multiplicity in (x_t, π_t): sunspot / self-fulfilling expectations

Determinacy: the problem(s)

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1. Sargent & Wallace (1975): an exogenous interest rate peg pins down $\mathbb{E}_t[\pi_{t+1}]$ (via the Fisher equation) but leaves the price level P_t indeterminate

2. The model has two jump variables and no initial condition
   $\Rightarrow$ policy must select a unique bounded equilibrium for (x_t, π_t)

▷ Simplest example: a ***real rate rule*** $i_t = \rho + \mathbb{E}_t\pi_{t+1} + \varepsilon_t \;\Rightarrow\; r_t = \rho + \varepsilon_t$

$$x_t = -\frac{1}{\sigma}\sum_{k=0}^{T-1}\mathbb{E}_t(r_{t+k} - \rho) + \mathbb{E}_t x_{t+T}$$

- $\lim_{T\to\infty}\mathbb{E}_t x_{t+T}$ is not pinned down: any $x_t + \xi_t$ with $\mathbb{E}_t\xi_{t+1} = \xi_t$ is an equilibrium
- "Natural" selection: $\lim \mathbb{E}_t x_{t+T} = 0$; obtained as limit of $i_t = \rho + \phi_\pi \mathbb{E}_t\pi_{t+1} + \varepsilon_t$, $\phi_\pi \searrow 1$

An interest rate peg implies indeterminacy

- Peg $i_t = \rho$ (and $r_t^n = \rho$). Write the system forward, using $\mathbb{E}_t \pi_{t+1} = (\pi_t - \kappa x_t)/\beta$:

$$\begin{pmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{pmatrix}}_{\equiv M} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$$

- Blanchard & Kahn (1980): unique bounded solution iff # eigenvalues of M outside the unit circle = # jump variables = 2
- Characteristic polynomial $p(\lambda) = \lambda^2 - \mathrm{tr}(M)\lambda + \det(M)$:

$$p(0) = \det M = \frac{1}{\beta} > 0, \qquad p(1) = -\frac{\kappa}{\sigma\beta} < 0$$

$\Rightarrow$ one root in $(0, 1)$, one root > 1: only one unstable eigenvalue

$\Rightarrow$ A continuum of bounded equilibria, indexed by a sunspot
(any initial π_0 consistent with the stable root)

The Taylor principle

- Rule $i_t = \rho + \phi_\pi \pi_t + \phi_y x_t + v_t$. Substituting into the DIS and NKPC:

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = A_T \begin{pmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi_{t+1} \end{pmatrix} + B_T \left(r_t^n - \rho - v_t \right), \qquad A_T = \Omega \begin{pmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{pmatrix}$$

  with $\Omega \equiv 1/(\sigma + \phi_y + \kappa\phi_\pi)$, $B_T = \Omega\,(1, \kappa)'$ $\quad$ [$M = A_T^{-1}$ from before]

- Determinacy $\Leftrightarrow$ both eigenvalues of A_T inside the unit circle $\Leftrightarrow$ (for $\phi_\pi, \phi_y \geq 0$)

$$\boxed{\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0}$$

- With $\phi_y = 0$: $\phi_\pi > 1$: Taylor principle
- Interpretation: a permanent increase $d\pi$ raises x by $\frac{1-\beta}{\kappa} d\pi$ (NKPC), hence i by

$$\left[\phi_\pi + \phi_y \frac{1 - \beta}{\kappa} \right] d\pi > d\pi$$

  $\Rightarrow$ the real rate rises when inflation rises

Taylor principle: intuition and evidence

- If $\phi_\pi < 1$: self-fulfilling inflation expectations

$$\mathbb{E}_t \pi_{t+1} \uparrow \Rightarrow \; r_t \downarrow \Rightarrow \; x_t \uparrow \Rightarrow \; \pi_t \uparrow \Rightarrow \; \text{validates expectations}$$

- If $\phi_\pi > 1$: any sunspot path would put inflation on an explosive path
  $\Rightarrow$ the only bounded equilibrium is the fundamental one (off-equilibrium threat)
- Taylor (1993) policy rule: unrelated, but ended up being related!

$$i_t = r^* + \pi_t + \tfrac{1}{2}(\pi_t - \pi^*) + \tfrac{1}{2}\,\tilde{y}_t \;\Rightarrow\; \phi_\pi = 1.5$$

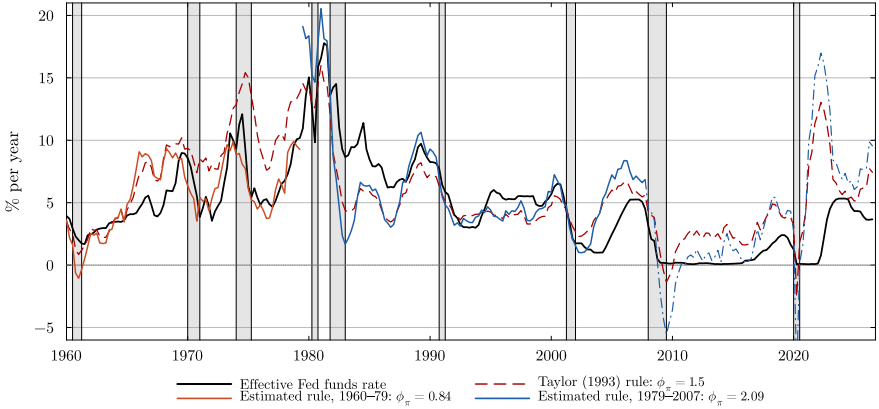
- Clarida, Galí & Gertler (2000): forward-looking Taylor rule estimated by GMM
  - pre-Volcker (1960–79): $\phi_\pi \approx 0.8 < 1$; Volcker–Greenspan: $\phi_\pi \approx 2$

  $\Rightarrow$ 1970s instability as sunspot-driven indeterminacy (also Lubik & Schorfheide, 2004)

Taylor principle: intuition and evidence

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Nominal Rigidities and Aggregate Demand Externality

Aggregate Demand externality – Blanchard–Kiyotaki

- ***Flexible prices***: monopolistic competition alone makes output too low, but not demand-determined

$$\left(Y_t^n\right)^{\sigma+\varphi} = \frac{Z_t^{1+\varphi}}{\mu} \; < \; \left(Y_t^e\right)^{\sigma+\varphi} = Z_t^{1+\varphi}$$

  classical dichotomy: path of r_t (or M/P in BK) has no effect on real allocations

- ***Sticky prices***: output is set by demand (DIS) since $P_t(i) > MC$, firms are willing to serve extra demand
- A demand expansion $dY \propto x_t = y_t - y_t^n$ at given prices raises welfare at first order (using $Y = ZN$, $\frac{W}{P} = N^{\varphi} C^{\sigma}$):

$$dU = C^{-\sigma} dY - N^{\varphi} dN = C^{-\sigma}\left(1 - mc_t\right) dY = C^{-\sigma-1}\left(1 - \tfrac{1}{\mu}\right) x_t > 0$$

  social value of a unit (price) > social cost (mc): the markup is a wedge on *every* unit
- Externality: firm i takes (Y_t, P_t) as given: no incentive to cut its own price
  individiual $P_t(i)$ has no effect on P_t and aggregate demand Y_t

$\Rightarrow$ no firm internalizes that a coordinated expansion ($r_t \downarrow$ or lower $P_t(i)$ for *all* i) raise all demand curves ($x_t \uparrow$)

The lesson of Blanchard–Kiyotaki, in the NK model

- Welfare change of demand expansion x_t is first order

$$dU = C^{-\sigma-1}\left(1-\tfrac{1}{\mu}\right)x_t = \frac{x_t}{\varepsilon} > 0$$

- Externality: firm i takes (Y_t, P_t) as given; its own price barely moves aggregate demand
  $\Rightarrow$ nobody internalizes that a coordinated expansion ($r_t \downarrow$) raises all demand curves
- ***Menu costs***: private loss Π of a non-optimal price is second order (envelope theorem)
  prices too high: lose customers / prices too low: earn less per customer. Around p^*: two effects cancel out

$$d\Pi \approx \tfrac{\varepsilon-1}{2}(\sigma+\varphi)^2 x_t^2$$

$\Rightarrow$ small frictions rationalize that Calvo stickiness have large real effects

- ***Policy***: welfare is linear in x_t, weight $\Phi \equiv 1-1/\mu$

$\Rightarrow$ central banks: temptation to increase x_t: inflation bias under discretion

$\Rightarrow$ solution: employment subsidy $\tau = 1/\varepsilon \Rightarrow \Phi = 0$, $Y^n = Y^e$: first-order gain disappear

Blanchard–Kiyotaki (1987): environment [Not covered]

- Static GE: m firms with differentiated goods, n households with differentiated labor
  Notation adapted: $(\varepsilon, \varepsilon_w, \nu)$ here $= (\theta, \sigma, \beta)$ in BK

- ***Technology***: $Y_i = N_i^{1/\alpha}$, N_i CES bundle of labor types (elasticity ε_w), $\alpha \geq 1$
- ***Preferences***: $U_j = C_j^{\gamma} \left(M_j'/P\right)^{1-\gamma} - N_j^{\nu}$, C_j CES bundle of goods (elasticity ε), $\nu \geq 1$
  $\alpha - 1$: elasticity of MC; $\nu - 1$: elasticity of the marginal disutility of work
  - money = non-produced good and numeraire: choosing goods vs. money breaks Say's law
  - utility linear in real wealth $\Rightarrow$ welfare = sum of surpluses

- ***Demand side*** (holds whether or not prices are set optimally):

$$Y = K \frac{M}{P}, \qquad Y_i = K_c \, Y \left(\frac{P_i}{P}\right)^{-\varepsilon}, \qquad N_j = K_n \, Y^{\alpha} \left(\frac{W_j}{W}\right)^{-\varepsilon_w}$$

  aggregate demand $\propto$ real balances ($K = \frac{\gamma}{1-\gamma}$); isoelastic goods demand; derived labor demand

Price setting, equilibrium, and the cost of market power [Not covered]

- ***Price*** and ***wage rules***, each setter taking (P, W, Y) as given:

$$\frac{P_i}{P} = \left[\frac{\varepsilon}{\varepsilon-1} K_p \frac{W}{P} Y^{\alpha-1}\right]^{\frac{1}{1+\varepsilon(\alpha-1)}}, \qquad \frac{W_j}{W} = \left[\frac{\varepsilon_w}{\varepsilon_w-1} K_w \frac{P}{W} Y^{\alpha(\nu-1)}\right]^{\frac{1}{1+\varepsilon_w(\nu-1)}}$$

  markup $\frac{\varepsilon}{\varepsilon-1}$ over MC; markup $\frac{\varepsilon_w}{\varepsilon_w-1}$ over the marginal disutility of work
- ***Symmetric equilibrium***: $\frac{P}{W} = \frac{\varepsilon}{\varepsilon-1} K_p Y^{\alpha-1}$, $\frac{W}{P} = \frac{\varepsilon_w}{\varepsilon_w-1} K_w Y^{\alpha(\nu-1)}$
  "labor demand" and "labor supply" pin down $(W/P, Y)$; then M/P from AD, M gives P
- vs. ***perfect competition*** (drop both markups):

$$\frac{Y^{MC}}{Y^{PC}} = \left(\frac{\varepsilon_w - 1}{\varepsilon_w} \cdot \frac{\varepsilon - 1}{\varepsilon}\right)^{\frac{1}{\alpha\nu-1}} < 1$$

  gap shrinks with $\varepsilon, \varepsilon_w$ (closer substitutes) and α, ν; explodes as $\alpha\nu \to 1$
- Money is neutral: homogeneous of degree 0 in (P, W, M)
  $\Rightarrow$ monopolistic competition *alone* does not make aggregate demand matter

The aggregate demand externality [Not covered]

- ***Experiment***: *coordinated* proportional cut in all P_i, W_j at given M
  $\Rightarrow$ relative prices unchanged, $P \downarrow$, $M/P \uparrow$, $Y = KM/P \uparrow$
- Utility = three surpluses ($\mu = \gamma^{\gamma}(1-\gamma)^{1-\gamma}$), all of which rise:

$$U_j = \underbrace{\left[\mu \frac{W_j}{P} N_j - N_j^{\nu}\right]}_{\substack{\text{labor demand shifts out,} \\ \mu W_j/P > \text{marg. disutility}}} + \underbrace{\mu \sum_i \frac{V_{ij}}{P}}_{\substack{\text{demand shifts out,} \\ P_i > \text{MC}}} + \underbrace{\mu \frac{M_j}{P}}_{\text{real balances}}$$

$\Rightarrow$ welfare rises unambiguously

- Why nobody does it: each setter takes P as given; one firm's price cut barely moves Y and the firm captures a negligible share of the gain $\Rightarrow$ externality
- Beyond the usual monopoly loss: if *all* firms priced competitively, every demand curve would also shift out (GE amplification of underproduction)

Menu costs: small frictions, large effects [Not covered]

- Small fixed cost of changing *nominal* prices and wages; shock $M \uparrow$
- ***Private*** cost of not adjusting is second order (envelope theorem):

$$\frac{dV_i^*}{dM} = \frac{\partial V_i}{\partial M} + \underbrace{\frac{\partial V_i}{\partial P_i}}_{=0} \frac{dP_i}{dM}$$

  $\Rightarrow$ nobody adjusts given others do not: M/P moves one-for-one with M
- ***Social*** effect of $M/P \uparrow$ at fixed relative prices is first order: the AD externality

$\Rightarrow$ second-order menu costs $\Rightarrow$ first-order effects of money on output and welfare

- Private loss (share of revenue) after a $100\Delta\%$ demand change:

$$L(\Delta) = \frac{(\alpha-1)^2(\varepsilon-1)}{2\left[1+\varepsilon(\alpha-1)\right]}\Delta^2 + o(\Delta^2)$$

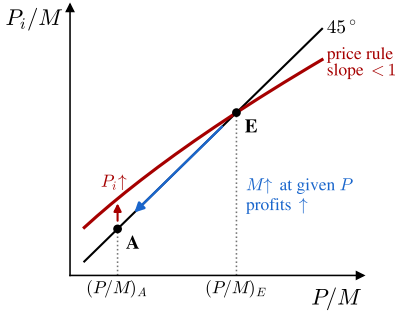
  $L = 0$ at $\alpha = 1$: with constant MC, the optimal price ignores multiplicative demand shifts
- $\varepsilon = 5$, $\alpha = 1.1$: 0.003% of revenue blocks adjustment to a 5% money shock; welfare gain $\approx 2\%$ of GNP vs. menu costs 0.03–0.13% of GNP

The price rule and the effect of money [Not covered]

- $\nu = 1$ (constant real wage):

$$\frac{P_i}{M} = k\left(\frac{P}{M}\right)^s, \quad s = \frac{1+(\varepsilon-1)(\alpha-1)}{1+\varepsilon(\alpha-1)} < 1$$

- E: rule $\cap$ 45° line (Nash, $P_i = P$)
- A: best symmetric point, $\frac{(P/M)_A}{(P/M)_E} = \left(\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{1}{\alpha-1}}$
- $M\uparrow$, prices fixed: $E \to A$, profits $\uparrow$
- No menu cost: at A, rule $> 45°$
  $\Rightarrow P_i \uparrow \Rightarrow$ back to E (neutrality)



- ***Strategic complementarity***: $s \in (0,1)$, relative-price motive beats demand motive
  $\Rightarrow$ Pareto-ranked equilibria for intermediate menu costs (coordination failure)
- ***Real rigidities***: $\alpha \to 1 \Rightarrow s \to 1$ and $L(\Delta) \to 0$ (Ball & Romer, 1990)

Blanchard–Kiyotaki: limits and links to the NK model [Not covered]

- ***Labor supply is the bottleneck***: wage setters' losses are small only if $\nu - 1$ is small (Frisch elasticity $1/(\nu - 1)$ of 1.7–5 in the examples)
- ***Initial distribution***: all prices start optimal; with inherited dispersion, some setters adjust even after small shocks (Caplin & Spulber, 1987)
- ***Welfare***: with mean-zero shocks, first-order gains in booms and losses in slumps cancel
- ***Complementarity*** rests on unit elasticity of AD w.r.t. M/P: not robust
- ***Links to NK***:
  - same Dixit–Stiglitz demand; Calvo replaces the menu cost
  - $\mu > 1 \Rightarrow Y^n$ inefficiently low $\Rightarrow$ employment subsidy; inflation bias under discretion (Barro & Gordon, 1983)
  - real rigidities flatten the NKPC (lower κ): the same force as $s \to 1$

Empirics of the Building Blocks

The Euler equation in the data: Hall (1978, 1988)

- Log-linear Euler equation (lognormal returns, constant variances):

$$\Delta \log c_{t+1} = \text{const} + \psi\, r_{t+1} + \varepsilon_{t+1}, \qquad \psi = 1/\sigma = \text{EIS}, \qquad \mathbb{E}_t \varepsilon_{t+1} = 0$$

- Hall (1978): with constant r, consumption is a random walk (+ drift)
  - lagged income does not predict Δc_{t+1}; (lagged stock prices do, mildly)
- Hall (1988): estimate ψ by IV, instruments in the time-t information set (r_{t+1} is correlated with news in ε_{t+1})
  - estimated EIS close to zero: "probably not above 0.1"
- Why it matters for the NK model: σ^{-1} = interest elasticity of aggregate demand
  $\Rightarrow$ governs the strength of the intertemporal substitution channel of monetary policy

GMM estimation of the Euler equation

- Non-linear Euler equation for each asset j, instruments $z_t \in \mathcal{I}_t$:

$$\mathbb{E}_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^j_{t+1} - 1\right] = 0 \qquad \Rightarrow \qquad \mathbb{E}\left[\left(\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^j_{t+1} - 1\right) z_t\right] = 0$$

- GMM (Hansen, 1982): $(\hat{\beta}, \hat{\gamma}) = \operatorname{argmin}\ g_T(\beta,\gamma)'\, W_T\, g_T(\beta,\gamma)$; Hansen's J-test of overidentifying restrictions
- Hansen & Singleton (1982): reasonable γ, but restrictions rejected with several returns (cf. equity premium puzzle, Mehra & Prescott, 1985)
- Campbell & Mankiw (1989): $\Delta c_t = \mu + \lambda\, \Delta y_t + (1-\lambda)\psi\, r_t + \varepsilon_t$
  - $\lambda \approx 0.5$: half of income goes to "rule-of-thumb" consumers $\Rightarrow$ excess sensitivity
- Yogo (2004): instruments are weak $\Rightarrow$ weak-IV robust inference: EIS small, significantly below 1 in most countries

EIS estimates and the aggregate Euler equation

- Micro and quasi-experimental estimates:
  - Attanasio & Weber (1993): cohort data, EIS ≈ 0.8 once aggregation / demographics are handled
  - Vissing-Jorgensen (2002): asset market participants: 0.3–0.4 (stockholders), 0.8–1 (bondholders)
  - Best, Cloyne, Ilzetzki & Kleven (2020): UK mortgage notches, EIS ≈ 0.1
  - Havranek (2015) meta-analysis: ≈ 0.3–0.4 after correcting for selective reporting
- NK calibrations: $\sigma = 1$ (EIS = 1); Smets–Wouters (2007): $\sigma \approx 1.4$
- The aggregate Euler equation fits poorly:
  - Euler-implied interest rate is *negatively* correlated with money-market rates (Canzoneri, Cumby & Diba, 2007)
  - consumption responds sluggishly / hump-shaped $\Rightarrow$ habits in medium-scale models

$\Rightarrow$ Heterogeneity, high MPCs, failure of Ricardian equivalence: HANK lectures

Measuring price rigidity in micro data

- BLS micro data underlying the CPI ($\approx 70\%$ of consumer expenditure) and the PPI, 1988–2005
- Bils & Klenow (2004): median duration ≈ 4.3 months, prices more flexible than thought
- Nakamura & Steinsson (2008): distinguish temporary sales and product substitutions
  - regular prices: median frequency 9–12% per month $\Rightarrow$ median duration 8–11 months
  - including sales: $\approx 20\%$ per month (≈ 4.5 months)
  - sales: temporary, prices return to the regular price $\Rightarrow$ arguably little macro relevance (Eichenbaum, Jaimovich & Rebelo, 2011; Kehoe & Midrigan, 2015)
- Large heterogeneity across sectors: gasoline, fresh food weekly vs. services > 1 year
- Calvo mapping: monthly frequency $10\% \Leftrightarrow$ quarterly $1 - \theta \approx 0.27$, $\theta \approx 0.73$

Five facts about prices

1. Median frequency of regular price changes 9–12% per month (excluding sales)
2. About one-third of regular price changes are price *decreases*
3. The frequency of price *increases* covaries strongly with inflation; the frequency of decreases and the size of increases do not
4. Price changes are highly seasonal: most frequent in the first quarter
5. Hazard of price changes slightly declining over the first months, then flat

- Plus: price changes are large (median absolute size $\approx 10\%$) relative to inflation
  $\Rightarrow$ idiosyncratic shocks dominate pricing decisions

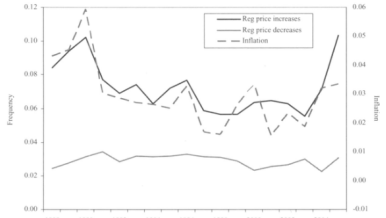


FIGURE II
Inflation and the Frequency of Regular Price Change for Consumer Prices
Notes: Inflation is the annual evolution of the weighted median frequency of regular price increases and decreases along with the CPI inflation rate.

Nakamura & Steinsson (2008): monthly frequency of regular price increases and decreases vs. CPI inflation, 1988–2005 (increases co-move with inflation, decreases flat)

Time- vs. state-dependent pricing: does it matter?

- ***Calvo***: exogenous timing. ***Menu costs***: adjust when $|p_i - p_i^*|$ exceeds a threshold
- Caplin & Spulber (1987): with (S,s) rules, money can be *neutral* in the aggregate
- Golosov & Lucas (2007): menu costs + idiosyncratic shocks calibrated to micro data
  - selection effect: firms that adjust are those far from their target
    $\Rightarrow$ real effects of money much smaller than in Calvo
- Midrigan (2011): multi-product firms, fat-tailed shocks $\Rightarrow$ many small changes, weak selection, close to Calvo
- Nakamura & Steinsson (2010): sectoral heterogeneity roughly triples non-neutrality
- Alvarez, Le Bihan & Lippi (2016): sufficient statistic for a small monetary shock δ

$$\text{Cumulative output effect} \approx \frac{\delta}{6} \frac{\text{Kurt}(\Delta p_i)}{N(\Delta p_i)}$$

(Calvo: Kurt = 6; Golosov–Lucas: Kurt = 1)

$\Rightarrow$ Calvo is a reasonable approximation at low inflation; state dependence matters for large shocks / high inflation (2021–23, see curvy PC below)

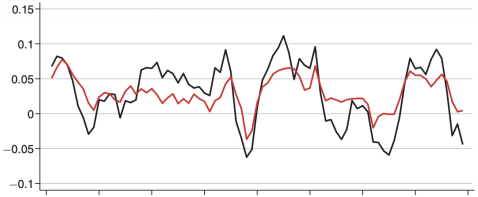
Estimating the NKPC: identification challenges

- Galí & Gertler (1999), Sbordone (2002): GMM on the marginal-cost NKPC, with labor share as $\widehat{mc}_t$
  - forward-looking term dominant, $\lambda > 0$; but weak instruments (Mavroeidis, Plagborg-Møller & Stock, 2014)
- Output-gap PC $\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t$: aggregate time series hard to use
  - ***Endogeneity***: central bank offsets cost-push shocks $\Rightarrow \mathbb{C}\text{ov}(x_t, u_t) < 0 \Rightarrow \hat{\kappa}$ biased down (McLeay & Tenreyro, 2020)
  - ***Trend inflation*** / shifting expectations confound the slope
- Regional data with time fixed effects: Hazell, Herreño, Nakamura & Steinsson (2022)
  - $\kappa \approx 0.006$ (quarterly): very flat; vs. Rotemberg & Woodford (1997): 0.024
  - flattening of the reduced-form PC since the 1980s mostly due to *anchored expectations*
- vs. calibrated $\kappa \approx 0.1$–0.3: a puzzle. Is it price rigidity, or link output $\rightarrow$ marginal cost?

Anatomy of the Phillips curve: Gagliardone et al (2025, AER)

- ***Bottom-up*** approach: quarterly Belgian micro data on prices, quantities and costs
- Dynamic pass-through regressions of firm prices on marginal cost and competitors' prices (IV, fixed effects for trends and expectations)
  - prices fixed 3–4 quarters on average
  - strategic complementarities halve the pass-through of marginal cost
- Slope of the cost-based NKPC: $\lambda \approx 0.05$–0.07 (baseline 0.052): steep
  - marginal cost alone accounts for $\geq 70\%$ of PPI inflation variation
  - validated with identified oil supply shocks (Känzig, 2021)
- Reconciliation: $\kappa = \lambda \times \Omega$, with Ω = elasticity of marginal cost to the output gap
  - Ω is low in the pre-pandemic sample $\Rightarrow$ low κ, consistent with the literature

  $\Rightarrow$ flat PC = weak link output $\rightarrow$ marginal cost, not weak link marginal cost $\rightarrow$ inflation



Gagliardone et al. (2025): model-implied vs. actual PPI inflation (aggregate marginal cost fed into the estimated NKPC)

The Phillips curve in developing economies: Herreño et al (2026)

- ***Sufficient statistics***: in a broad class of NK models, $\kappa_y = \kappa_{mc} \times \Omega$
- κ_{mc}: firm-level pass-through of input cost shocks (shift-share IV, as in Amiti, Itskhoki & Konings, 2019), rescaled by the frequency of price changes
  - India, Annual Survey of Industries 1998–2017: pass-through 0.22 (annual); price-change frequency 0.45 per quarter
  - $\kappa_{mc} \approx 0.093$: $\approx 2\times$ advanced economies, entirely due to less rigid prices
- Ω: elasticity of marginal cost to output with a demand-shifter design
  - firm level 0.2; industry / regional level 0.7–0.8: $\geq 3\times$ advanced-economy estimates
- $\kappa_y \approx 0.074$ (quarterly): one order of magnitude steeper than in the US
  - micro real rigidities reduce the slope by a factor ≈ 4 (similar to advanced economies)
  - difference explained by weaker *macro* real rigidities and less rigid prices; misallocation channel small
- Policy: domestic demand matters for inflation in EMs $\Rightarrow$ relevance of inflation targeting

Non-linear Phillips curves I

- 2021–23 inflation surge: hard to rationalize with $\kappa \approx 0.006$
- Slack measured by labor market tightness $\theta_t = v_t/u_t$ (vacancies / unemployment)
- Inverse-L Phillips curve: slope jumps once θ_t crosses the ***Beveridge threshold*** $\theta^* \approx 1$ (labor shortage). Schematically:

$$\pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, \hat{\theta}_t + \kappa^{tight} \max\{\hat{\theta}_t - \hat{\theta}^*, 0\} + u_t, \qquad \kappa^{tight} \gg \kappa$$

- Beveridge curve nearly vertical beyond the threshold: tightening reduces vacancies with little rise in unemployment (2022–24 "soft landing")
- Same pattern in five other inflation surges over the past 111 years

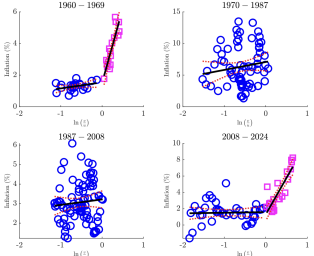


Figure 1: Inflation and Labor Market Tightness in the United States, 1960–2024. The y-axis represents

Beveridge curve & threshold unemployment rate. Shocks trigger sharp inflation. US price inflation vs. v/u. flat for $v/u < 1$ and steep for $v/u > 1$,

Non-linear Phillips curves II

- ***Smith, Timmermann & Wright (2024)***, "Nonlinear Phillips Curves"
  - the slope flattened around 2000, but the PC is also kinked: steeper when the labor market is tight
  - kink large enough that the flattening barely changed the slope in tight markets
  - holds for price and wage PCs, in the US and the EU

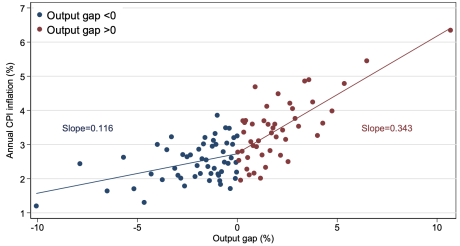
- ***Bunn, Anayi, Barnes, Bloom, Jalca, Mizen, Thwaites & Yotzov (2024)***, "How Curvy Is the Phillips Curve?"
  - firm surveys (UK Decision Maker Panel, US): price responses to demand shocks are convex (hypothetical scenarios, realized sales, Covid shocks; also cost shocks)
  - stronger where inflation is higher; fades after ≈ 2 years
  - menu cost model: higher trend inflation brings firms closer to their adjustment thresholds
  - non-linearity explains $\approx 1/4$ of the 2022 inflation overshoot

$\Rightarrow$ state-dependent slope κ: asymmetric cost of "running the economy hot"

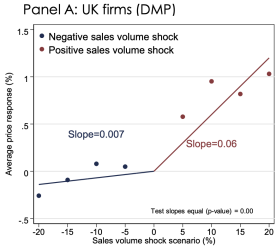
Non-linear Phillips curves III

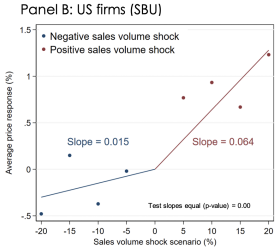
- ***Bunn, Anayi, Barnes, Bloom, Jalca, Mizen, Thwaites & Yotzov (2024)***, "How Curvy Is the Phillips Curve?"

Figure 1: Macro evidence on inflation and output gap



Notes: This figure is a binned scatterplot of annual CPI inflation rates on output gap estimates for 38 countries over the period 1990-2024. Data on annual inflation rates is taken fro the World Bank cross-country database of inflation (Ha et al. 2023). Data on output gap estimates is obtained from the OECD. The total number of country-year observations in the figure is 1,205.





Notes: This figure reports responses to the question "Suppose that your business's sales volume over the next 12 months is X per cent higher/lower than you currently expect. How would that affect the average price you charge, relative to what you currently expect?" Firms are randomised into one of four scenarios for sales volume: ±5%, ±10%, ±15%, ±20%. Firms are presented with both the positive and negative values for a given scenario. Panel A is based on 6,394 observations from 2,486 UK firms in the Decision Maker Panel. Panel B is based on 1,570 observations from 787 US firms in the June 2024 survey wave of the Survey of Business Uncertainty. Across the two panels, we cannot reject the equality of the slopes on positive sales volume shocks (t-stat = 0.39) or the equality of the slopes on negative sales volume shocks (t-stat=0.62).

Evidence on Monetary Non-Neutrality

Identifying monetary policy shocks

- Standard story (Friedman, 1968): money $\uparrow \Rightarrow i \downarrow$ (liquidity effect), prices sticky $\Rightarrow r \downarrow$ $\Rightarrow$ spending, employment $\uparrow$; in the long run, only prices and i are higher
- Why study monetary *shocks*? Not to understand policy (the systematic part matters more), but to test models: NK vs. neoclassical (neutral) models
- Main difficulty: the policy instrument S_t (Fed funds rate, reserves) is endogenous

$$S_t = f(\Omega_t) + \varepsilon_t^m$$

f: policy rule given the information set Ω_t; ε_t^m: shift in preferences, technical mistakes... A rate hike may be a shock, or the response to a booming economy

- ***Four approaches***:
  1. VARs with recursive identification
  2. Narrative approach
  3. High-frequency identification
  4. Case studies / natural experiments

1. VARs with recursive identification

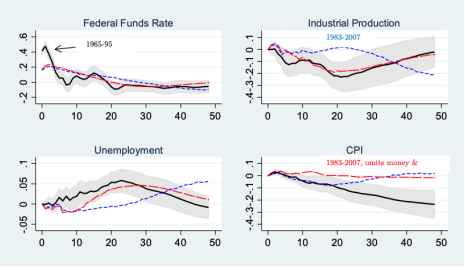
- Reduced form $Z_t = B_1 Z_{t-1} + \cdots + B_q Z_{t-q} + u_t,\ \mathbb{E}[u_t u_t'] = V$
- Structural shocks $A_0 u_t = \varepsilon_t, \mathbb{E}[\varepsilon_t \varepsilon_t'] = I \Rightarrow V = A_0^{-1}(A_0^{-1})'$: not enough to pin down A_0
- ***Recursiveness assumption***: $Z_t = (X_{1t}, S_t, X_{2t})$

$$A_0 = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

  - X_{1t} (output, prices, commodity prices): known by the CB, do not react within the period
  - X_{2t} (reserves, money): react contemporaneously
  - $\approx$ regress S_t on Ω_t, use residuals
- CEE (1999) results for a contractionary FF shock:
  - output falls after $\approx$ 2 quarters, trough after 1–1.5 years; unemployment rises
  - prices flat for $\approx$ 1.5 years, then decline
  - price puzzle if commodity prices are omitted (Sims, 1992): CB reacts to anticipated inflation

VAR evidence: robustness

Figure 3.1. Christiano, Eichenbaum, and Evans (1999) Identification
1965m1-1995m6 full specification: solid black lines; 1983m1-2007m12 full specification: short dashed blue lines; 1983m1-2007m12, omits money and reserves: long dashed red lines)



Ramey (2016) Fig. 3.1: CEE (1999) identification, IRFs of FF rate, industrial production, unemployment, CPI, commodity prices, nonborrowed reserves. 1965m1–1995m6 vs. 1983m1–2007m12 (with and without money and reserves)

- Post-1983 sample: weaker effects, output response sometimes positive, price puzzle returns
- Fundamental issue: the CB's information set (Greenbook) exceeds the econometrician's
- Small contribution: MP shocks explain $\approx 21\%$ (FF) / 7% (NBR) of the 1-year-ahead forecast error variance of output, ≈ 0 for prices

2. Narrative approach: Romer & Romer (1989, 2004)

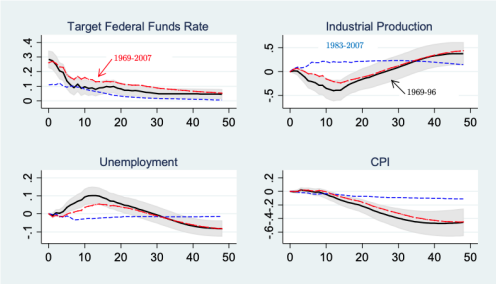
- Romer & Romer (1989): read FOMC records ⇒ dates of deliberate tightening to reduce inflation ("Romer dates")
- Romer & Romer (2004): intended FF change at each FOMC meeting m, purged of the Fed's information using Greenbook forecasts:

$$\Delta ff_m = \alpha + \beta\, ff_m^b + \sum_{i=-1}^{2} \gamma_i\, \Delta\tilde{y}_{m,i} + \sum_{i=-1}^{2} \lambda_i \big(\Delta\tilde{y}_{m,i} - \Delta\tilde{y}_{m-1,i}\big)$$
$$+ \sum_{i=-1}^{2} \varphi_i\, \tilde{\pi}_{m,i} + \sum_{i=-1}^{2} \theta_i \big(\tilde{\pi}_{m,i} - \tilde{\pi}_{m-1,i}\big) + \rho\, \tilde{u}_{m,0} + \varepsilon_m \qquad \text{RR shock} = \hat{\varepsilon}_m$$

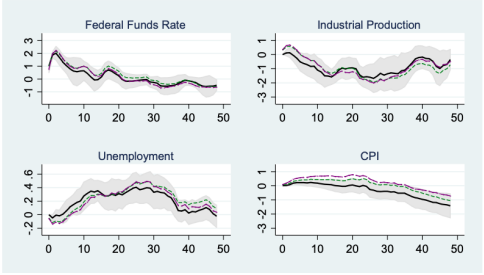
- 1 pp shock: industrial production falls by up to $\approx 4.3\%$ after ≈ 2 years; prices start falling after ≈ 22 months
  ⇒ larger and faster than recursive VARs
- Caveats: Fed info beyond the Greenbook; driven by 1970s–early 80s (Coibion, 2012); puzzling post-1983

Narrative shocks: impulse responses

A. Coibion VAR 1969m3–1996m12: solid black lines; 1983m1-2007m12: short dashed blue lines; 1969m3-2007m12: long dashed red lines.



B. Jordà Local Projection Method, 1969m3–1996m12 Recursiveness assumption: solid black lines; No recursiveness assumption: dashed green lines; No recursiveness assumption, FAVAR controls: dashed purple lines.



- Full sample: contraction in IP, rise in unemployment, slow decline of CPI
- Post-1983: few large shocks, imprecise and sometimes wrong-signed responses

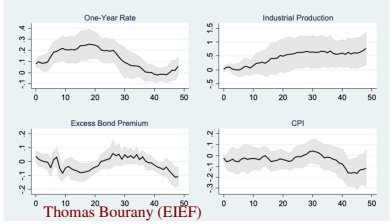
3. High-frequency identification

- Idea: in narrow window (e.g. 30 min) around FOMC announcement, the only news is the policy decision
- Kuttner (2001): surprise from current-month Fed funds futures (day d of a D-day month)

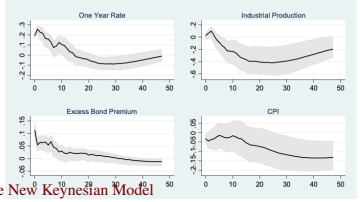
$$\Delta i_t^u = \frac{D}{D-d}\left(f_t^0 - f_{t-\Delta t}^0\right)$$

- Gürkaynak, Sack & Swanson (2005): two factors: target and path (forward guidance)
- Gertler & Karadi (2015): 3-month-ahead FF futures surprise as an external instrument (proxy SVAR) for the 1-year rate
  - tightening: IP ↓, CPI ↓, excess bond premium ↑ (credit channel)
- Caveats (Ramey, 2016): GK surprises serially correlated, predictable by Greenbook, fragile to specs.

Figure 3.3B Gertler-Karadi Monetary Shock, Jordà 1990m1-2012m6



A. Gertler-Karadi's Monetary Proxy SVAR, VAR from 1979m7-2012m6, instrument from 1991m1–2012m6



High-frequency identification of non-neutrality

- 30-minute windows around FOMC announcements, 1995–2014 (incl. ZLB)
- Policy news shock = first principal component of changes in 5 interest rate futures (FF current and next meeting, eurodollar 2–4 quarters ahead)
- Test of neutrality: do real rates move when the Fed moves nominal rates?
  - TIPS real yields / forwards respond almost one-for-one with nominal yields up to $\approx$ 2–3 years
  - breakeven inflation barely moves at short horizons

  $\Rightarrow$ neutral models: $r_t = r_t^n$, unaffected by policy – rejected
- ***Fed information effect***: after a surprise tightening, private forecasts (Blue Chip) of output growth *rise*
  - announcements reveal Fed's info; NK model w/ information effect: large non-neutrality
  - debate: "Fed response to news" rather than information (Bauer & Swanson, 2023)

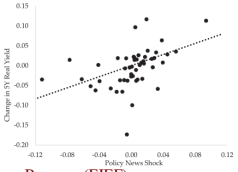


FIGURE I

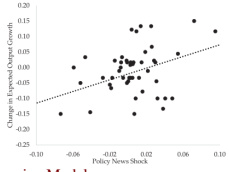


FIGURE II

4. Case studies and natural experiments

- Velde (2009), "Chronicle of a deflation unforetold": France, 1724
  - value of gold and silver coins in *livres* set by decree; government cuts it by 45% in three steps: unexpected, well publicized, proportional to money holdings
  - exchange rate in London adjusts immediately
  - prices of commodities and manufactured goods, industrial wages: fall slowly and incompletely
  - industrial output (textiles) contracts by $\approx 30\%$
- Other classics:
  - Friedman & Schwartz (1963): monetary history of the US, Great Depression
  - Mussa (1986): RER volatility jumps from fixed to floating: *nominal* regime affects a *real* variable
  - Volcker disinflation (1979–82): deep recession

Monetary non-neutrality: summary

- Robust *qualitative* picture after a contractionary shock:
  - nominal *and* real rates rise
  - output falls with a delay, peak effect after 1–2 years
  - prices sluggish, fall only slowly
- *Magnitudes* uncertain: identifying a clean monetary shock is hard (CB private information, forward guidance, anticipation)
- MP shocks explain a small share of fluctuations
  $\Rightarrow$ but the systematic part of policy can matter a lot
- For the NK model:
  - non-neutrality: ✓
  - delayed, hump-shaped responses and inflation persistence: need richer models (CEE, 2005; Smets & Wouters, 2007) or richer heterogeneity (HANK)

Policy Implications

Efficient allocation and the role of policy

- Two distortions:
  - (i) monopolistic markups $\mu > 1$: natural output too low (cf. Blanchard–Kiyotaki)
  - (ii) sticky prices: output gap $Y_t \neq Y_t^n$ and price dispersion $\Delta_t > 1$
- Planner: $-U_N/U_C = Z_t$. Flex-price equilibrium: $-U_N/U_C = Z_t/\mu$
- (i) is a *steady-state* distortion: employment subsidy τ financed lump-sum

$$(1-\tau)\mu = 1 \qquad \Leftrightarrow \qquad \tau = 1/\varepsilon \qquad \Rightarrow \qquad Y_t^n = \text{efficient output}$$

- (ii) If $\pi_t = 0$ forever: no firm wants to change its price, $\Delta_t = 1$, mc_t constant
  $\Rightarrow$ the sticky-price economy replicates the flexible-price allocation

$\Rightarrow$ Optimal policy (with subsidy): $\pi_t = 0,\ x_t = 0 \ \Leftrightarrow\ i_t = r_t^n$
strict inflation targeting is optimal

Divine coincidence

- In the basic NK model, stabilizing inflation $\Leftrightarrow$ stabilizing the welfare-relevant output gap $\Rightarrow$ no trade-off: "divine coincidence"
- Any shock that moves r_t^n (technology, demand) can be fully offset by $i_t = r_t^n$
  - Moll / Mankiw–Weinzierl two-period model: for *any* demand shock there is an i_1 restoring first best
- Breaks with:
  - cost-push shocks u_t (time-varying markups, taxes)
  - sticky wages and prices (Erceg, Henderson & Levin, 2000); real wage rigidities (Blanchard & Galí, 2007)
  - zero lower bound, open economy, heterogeneity…
- ***Implementation***: the rule $i_t = r_t^n$ alone yields indeterminacy! Instead:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y x_t, \qquad \kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

$\Rightarrow \pi_t = x_t = 0$ is the unique bounded equilibrium: threat of a strong response off equilibrium

Welfare loss and the policy trade-off

- Second-order approximation of household welfare around the efficient steady state:

$$\mathbb{W} \approx -\frac{1}{2}\frac{\varepsilon}{\lambda}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \alpha_x x_t^2\right], \qquad \alpha_x = \frac{\kappa}{\varepsilon}$$

  (in units of steady-state consumption)

- Inflation is costly because of price dispersion $\Rightarrow$ misallocation; weight $\propto \varepsilon/\lambda$: large
- Calibration: $\alpha_x \approx 0.34/6 \approx 0.06$: inflation stabilization dominates
- With a cost-push shock u_t:

$$\pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, x_t + u_t$$

  $\Rightarrow$ cannot achieve $\pi_t = x_t = 0$: genuine trade-off between inflation and output gap

- Optimal policy problem: $\min\ \mathbb{E}_0 \sum_t \beta^t \left[\pi_t^2 + \alpha_x x_t^2\right]$ s.t. the NKPC
  (the DIS then determines the i_t that implements the optimum)

Optimal policy: discretion vs. commitment

- ***Discretion***: CB re-optimizes each period, takes expectations as given:

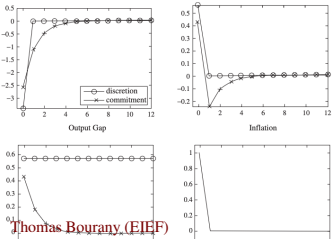
$$x_t = -\frac{\kappa}{\alpha_x}\pi_t \qquad \text{(“lean against the wind”)}$$

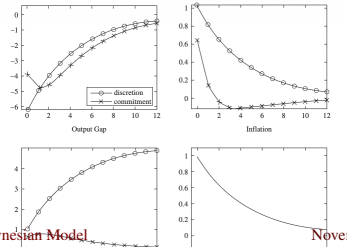
with u_t AR(1), persistence ρ_u: $\pi_t = \frac{\alpha_x}{\kappa^2+\alpha_x(1-\beta\rho_u)}u_t$, $x_t = -\frac{\kappa}{\kappa^2+\alpha_x(1-\beta\rho_u)}u_t$

- ***Commitment*** (timeless perspective): Lagrangian with multiplier ξ_t on the NKPC

$$x_t - x_{t-1} = -\frac{\kappa}{\alpha_x}\pi_t \qquad \Rightarrow \qquad \hat{p}_t \equiv p_t - p_{-1} = -\frac{\alpha_x}{\kappa}x_t$$

  - history dependence: promise of future restraint lowers expected inflation today
  - price-level targeting: inflation is partially reversed
  - discretion suffers a *stabilization bias* (and an inflation bias if $\mu(1-\tau) > 1$)





Fiscal policy: government spending

- Spending G_t financed by lump-sum taxes; $C_t + G_t = Y_t$; $g_t \equiv dG_t/Y$

$$\text{(DIS)} \qquad y_t - g_t = \mathbb{E}_t\big[y_{t+1} - g_{t+1}\big] - \sigma^{-1}\big(i_t - \mathbb{E}_t\pi_{t+1} - \rho\big)$$

$$\text{(NKPC)} \quad \pi_t = \kappa\Big(y_t - \zeta\, g_t - \frac{1+\varphi}{\sigma+\varphi} z_t\Big) + \beta\, \mathbb{E}_t\pi_{t+1}, \qquad \zeta \equiv \frac{\sigma}{\sigma+\varphi} < 1$$

  ζ: neoclassical multiplier (negative wealth effect raises labor supply $\Rightarrow y_t^n$ rises)

- ***Real-rate rule*** ($r_t = \rho$): $c_t = 0 \;\Rightarrow\; y_t = g_t$: multiplier exactly one
  - no Keynesian cross: RA consumption depends on real rates only (MPC out of income ≈ 0)
  - Moll / Mankiw–Weinzierl: same result, G "puts idle resources to work"

- ***Taylor rule*** (unanticipated i.i.d. g_0), $\Psi \equiv \frac{\sigma^{-1}\phi_\pi\kappa}{1+\sigma^{-1}\phi_\pi\kappa} \in (0,1)$:

$$c_0 = -\Psi(1-\zeta)\, g_0, \qquad y_0 = \big(1 - \Psi(1-\zeta)\big)\, g_0 \in (\zeta\, g_0,\, g_0)$$

  active policy raises real rates $\Rightarrow$ crowds out consumption. Fixed i (e.g. ZLB): multiplier > 1 (Christiano, Eichenbaum & Rebelo, 2011)

Transfers, Ricardian equivalence, and the pecking order

- Deficit-financed transfers ($T_t \downarrow$, $T_{t+k} \uparrow$): present value unchanged
  $\Rightarrow$ RA consumption unchanged: Ricardian equivalence, transfers are irrelevant
- Government spending stabilizes *output* but is second-best: private consumption not restored (useless G = "digging ditches")
- Monetary policy restores the first best (divine coincidence)

  $\Rightarrow$ pecking order: monetary policy first, fiscal policy only if MP is constrained (ZLB)

- Breaks with high-MPC households, e.g. TANK (Campbell–Mankiw spenders, share λ, income share γ^{sp}):

$$\frac{\partial Y_1}{\partial(-T_1^{sp})} = \frac{\lambda}{1-\lambda\gamma^{sp}} > 0, \qquad \frac{\partial Y_1}{\partial G_1} = \frac{1}{1-\lambda\gamma^{sp}} > 1$$

- HANK: under some conditions, interest rate cuts and stimulus payments are equivalent for aggregate demand management (Wolf, 2025) $\rightarrow$ HANK lectures

The forward guidance puzzle (preview)

- Credible announcement at date 0: real rate cut by Δ at date T only
- From the DIS solved forward, $y_t = \sigma^{-1}\Delta$ for $t \leq T$, then 0:

$$y_0 = \sigma^{-1}\Delta \qquad \text{independent of } T\,!$$

- From the NKPC solved forward:

$$\pi_0 = \kappa\,\sigma^{-1}\Delta \sum_{k=0}^{T} \beta^k \qquad \text{increasing in } T\,!$$

- Promises about the far future are as powerful as (or more than) current cuts: implausible (even worse at the ZLB, where y_0 also grows with T)
- Root: RA Euler equation (no discounting of future income, perfect information)
  Fixes: incomplete markets (McKay, Nakamura & Steinsson, 2016), behavioral discounting (Gabaix, 2020), bounded rationality (García-Schmidt & Woodford, 2019; Farhi & Werning, 2019)

Conclusion

- NK model = RBC core + monopolistic competition + nominal rigidities
  - non-linear model with Calvo pricing, price dispersion as the cost of inflation
  - log-linear 3-equation model: DIS, NKPC, Taylor rule
- Determinacy requires the Taylor principle: real rates must rise with inflation
- Aggregate demand externality: why nominal rigidities matter for welfare
- Empirics of the building blocks:
  - Euler equation: small and imprecise EIS, poor aggregate fit $\rightarrow$ HANK
  - prices change every 8–11 months, state dependence at high inflation
  - Phillips curve: flat in output gap, steep in marginal cost, non-linear, steeper in EMs
- Monetary non-neutrality: robust qualitatively, magnitudes debated
- Policy: divine coincidence, strict inflation targeting, commitment vs. discretion, multiplier of one, Ricardian equivalence $\Rightarrow$ pecking order

$\Rightarrow$ Next: critiques of the NK model (forward guidance puzzle, ZLB, Neo-Fisherianism, determinacy), then heterogeneous agents and HANK

Appendices

Rotemberg pricing

- Each firm pays a quadratic cost of changing its price, in units of the final good:

$$\max_{\{P_t(i)\}} \mathbb{E}_0 \sum_t \Lambda_{0,t}\Big[P_t(i)Y_t(i) - W_t\frac{Y_t(i)}{Z_t} - P_t\frac{\psi}{2}\Big(\frac{P_t(i)}{P_{t-1}(i)} - 1\Big)^2 Y_t\Big]$$

- Symmetric equilibrium, non-linear Phillips curve:

$$(\Pi_t - 1)\Pi_t = \frac{\varepsilon}{\psi}\Big(mc_t - \frac{1}{\mu}\Big) + \mathbb{E}_t\Big[\beta\Big(\frac{C_{t+1}}{C_t}\Big)^{-\sigma}\frac{Y_{t+1}}{Y_t}(\Pi_{t+1} - 1)\Pi_{t+1}\Big]$$

- Log-linearized: $\pi_t = \beta\,\mathbb{E}_t\pi_{t+1} + \frac{\varepsilon-1}{\psi}\widehat{mc}_t$
- Same as Calvo to first order if $\frac{\varepsilon-1}{\psi} = \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$
- Differences: no price dispersion (representative firm), adjustment costs are a real resource cost

Determinacy with a Taylor rule: derivation

- Plug $i_t = \rho + \phi_\pi \pi_t + \phi_y x_t + v_t$ into the DIS:

$$(\sigma + \phi_y) x_t + \phi_\pi \pi_t = \sigma \mathbb{E}_t x_{t+1} + \mathbb{E}_t \pi_{t+1} + (r_t^n - \rho - v_t)$$

- With the NKPC $\pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1}$, solve for (x_t, π_t):

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \begin{pmatrix} \sigma & 1 - \beta \phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{pmatrix} \begin{pmatrix} \mathbb{E}_t x_{t+1} \\ \mathbb{E}_t \pi_{t+1} \end{pmatrix} + \dots$$

- Both variables are non-predetermined $\Rightarrow$ need both eigenvalues of A_T inside the unit circle
- For a 2×2 matrix: $|\det A_T| < 1$ and $|\operatorname{tr} A_T| < 1 + \det A_T$
  $\det A_T = \frac{\beta\sigma}{\sigma + \phi_y + \kappa\phi_\pi}$, $\operatorname{tr} A_T = \frac{\sigma + \kappa + \beta(\sigma + \phi_y)}{\sigma + \phi_y + \kappa\phi_\pi}$
- The second condition reduces to $\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$ (Bullard & Mitra, 2002)