Fiscal policy in a monetary union: Optimal policy and cross-country spillovers

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Abstract

What are the effects of fiscal policy in a monetary union? This article explores the role of government spending, the optimal policy design and the various spillovers of public spending shocks in an integrated union. To this purpose, we develop a two-countries model with "large economies", updating the conclusions at stake in "small open economy" models, and providing a general framework where countries differ on many aspects – home-bias, agents preferences, price rigidities and labor supply. We show first that interaction effects and structural heterogeneity matter for optimal policy: the clear separation between central bank stabilizing the union and fiscal policies stabilizing country-specific shocks will not hold in this setting. Our second set of results is to identify the main transmission mechanisms of fiscal policy, with first a trade channel, through relative prices, and second a monetary response from the union central bank. Our main conclusion is that spillovers of fiscal policy shocks crucially depend on the central bank mandate and the second channel largely dominates the first in this setting. This framework provides arguments supporting coordination between union central bank and fiscal authorities in the context of the European Monetary Union.

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1 Introduction

The recent crisis has revealed that one of the major challenges of European Union is fiscal. If one hope to *complete the EU*, in particular in Eurozone, its main currency and monetary union, one would have to rethink the role of fiscal policy. According to the "Five Presidents' Report", the Euro Area needs to introduce an advisory European Fiscal Board "coordinating and complementing national fiscal councils", before implementing a "common stabilisation function" with budgetary instruments. When monetary policy fails to stabilize large adverse shocks in situation of liquidity trap, such fiscal framework would strengthen the policy responses, while leaving idiosyncratic national shocks to the member states.

However, such clear-cut separation between asymmetric vs. aggregate stabilization – performed at country vs. union-level – may fail to address several issues at stake: the European crisis has shown powerful cross-country transmission channels, due to trade and financial integration. In particular, fiscal policy may react to shocks originated from neighboring countries or may itself imply policy spillovers on the rest of the Union.

This papers addresses these two questions: what are the transmission channels of government spending shocks and should fiscal policy be used to stabilize asymmetric shocks in a monetary union? The contribution is this paper is thus twofold: we first identify the cross-country spillovers of fiscal policy, and then determine the optimal policy in this monetary union. To this purpose, I develop a two-countries DSGE model with two *large* economies: I advocate that Small-Open-Economy models fail to identify cross-country transmission channels due to the lack of interaction effects: shock in one country affects the other country's decisions, which in turn amplifie the original shock, and so on. Moreover, I provide a general framework where the two countries differ on many aspects: home-bias, agents preferences, price rigidities, labor supply, etc. As cross-country effects are not symmetrical anymore, the consequences of an aggregate shock or central bank policy may affect one country more severely than the other.

With these two characteristics, I provide a more accurate vision of the spillovers of government spending, and how interact the two countries' households, local fiscal authorities and the union-central bank. I choose to focus on a tractable New-Keynesian model, adding governments that provide public goods to representative households. This very general environment differs from the preexisting literature mainly to account for structural heterogeneity and interaction effects.

This paper offers both normative and positive analyses. First, we establish general results about optimal policies – at the first and second best – and we build upon this analysis to discuss policy implications for the EMU. The main message is that country structural heterogeneity complicate the welfare investigation and require a greater cooperation between the different stabilisation instruments. In particular, both the first-best allocation and the union loss function display novel "wedges" and "weights" depending on home-bias and labor supply elasticity. Using this welfare criterion for policy analysis, the *optimal policy* for central bank and national fiscal authorities reduces fourfold the welfare loss in case of asymmetric supply shocks. Fiscal policies can reduce trade imbalances and relative price differential that overload labor markets of one country. Moreover, we show that optimal policy can be *replicated* by means of simple monetary and fiscal policy *rules* – where central bank follows the standard Taylor principle and government spendings react to terms-of-trade and output gaps. The presence of structural heterogeneity provide a rationale for cooperation: to reach the optimum, the central bank should account for government spending adjustment and conversely.

Second, we turn to more positive implications and identify spillovers of fiscal policy shocks. The main result is the presence of two transmission channels: public spending in one country results in domestic inflationary pressures: This policy distorts terms-of-trade, as foreign goods are relatively cheaper, and private consumption for domestic-good decreases. This first *trade channel* appears both in small-open economy and large economy framework. The key insight when focusing on large-economies is to understand the impact of *monetary policy reaction.* In small-open economy, the domestic country, being negligible, has no impact on the union, while with two large economies, the effects of government purchases affects the union inflation: the central bank will therefore react to spending shocks, raising interest rate. As a result, this interest hike affects negatively the foreign country, where consumption decreases, output gap widens and prices fall. Therefore, the fiscal policy in a country may have a deflationary impact on its neighbor through central bank reaction. Such transmission channel is at the heart of our result and undermine common assessments from the small-open economy framework. The spillovers of fiscal policy of one country for its neighbors show a positive trade effect due to distorted terms-of-trade and a negative impact of central bank reaction, the second being much stronger in this setting.

Therefore, the nature of cross-country spillovers depends crucially on the central bank mandate, as well as country structural parameters: (i) if the monetary policy accounts for both consumption gap and inflation, i.e. following a Taylor rule, or if it sets interest rate with a high degree of inertia, then the outcome of a domestic fiscal policy shock may not cause foreign depression or deflation as in the benchmark model, (ii) when countries are heterogeneous, transmission of inflation may be stronger – through price flexibility, labor rigidity and openness – and both monetary and fiscal policy should focus more on stabilizing the inflationary country. Our general conclusion is that the Foreign country does not really "benefit" from a Home spending shock. Under some specific restriction the spillover can be positive in terms of consumption, but at a cost of an higher inflation.

Literature: Before turning to the model, I briefly review how this paper stands in the literature. Since the appearance of the concept of *"Optimum currency area"*, how to improve resilience of economic unions to asymmetric shocks, in particular with fiscal policies coordination, has been a ongoing question. In the Public economics literature, since Tiebout's model to framework including game-theoretical features, many articles investigated the conditions for tax competition, public goods (mis-)allocation and interaction effects between regional units of federations, cf. Gordon (1983), Wilson (1986), Wildasin (1988) and Frankel and Rockett (1988), as well as Kehoe (1987), Devereux (1991) who adopt a macroeconomics approach. Considering the three public sector tasks (cf. Musgrave (1959)), we focus on government purchasing and providing public goods, both as an *allocation* and a *stabilization* policy.

This model is primarily interested in describing a microfounded New-Keynesian framework to analyze the nature of optimal policy in a currency union and is close to four articles from the New-Keynesian literature. First, Galí and Monacelli (2008) is a standard model investigating the role of fiscal and monetary policy in a "small-open economy" framework. However, as the union is made of a continuum of infinitesimal countries, a country policy have no impact on union inflation, output and central bank policy. We will show that such an environment is not suitable to identify cross-country policy transmission.

Beetsma and Jensen (2005) is the equivalent model with two countries and fiscal-monetary interaction, and is directly inspired from Benigno (2004). In terms of policy analysis, this article is very close to our setting. However, the main difference lies in the question of openness and Home-bias: as Benigno (2004), they consider "fully-open" households, without Home-bias and, as a result, both countries have identical consumption and CPI because of risk-sharing. This restriction changes the general result, as trade transmission channel is absent. For this reason, the gains from stabilization of asymmetric shocks by fiscal policy are likely to be underestimated. Hence, we differ from this model, and adopt a more general setting.

Nevertheless, they also study country heterogeneity on policy tradeoffs, focusing on nominal rigidities. They conclude that monetary policy should implement a weighted-average inflation targeting rule, with more weight on the region with higher degree of price stickiness. We obtain a similar – but more general – result with weighted average, the weight accounting for supply-side factors: price stickiness but also labor-supply elasticity and agent preferences. In a different fashion, Liu and Pappa (2008) emphasize the importance of dissimilarity in trading structures – tradable/non-tradable sector – for macroeconomic coordination.

This paper also stand in the literature focusing on the effects of government spending and fiscal multipliers. Christiano, Eichenbaum and Rebelo (2011), Galí, López-Salido and Vallés (2007) and Farhi and Werning (2012*b*) emphasize the importance of either non-separable preferences, presence of credit-constrained households or the absence of monetary reaction – at the ZLB – to generate high fiscal multipliers: inflationary public spending, in situation of positive wealth effects of labor supply or liquidity trap, reduces real interest rate and consumption reacts positively. In a *currency union* however, these effects are strikingly different, due to terms-of-trade distortion. When government spending appreciates terms-of-trade, this dampens consumption and the multiplier can be lower than one. Most of our result are analogous to the analysis of Farhi and Werning (2012*b*), but our framework display the interaction effects and implications from structural heterogeneity, absent from the small-economy model of their article.

Several articles such as Correia, Farhi, Nicolini and Teles (2013), Farhi, Gopinath and Itskhoki (2014), Farhi and Werning (2012*a*), also discussed innovative actions for government intervention. Using distortive taxes, government can "mimic" the path on interest rate or exchange rate, or fiscal-union can provide insurance via contingent transfers to cover against asymmetric shocks. However, distortive taxation can undermine greatly the positive impact of government spending, as discussed in Drautzburg and Uhlig (2015). Ferrero (2009) use a framework similar to Galí and Monacelli (2008) but including distortive taxation and government debt. However, I choose to focus solely on government spending policy, as taxes are always slower to react for stabilization motives.

Considering the research question and motivation we draw inspiration from the recent article by Blanchard, Erceg and Lindé (2017). This article also measure the effect of a fiscal expansion in the "Core" economies of Euro Area to improve economic outcomes of the "Periphery" using a medium and a large-scale DSGE model, they show that trade channel is at the center of the mechanism, along with the lack of reaction by central bank – i.e. liquidity trap here. As they use quantitative models with a large number of features, it can be relevant to describe the transmission channels with a relatively simple and tractable model.

A last word on the empirical relevance of the two channels of fiscal policy in a currency union. As described in Beetsma and Giuliodori (2011) a fiscal shock can be followed by a rise in output and consumption but a reduction in trade-balance. This characterizes the "leakage effect" we describe, due to terms-of-trade distortions. Carlino and Inman (2013) also study the spillover of state-deficit policy in United States in terms of employment. This question is also addressed in Hebous and Zimmermann (2013), who use "Global VAR" to study the spillover of a fiscal shock in Euro Area, and compare unilateral vs. coordinated fiscal actions and show that the latter have much larger effect. We can also consider evidences from Poghosyan et al. (2014) on how fiscal transfers can smooth regional shock in established federation.

The rest of the paper is organized as follow: the next section develop the two-country DSGE model. In the third section, we step aside to determine the optimal allocation at the first-best. The efficient allocation is necessary to understand what kind of policy trade-offs the economy faces in presence of nominal rigidities and how to derive the welfare criterion, as we shall see in the fourth section and the fifth, where we derive the optimal policy by mean of simple rule. The sixth and seventh chapters present the policy experiments, the effects of the models subject to respectively technology shock and fiscal policy shocks. The last develop on the role for structural heterogeneity, which is the main rationale of this paper for cooperation between the union central banks and fiscal authorities.

2 A two-countries DSGE model

In this model, the currency union is a closed system made of two countries (Home and Foreign) of identical "unit" size. The "Home" economy is denoted with standard variables, while Foreign is denoted with stars (*). As presented in introduction, the two countries are ***not*** isomorphic, and that explains why the following formulas might be slightly heavier than in *Small-Open Economy* models. Here are the main features of the model:

Households

Each country is made of a ***representative-household*** maximizing utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, G_t, N_t)$ positive in private (C_t) or public (G_t) consumption and negative in labor supply (N_t).

Consumption C_t (or C_t^*) are subject to ***Home-bias***. As a CES aggregator, it is composed of domestic ($C_{H,t}$) or imported ($C_{F,t}$) goods, with the Home-goods share represented by an index α. If $\alpha < 1/2$, this reflects a *Home-bias*, while $\alpha, \alpha^* \rightarrow 1/2$ represents full openness. Note that *the bias may be different* for the foreign country (if $\alpha < \alpha^*$, Home is less open).

Goods produced in each country ($C_{J,t}$, $J = H, F$) are made of a ***continuum of varieties*** (again CES aggregators) and where ϵ represents elasticity of substitution across varieties.

Household face a ***budget constraint*** (j representing Home or Foreign – with $*$):

$$\int_0^1 C_{H,t}^j(i) P_{H,t}(i) di + \int_0^1 C_{F,t}^j(i) P_{F,t}(i) di + \mathbb{E}_t\{Q_{t,t+1} D_{t+1}^j\} \leq D_t^j + W_t^j N_t^j - T_t^j$$

where $P_{H,t}$ & $P_{F,t}$ represent production prices at Home and Abroad, D_{t+1}^j the payoff of the Household portfolio, $Q_{t,t+1}$ the stochastic discount factor for one-period ahead nominal payoff, W_t^j the nominal wage and T_t^j the lump-sum tax. Here we suppose that *law of one price* holds and $P_{H,t}(i) = P_{H,t}^*(i)$.

The first-step optimization yields ***demand-schedules*** where consumption levels are inversely related to prices, and we then define aggregate price indices and CPI (consumer price index P_t and P_t^*) are Cobb-Doublas depending on openness α (or α^*).

The union economy is a Currency-Union and exchange rate is pegged. ***Terms-of-trade*** variable is key and defined as $\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}}$. The Real-Exchange Rate follows: $\mathcal{R}_t = \frac{P_t^*}{P_t} = \mathcal{S}_t^{(1-\alpha-\alpha^*)}$

Household utility is separable and logarithmic in consumption of private and public goods and CRRA in labor: $U(C_t, G_t, N_t) \equiv (1-\Gamma)\log(C_t) + \Gamma \log(G_t) - \frac{N_t^{1+\varphi}}{1+\varphi}$, where $\Gamma \in [0,1)$ is the weight attached to public goods and φ the disutility of labor supply by household (i.e. the inverse of Frisch elasticity). These two parameters *are heterogeneous* across countries: the "taste" of household for public good (e.g. Social service, Health or Education) can differ i.e. $\Gamma^* \neq \Gamma$, as well as disutility of working (e.g. due to labor supply constraints) i.e. $\varphi^* \neq \varphi$.

Deriving a second-step optimization, we obtain the two standards relations describing Household behavior: ***Euler equation*** (LHS) and ***Consumption-Leisure tradeoff*** (RHS):

$$Q_{t,t+1} = \beta\Big(\frac{P_t}{P_{t+1}}\Big)\Big(\frac{C_t}{C_{t+1}}\Big) \qquad\qquad C_t\, N_t^{\varphi} = (1-\Gamma)\frac{W_t}{P_t}$$

In this union, financial markets are ***complete***, i.e. this currency union is perfectly financially integrated as there exists a full set of state-contingent Arrow-Debreu securities traded across countries. The stochastic-discount factor are equalized between Home and Foreign. This leads to:

$$\frac{C_{t+1}}{C_t} = \frac{C^*_{t+1}}{C^*_t}\Big(\frac{\mathcal{R}_{t+1}}{\mathcal{R}_t}\Big) \qquad\qquad \Rightarrow \quad C_t = \vartheta_0\, C^*_t\, \mathcal{R}_t \;=\; \vartheta_0\, C^*_t\, \mathcal{S}_t^{1-\alpha-\alpha^*}$$

ϑ_0 represents NFA position differentials: this parameter introduces a wedge between Home and Foreign consumption due to heterogeneous preferences. The determination will be discussed latter, but ϑ_0 will intuitively stand for the asymmetry in private/public goods preferences (and directly, government size) under the ratio : $\vartheta_0 = \frac{1-\Gamma}{1-\Gamma^*}$. This value will be consistent with a null-trade-balance at steady-state between the two countries[1].

Government and monetary policy

The currency-union is a ***monetary-union***. There is a single ***Central-bank*** and short term interest-rate i_t that apply to both Home and Foreign countries.

Governments produce public goods G_t (or G^*_t) in the two countries[2]. Public goods are produced from a CES aggregation of intermediary goods bought to firms. The public good output is entirely consumed by Households, either at Home or Abroad: the Home Household can consume "Foreign" public goods with a share α_G (and "Home" public goods with a share $1-\alpha_G$). Again,

[1]This wedge is consistent with a feature developed in Liu and Pappa (2008) introducing such a wedge to account for asymmetry in trading structures between the two countries. In their article, ϑ_0 is the ratio of tradable sector shares, while ours is the ratio of private sectors

[2]The optimal level of "government purchase" will be in the center of our normative analysis in the next section

Foreign household may consume a different share of Home public goods ($\alpha_G^* \neq \alpha_G$). Therefore, government spending G_t can be represented by a Cobb-Douglas function with parameter α_G. Demand schedules for public goods and government price indices are similar to private goods, but depending on "government openness"[3].

Government purchase is financed through ***lump-sum taxes*** and government has a ***balanced budget***. Here, we abstract from central features for fiscal multiplier determination (namely debt issuance and taxes adjustment) and these points might deserve extensions.

Firms and production

A ***continuum of firms*** is producing intermediary goods under ***monopolistic-competition*** in each country. The technology is ***linear*** in labor N_t, and subject to productivity shocks A_t. This TFP (A_t or A_t^*) follow an AR(1) process with persistence $\rho_a \in [0, 1]$ and white-noisy shocks $\{\varepsilon_t^a\}$.

Aggregation of firms output using CES technology yield the standard $Y_t = A_t\, N_t\, \Delta_t$, where Δ_t is the coefficient of price dispersion.

Marginal cost is defined as real wages $W_t/P_{H,t}$, accounting for employment subsidy τ and divided by the marginal productivity. Because production is linear, all firms have the same marginal cost.

$$MC_t = (1-\tau)\frac{W_t}{P_{H,t}}\,\frac{N_t}{Y_t} = (1-\tau)\frac{W_t}{P_t}\,\frac{N_t}{Y_t}\,(S_t)^{\alpha}$$

Here, terms of trade impact positively marginal costs as real wage – relatively to consumer prices – are appreciated when terms of trade are depreciated, increasing the cost.[4].

Prices are subjects to ***Calvo-Yun*** rigidities, with a share θ of firms not allowed to reset their prices. ***Price stickiness*** θ is heterogeneous between Home and Abroad (i.e. $\theta \neq \theta^*$), and firms resetting prices face profit-maximization problem. First-order condition yields the optimal price level as a function of marginal cost and mark-up ($\mathcal{M} = \frac{\epsilon}{\epsilon-1}$), and as a result, under flexible price we obtain $\mathcal{M}^{-1} = MC_t \quad \forall t$.

Log-linearization of the main relations

We will approximate the main equations of this model around the Steady-State. For each variable Z_t, this log-deviation is denoted $\hat{z}_t \equiv \ln Z_t - \ln Z = z_t - z$ where Z represent the steady-state value and z_t its log-value.

[3] This parameter that will be important for transmission of inflation from one country to the other

[4] Note that this formula is similar for the Foreign country, adding stars to the other variables, with $\tau \neq \tau^*$

$$\textit{Relation Inflation-T.o.T.} \qquad \pi_t = \pi_{H,t} + \alpha\,\Delta\hat{s}_t \quad (\ \&\ resp.\ \pi^*_t = \pi_{F,t} - \alpha^*\ \Delta\hat{s}_t\ )$$

$$\textit{Relation R.E.R. -T.o.T.} \qquad \hat{r}_t = (1-\alpha-\alpha^*)\,\hat{s}_t$$

$$\textit{Consumption-Leisure tradeoff} \qquad \hat{w}_t - \hat{p}_t = \hat{c}_t + \varphi\,\hat{n}_t - \ln(1-\Gamma)$$

$$\textit{Euler equation} \qquad \hat{c}_t = \mathbb{E}_t(\hat{c}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{t+1}) - \rho)$$

$$\textit{Risk-sharing equation} \qquad \hat{c}_t = \hat{c}^*_t + \hat{r}_t = \hat{c}_t{}^* + (1-\alpha-\alpha^*)\,\hat{s}_t$$

with production inflation $\pi_{H,t+1} \equiv \hat{p}_{H,t+1} - \hat{p}_{H,t}$, CPI inflation $\pi_{t+1} \equiv \hat{p}_{t+1} - \hat{p}_t$, discount factor $\rho = -ln\beta$ and nominal interest: $i_t = -\ln Q_t$ (keeping in mind that asset price $Q_t = \mathbb{E}_t[Q_{t,t+1}]$). Terms of trade relate simply as $\hat{s}_t = -\hat{s}_t{}^* = \hat{p}_{F,t} - \hat{p}_{H,t}$

The relations for firms are similar to the usual "closed economy" model because there is no "supply" spillovers on firms of the other country in this model. Note that (i) output does not account for price-dispersion (i.e. $\delta_t = \ln[\Delta_t] \approx 0$) which is of second-order, (ii) denote the marginal cost as log (and not log-deviation from steady state)[5].

$$\textit{Output expression} \qquad \hat{y}_t = \hat{a}_t + \hat{n}_t$$

$$\textit{Marginal cost} \qquad mc_t = \ln(1-\tau) + (w_t - p_{H,t}) - (y_t - n_t)$$

$$\textit{Optimal price} \qquad \overline{\overline{p_{H,t}}} = \mu + (1-\beta\theta)\,\sum_{k=0}^{\infty} (\beta\theta)^k\,\mathbb{E}_t[mc_{t+k} + p_{t+k}]$$

$$\textit{Inflation dynamics} \qquad \pi_{H,t} = \beta\,\mathbb{E}_t\,\pi_{H,t+1} + \chi(mc_t + \mu)$$

Here $\chi = (1-\theta)(1-\beta\theta)/\theta$ is a coefficient between marginal cost and inflation. In the Philipps Curve, it shows how inflation reacts to marginal cost and it decreases in price stickiness: if prices are fully flexible, the coefficient is infinite, and inflation reacts instantaneously to change in marginal cost. Again, the foreign version is analogous, substituting $\pi_{F,t}$ for $\pi_{H,t}$ and $\chi* \neq \chi$.

Market-clearing, aggregate demand and terms-of-trade

Until now, the setting is really similar to closed or small-open economy New-Keynesian models. However, the presence of two-countries instead of a continuum, and the heterogeneity in structural parameters (recall: α, α_G, φ, Γ, θ and χ) make the main equations relatively heavier.

Using the Demand schedules, risk-sharing condition, expressing prices with "terms-of-trade" and aggregating, we obtain Home-goods output and Union-level output:

[5] As we will need this formula to compute the output gap

$$Y_{H,t} = \left[(1-\alpha) + \vartheta_0^{-1}\,\alpha^*\right] \mathcal{S}_t^{\alpha}\, C_t + (1-\alpha_{\mathrm{G}})\mathcal{S}_t^{\alpha_{\mathrm{G}}}\, G_t + \alpha_{\mathrm{G}}^*\, \mathcal{S}_t^{1-\alpha_{\mathrm{G}}^*}\, G_t^*$$

$$Y_{W,t} \equiv Y_{H,t} + Y_{F,t} = \mathscr{S}_t^{W_{H,C}}\, C_t + \mathscr{S}_t^{W_{F,C}}\, C_t^* + \mathscr{S}_t^{W_{H,G}}\, G_t + \mathscr{S}_t^{W_{F,G}}\, G_t^*$$

where the variables $\mathscr{S}_t^{W_{H,C}}$, $\mathscr{S}_t^{W_{F,C}}$, $\mathscr{S}_t^{W_{H,G}}$ and $\mathscr{S}_t^{W_{F,G}}$ are functions[6] of terms-of-trade and shift aggregate demand due to price differentials. For example, for Home-consumption, this wedge decreases exponentially (with power $1-\alpha$) when terms of trade are appreciated (i.e. $\mathcal{S}_t < 1$) and increases logarithmically (power α) when depreciated (i.e. $\mathcal{S}_t > 1$). Intuitively, this explains why, when Home-goods are relatively cheaper ($\mathcal{S}_t > 1$), the Home-household consume more of its own goods, increasing demand for Home-production. This effect comes directly from Home-bias in consumption is undermined if country are more open. Therefore, with *balanced* terms-of-trade ($\mathcal{S}_t = 1$), $\mathscr{S}_t^{W_{\cdot,\cdot}}$ collapse to 1 and $Y_{W,t} = C_t + C_t^* + G_t + G_t^*$.

When ***log-linearized*** around a symmetric steady-state, we use some ***specific notations***. To avoid multiplying the parameters, the *steady-state demand aggregates* can be expressed as share of world output ($Y_w = C + C^* + G + G^*$) and are denoted γ_{C}, γ_{C^*}, γ_{G} and γ_{G^*} respectively for C, C^*, G, G^* over Y_w. However, we also make the ***assumption*** **(A1)** that the two countries have the same size (unity). As a result, steady-state demand should not be higher that country size and we will suppose symmetry in steady-state output (which is not as restrictive as homogeneity) and we have $Y = Y^* = {}^1\!/\!{}_2 Y_w$. These two points will slightly constrain parameters values but the formula obtained is reduced to:

$$^1\!/\!_2\ \hat{y}_{H,t} = (1-\alpha)\,\gamma_{\mathrm{C}}\ \hat{c}_t\ +\ \alpha^*\,\gamma_{\mathrm{C}^*}\ \hat{c}_t^*\ +\ (1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}}\ \hat{g}_t\ +\ \alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}\,\hat{g}_t^* + \omega\,\hat{s}_t$$

where ω is a parameter[7] increasing in openness and zero in complete autarky. Once again, this represents the distortion implied by terms of trade differences: if terms of trade are depreciated, home goods are relatively cheaper for all the agents, increasing demand for Home-goods by a factor ω[8]. Note that $\omega = \omega^*$ this effect is symmetric on Home and Foreign.

We know from the previous section that "Home goods" are consumed by the "World-household" with a share[9] $\{(1-\alpha)\,\gamma_{\mathrm{C}} + \alpha^*\,\gamma_{\mathrm{C}^*}\}$. As we will need it in a latter section, we introduce a ***new notation*** with the parameter $\Xi^H \equiv (1-\alpha)\,\gamma_{\mathrm{C}} + \alpha^*\,\gamma_{\mathrm{C}^*}$ and $\Xi^F \equiv (1-\alpha^*)\,\gamma_{\mathrm{C}^*} + \alpha\,\gamma_{\mathrm{C}}$.

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[6] For example for Home consumption: $\mathscr{S}_t^{W_{H,C}} \equiv (1-\alpha)\mathcal{S}_t^{\alpha} + \alpha\mathcal{S}_t^{1-\alpha}$, and other functions are similar.

[7] This parameter is defined as $\omega \equiv (1-\alpha)\alpha\,\gamma_{\mathrm{C}} + (1-\alpha^*)\alpha^*\,\gamma_{\mathrm{C}^*} + (1-\alpha_{\mathrm{G}})\,\alpha_{\mathrm{G}}\,\gamma_{\mathrm{G}} + (1-\alpha_{\mathrm{G}}^*)\,\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}$

[8] For example, if the Home-household is completely open, the terms of trade appreciation ($\hat{s}_t$ negative) will dampen the home-good demand, both in quantity (large share of foreign goods consumed) and price (demand schedule strongly affected by relative prices)

[9] A share $(1-\alpha)\,\gamma_{\mathrm{C}}$ domestically and $\alpha^*\,\gamma_{\mathrm{C}^*}$ imported by the Foreign household.

Using this new parameter, we make another ***assumption*** **(A2)**. Because of the risk-sharing condition, the two households – in Home and Foreign – should consume the same amount, modulo a term-of-trade difference. Therefore, the share of home-good Ξ^H in the world-household basket should be equivalent to the share of Home-consumption i.e. $\gamma_C = {}^{C}/_{Y^w}$ as described above (and similarly for the Foreign country: $\Xi^F = \gamma_{C^*}$). Manipulating the terms, we obtain necessary the following restriction $\alpha^* \gamma_{C^*} = \alpha \gamma_C$. The intuition behind this constraint is an *equal size private-goods trade* between the two countries *at steady-state*. The two countries should consume the same amount of imported consumption goods at steady-state. If one country run a trade surplus for some time, he will save more, and consume more in the future (Home and Foreign) goods. In absence of other shocks, this adjustment will bring trade back to a "balanced" equilibrium. This second condition on parameters (implied by the risk-sharing assumption), is relatively stronger, but it is still valid to preserve heterogeneity.

Euler equations and Philipps Curve

We will recall and extend the main equations driving demand and supply in such a New-Keynesian model. First, the dynamic (forward-looking) ***Euler equations*** at Home[10] and at the Union-level[11] rewrites:

$$\hat{c}_t = \mathbb{E}_t(\hat{c}_{t+1}) - (i_t - \mathbb{E}_t(\pi_{H,t+1}) - \rho) + \alpha\, \mathbb{E}_t(\Delta \hat{s}_{t+1}) \tag{1}$$

$$\hat{c}_{w,t} = \mathbb{E}_t\, \hat{c}_{w,t+1} - \{(\gamma_C + \gamma_{C^*})[i_t - \rho] - (\Xi^H\, \mathbb{E}_t\, \pi_{H,t+1} + \Xi^F\, \mathbb{E}_t\, \pi_{F,t+1})\}$$

Aside the usual determinants of consumption (real interest rate, future consumption), the Home-consumption is also driven by terms-of-trade depending on openness α. Furthermore, the union-level equation, which is a key relation for Union-central-bank mandate shows the standard Euler equation, showing union-level inflation with weights Ξ^H and Ξ^F (i.e. share in the union consumption bundle).

Second, we turn to the supply-side and ***Philipps-Curve***. Using descriptions of output, technology and labor-consumption trade-off, we obtain the formula determining marginal cost for producing Home-goods[12]:

$$mc_t = \ln\left(\frac{1-\tau}{1-\Gamma}\right) + \left[1 + 2\Xi^H \varphi\right] c_t + 2\varphi\left[(1-\alpha_G)\,\gamma_G\, g_t \;+\; \alpha_G^*\,\gamma_{G^*}\, g_t^*\right] + \left[\alpha + 2\varphi(\omega - \alpha^*\,\gamma_{C^*}(1-\alpha-\alpha^*))\right] s_t - (1+\varphi)a_t$$

[10] And similarly in the Foreign country, using Foreign parameters and an opposite sign for terms-of-trade

[11] Aggregating simply with: $\hat{c}_{w,t} \equiv \gamma_C\ \hat{c}_t + \gamma_{C^*}\ \hat{c}_t^*$

[12] Again, this formula hold symmetrically for the foreign country, changing parameter values, and sign for s_t

In this relation, marginal cost is function of real wages, as households request higher wages when they can consume more or work supplementary hours. As a result, aggregate demand has a direct effect on marginal costs: higher consumption at Home or Abroad, higher government spending or terms-of-trade depreciation, through a rise of export, will all imply a higher work effort and disutility of labor (by a factor φ). Another indirect effect of terms-of-trade depreciation appears through the "relative price" of real wage, especially in "open" economy (high α). Concretely, wages are chosen by workers on account of CPI level, increasing in terms-of-trade. Therefore, the more open the country and the more depreciated the relative prices, the higher the requested wages and costs for firms, compared to producer level $P_{H,t}$. Finally, productivity and employment subsidy both reduce marginal costs as in the usual setting.

To obtain inflation dynamics, it is possible to replace the previous formula in the expression for inflations:

$$\pi_{H,t} = \beta\ \mathbb{E}_t\, \pi_{H,t+1} + \chi\Big[\ln\big(\frac{1-\tau}{1-\Gamma}\big) + \Big[1 + 2\Xi^H\varphi\Big]\, c_t + 2\varphi\big\{(1-\alpha_G)\,\gamma_G\, g_t \ +\ \alpha_G^*\,\gamma_{G^*}\, g_t^*\big\} \\ + \big[\alpha + 2\varphi(\omega - \alpha^*\,\gamma_{C^*}(1-\alpha-\alpha^*))\big]\, s_t - (1+\varphi)a_t\Big] \tag{2}$$

This latest formula is the ***New Keynesian Philipps-Curve***, reflecting the forward-looking price setting behavior of firms. Inflation reacts to common macro variables, through their impact on marginal cost – by a coefficient χ – i.e. which is higher with more flexible prices. In particular, the NKPC is vertical when prices are fully-flexible (i.e. $\theta \to 0$ and $\chi \to \infty$). This relation holds with a similar expression for the foreign country, changing the parameters and reverting the sign for terms-of-trade. We can also build the Union-wide Philipps Curve, by the aggregation of the two inflations (cf. section section 4).

Equilibrium determination

We have a set of equilibrium relations, both at country-level, and union level: (i)-(iii) Euler equations and risk-sharing condition determining the two countries consumption, (iv)-(vi) the New Keynesian Philipps Curves at Home, Foreign and for the union, (vii) the definition of terms-of-trade as inflation differential between the two countries. Therefore, we obtain seven linear relations, with seven endogenous variables $\pi_{w,t}, \pi_{H,t}, \pi_{F,t}, c_t, c_t^*, c_{w,t}, s_t$ given the path of the exogenous productivity shocks a_t, a_t^* and policy variables of central bank i_t and fiscal authorities g_t and g_t^*. The following sections will formalize the optimal allocation for these policy variables.

3 Efficiency and optimal policy

In this section, I follow closely the normative analysis made in Galí and Monacelli (2008). The aim is to determine the socially-optimal allocation of resources in a no-distortion-economy. We first analyze the decision of a Social Planner weighting the two countries equally – what we call a *cooperating optimum* – and then the decision of two Social Planners, one for each country – what we call a *Nash-bargaining allocation*. Third, we show how the cooperating equilibrium can be decentralized in a flexible-price economy. This cooperating optimum will constitute the benchmark for policy analysis in presence of nominal rigidities.

Social-planner allocation – cooperating optimum

The cooperating optimal allocation is solving the social planner's problem. The union's planner seeks to maximize the weighted sum of Household utility $\omega_H U_H(C_t, N_t, G_t) + \omega_F U_F(C_t^*, N_t^*, G_t^*)$ subject to both resource constraints of the two countries, $Y_{J,t} = C_{J,t} + C_{J,t}^* + G_{J,t} + G_{J,t}^*$ for $J = H, F$ and technological constraint: $Y_{J,t} = A_t^J\, N_t^J$

Drawing the constrained optimization problem, we derive optimality conditions as the allocation equalizing the marginal disutility of producing a good with the marginal utility of consuming the good by the four agents (as public or private good, and at Home or Foreign). This implies: $-\frac{\partial U_W}{\partial Y_H} = \frac{\partial U_W}{\partial C_H} = \frac{\partial U_W}{\partial C_H^*} = \frac{\partial U_W}{\partial G_H} = \frac{\partial U_W}{\partial G_H^*}$ and similarly for the foreign good. We obtain eight conditions, along with the constraints, and we obtain uniquely the allocation:

$$C_{H,t} = \omega_H\,(1-\Gamma)(1-\alpha)\,A_t \eta^{-\frac{\varphi}{1+\varphi}} \qquad C_{F,t}^* = \omega_F\,(1-\Gamma^*)(1-\alpha^*)\,A_t^*\,\eta*^{-\frac{\varphi*}{1+\varphi*}}$$

$$C_{H,t}^* = \omega_F\,(1-\Gamma^*)\alpha^*\,A_t\,\eta^{-\frac{\varphi}{1+\varphi}} \qquad C_{F,t} = \omega_H\,(1-\Gamma)\alpha\,A_t^*\,\eta*^{-\frac{\varphi*}{1+\varphi*}}$$

$$G_{H,t} = \omega_H\,\Gamma(1-\alpha_G)\,A_t\,\eta^{-\frac{\varphi}{1+\varphi}} \qquad G_{F,t}^* = \omega_F\,\Gamma^*(1-\alpha_G^*)\,A_t^*\,\eta*^{-\frac{\varphi*}{1+\varphi*}}$$

$$G_{H,t}^* = \omega_F\,\Gamma^*\,\alpha_G^*\;A_t\,\eta^{-\frac{\varphi}{1+\varphi}} \qquad G_{F,t} = \omega_H\,\Gamma\,\alpha_G\,A_t^*\eta*^{-\frac{\varphi*}{1+\varphi*}}$$

$$N_t = \eta^{\frac{1}{1+\varphi}} \qquad \eta = \omega_H\,(1-\Gamma)(1-\alpha) + \omega_F\,(1-\Gamma^*)\alpha^* + \omega_H\,\Gamma(1-\alpha_G) + \omega_F\,\Gamma^*\,\alpha_G^*$$

$$N_t^* = \eta^{*\,\frac{1}{1+\varphi}} \qquad \eta^* = \omega_F(1-\Gamma^*)(1-\alpha^*) + \omega_H\,(1-\Gamma)\alpha + \omega_F\,\Gamma^*(1-\alpha_G^*) + \omega_H\,\Gamma\,\alpha_G$$

We define a new parameter η (and resp. η^*) as "preferences" for Home (resp. Foreign) goods. The important point here is that ***labor supply*** is only function of structural parameters (this new η and labor disutility φ) and is consequently a function of the "tastes" of the four agents (Home/Foreign, Household/Government).

As a result, the optimal allocation of ***consumption*** in *each country* relies on the preferences of *both countries.* For example, Home consumption of Home goods $C_{H,t}$ depends on Foreign country tastes α^* and α_G^* (by household or government) through N_t and η. If these preferences are "high", for instance due to "cultural" reasons, labor supply adjusts to satisfy both Households, and in particular as exports to Foreign. Because of increasing marginal labor disutility, this will, in some ways, *"crowd-out"* Home-consumption. Such phenomenon would not appear without heterogeneous preferences (i.e. $\eta \neq \eta^*$) as the import/export motives will cancel out due to symmetry. Furthermore, such "crowding-out effect" of consumption is strengthened with an inelastic labor supply (and a small φ).

If we assume "perfect" coordination, with a social-planner weighting equally the two countries ($\omega_H = \omega_F = 1$), we could contrast our result with Galí and Monacelli (2008). As in their framework, the diverse aggregates are determined by productivity (A_t or A_t^*). However, in our general setting, the optimal level of output, and by extension consumption, public good provision and terms-of-trade, relies heavily on "preferences" η and labor rigidity φ.

The terms-of-trade term $\mathcal{S}_t$ is worth commenting: in this cooperating optimum, if the tastes differ between Home and Foreign ($\eta \neq \eta^*$), the ratio displays $\mathcal{S}_t$ a long-run ***price differential*** between the two countries, *even in absence of productivity differential.* This permanent "unbalanced" terms-of-trade arise in a cooperating equilibrium especially because agents interiorize both Home and Foreign resource and technological constraints.

To end this section, we need to know if this cooperating allocation is consistent with the hypothesis of ***"union-risk-sharing"***, as discussed in point 9 of the previous section. Recall that we obtained $C_t = \vartheta_0 \, C_t^* \, \mathcal{S}_t^{1-\alpha-\alpha^*}$. In this case, the value of the "risk-sharing-wedge" ϑ_0 should be consistent with both a long-run terms-of-trade level and efficient allocation.
This yields a wedge of the form: $\vartheta_0 = \frac{1-\Gamma}{1-\Gamma^*}$, weighting relative preferences for private goods between the two countries (essentially because risk-sharing happens only between Households, but not governments). Therefore, if $\Gamma > \Gamma^*$, the Home-household have higher preference for public goods, and $\vartheta_0 < 1$ the private consumption will always be higher in the foreign country than at Home, consistent with intuitions as well as structural parameter heterogeneity.

Social-planner allocation – Nash-bargaining allocations

We now consider an allocation where the two countries do not coordinate. Formally, this implies the existence of two social planners, each of them maximizing the utility of the Household in "its" country, taking into account its resource and technological constraints, and the one of the world economy. The main result of this section is that there exists an infinite number of allocations that are optimal from this point of view. The main point to remember is that, in this setting, the demand schedule for Home and Foreign goods may not interiorize the resource/technology constraints of both Home and Foreign countries. As a result, this may cause misallocation of public goods – relatively to the cooperating case – because of productivity differentials.

Decentralization and flexible-price equilibrium

We want to know how the union can reach (cooperating-)optimality in a decentralized equilibrium. In an equilibrium under flexible price, i.e. absent nominal rigidities (and denoted with upper bar), monopolistic competition is the only remaining distortion. For the production to reach optimum level, marginal cost should equal firm's mark-up, i.e.:

$$\mathcal{M}^{-1} = \frac{\epsilon - 1}{\epsilon} = \overline{MC_t} = (1-\tau)\frac{\overline{W_t}}{\overline{P_t}}\,(\overline{S_t})^{\alpha}\,\frac{\overline{N_t}}{\overline{Y_{H,t}}}$$

Replacing all the variables known and rearranging, we now suppose that (i) employment subsidy counterbalances monopolistic competition by setting $\tau = \frac{1}{\epsilon}$ and (ii) governments provide the optimal level of public good, as described above. As a result, we obtain:

$$\eta = (1-\Gamma)(1-\alpha) + (1-\Gamma^*)\,\alpha^* + \Gamma(1-\alpha_G) + \Gamma^*\,\alpha_G^*$$

which is exactly the definition of the term η we stated above. Note that, performing exactly symmetrical computation for the Foreign country, we obtain an analogous result.

Therefore, *decentralized equilibrium can reach optimal allocation.* What do we learn then on the link between preferences η and firms' marginal costs? First, decentralization is optimal only when both governments purchase the right amount of public goods and do not "*crowd out*" private consumption by "overloading" the two countries' labor supply. As we detailed above, public goods provision must account for preferences η and labor disutility φ. In such situation, Households set appropriately their labor supply and marginal cost naturally equals firms mark-up. Under the two conditions concerning employment subsidy and optimal allocation of public goods, the flexible-price equilibrium reaches optimality, as shown in Galí and Monacelli (2008).

Before returning to the log-linearized version of our model, we find more convenient to make ***another assumption* (A3)**. To simplify the result, we make the hypothesis that $\eta = \eta^* = 1$. Without this assumption, structural heterogeneity can lead to persistent differences in output, labor supply, consumptions and terms-of-trade, implying a persistent increase in the market share of one country. This is inconsistent with (A1) the assumption of equal size we made in the previous section and to the idea of log-linearization around a symmetric steady-state (where $\mathcal{S}_t = 1$). Therefore, we will *rule out this case*, and set $\eta = \eta^* = 1$. Rearranging the terms, we obtain a new constraint on the structural parameters for preferences:

$$(\alpha^* - \alpha) + \Gamma(\alpha - \alpha_{\mathrm{G}}) = \Gamma^*(\alpha^* - \alpha^*_{\mathrm{G}})$$

This equation makes the preferences between the two countries closer, i.e. more homogenous. Said differently, Home and Foreign Households should not have "too different" tastes in order to impose unitary labor supply and equal size in each country.

What intuition lies behind this assumption? If asymmetry exists in one parameter, for example, if private consumption home-bias is much stronger at Home than in Foreign ($\alpha < \alpha^*$), it should be offset by other heterogeneities: either Foreign household should have high preferences for public goods and higher Home-bias in public goods, or Home Household have lower preferences for Public goods. Without such condition, then either Households will not be satisfied with the binding resources constraint – because of the unitary labor supply – or the optimal allocation exceed the limited capacity of one country. As a consequence, this condition will impose that trade-balance is *null at steady-state*. For example, a trade surplus of private consumption goods should be balanced by a trade deficit of public consumption.

This constraint is automatically satisfied in two common situations: (i) the two households have the same home-bias ($\alpha = \alpha^*$) and government buys only domestic goods ($\alpha_{\mathrm{G}} = \alpha^*_{\mathrm{G}} = 0$) – case met almost-everywhere in the open-economy literature – or (ii) the two households have the same home-bias ($\alpha = \alpha^*$), the same preferences for public goods ($\Gamma = \Gamma^*$), and each government have the same Home-bias ($\alpha_{\mathrm{G}} = \alpha^*_{\mathrm{G}}$). We argue here that we can still preserve heterogeneity in preferences and keep such constraint satisfied, in order to analyze how heterogeneity affects transmission of fiscal shocks, when the two policies have diverging optimal allocation paths.

With this simplification, it is easy to rewrite optimal allocation, which is now much closer to the one described in Galí and Monacelli (2008)

$$
\begin{aligned}
&C_t = (1-\Gamma)\, A_t^{1-\alpha}\, A_t^{*\,\alpha} && C_t^* = (1-\Gamma^*)\, A_t^{*\,(1-\alpha^*)}\, A_t^{\alpha^*} \\
&G_t = \Gamma\, A_t^{1-\alpha_{\rm G}}\, A_t^{*\,\alpha_{\rm G}} && G_t^* = \Gamma^*\, A_t^{*\,1-\alpha_{\rm G}^*}\, A_t^{\alpha_{\rm G}^*} \\
&\mathcal{S}_t = {}^{A_t}/_{A_t^*} && Y_{H,t} = A_t \qquad Y_{F,t} = A_t^* \qquad N_t = N_t^* = 1
\end{aligned} \tag{3}
$$

To summarize, we recall the diverse assumptions we made throughout this model: Assumption (A1) sets equal country size, Assumption (A2) – due to risk-sharing condition – implies a private current account balance $\gamma_{\rm C}\, \alpha = \gamma_{\rm C^*}\, \alpha^*$; Assumption (A3) – setting unitary structural preferences and steady-state labor supply – implies a public-private goods trade balance $(\alpha^* - \alpha) + \Gamma(\alpha - \alpha_{\rm G}) = \Gamma^*(\alpha^* - \alpha_{\rm G}^*)$. We shall insist on the fact that these assumptions: first do not prevent decentralization to reach optimal allocation as argued above ; second, are compatible with a "risk-sharing wedge" $\vartheta_0 = \frac{1-\Gamma}{1-\Gamma^*}$ and, third, do not eliminate heterogeneous preferences – in particular *Home-bias* – between the two countries.

Moreover, when governments provide the efficient levels of public goods, absent productivity shocks, the optimal size of government in the two countries corresponds to Household preferences $\frac{G}{Y_H} = \Gamma = 2\ \gamma_{\rm G}$ and $\frac{G^*}{Y_F} = \Gamma^* = 2\ \gamma_{\rm G^*}$. However, adding nominal rigidities complicates the analysis of optimal policy.

4 Nominal rigidities and policy tradeoffs

In the previous section, we determined the flexible-price optimal allocation – expressed mainly in terms of productivity and Households preferences. In presence of nominal rigidities however, interactions between fiscal and monetary authorities can bring about a number of policy trade-offs. Moreover, in the two heterogeneous countries, the dynamics of output and employment are changed due to price stickiness and wage-bargaining and therefore terms-of-trade adjustment. To analyze the trade-offs and spillovers created by supply and policy shocks, we introduce a (standard) notation. We define a ***"gap"*** as the log-linear approximation of the deviation from efficient allocation: $\widetilde{z}_t = \hat{z}_t - \hat{\bar{z}}_t = z_t - \bar{z}_t$ where in the first equality $\hat{z}_t$ is the deviation from steady-state, $\hat{\bar{z}}_t$ the efficient (optimal allocation) deviation from steady-state, and $z_t, \bar{z}_t$ correspond to log-variables in the second equation. Therefore, we have for example the *output gap* defined as $\widetilde{y}_{H,t} = \hat{y}_{H,t} - \hat{\bar{y}}_{H,t}$, *government spending gap*: $\widetilde{g}_t = \hat{g}_t - \hat{\bar{g}}_t$ or consumption gap: $\widetilde{c}_t = \hat{c}_t - \hat{\bar{c}}_t$.

Policy trade-offs at country level

We can now express the equilibrium relations to underline the policy trade-offs each economy faces under sticky prices. First, we start with inflation dynamics. Considering firms' marginal cost, we know that it equals firm's mark-up in flexible-price equilibria. Subtracting the efficient level from the previous formula ($\widehat{mc_t} = mc_t - \overline{mc_t} = mc_t + \mu$) and thanks to the relation between inflation and marginal cost, we obtain a redefinition of the New Keynesian ***Phillips Curve***:

$$\pi_{H,t} = \beta\ \mathbb{E}_t\ \pi_{H,t+1} + \chi\left[1 + 2\Xi^H\ \varphi\right]\widetilde{c}_t + 2\chi\varphi\left\{(1-\alpha_G)\,\gamma_G\,\widetilde{g}_t\ +\ \alpha_G^*\,\gamma_{G^*}\,\widetilde{g}_t^*\right\}+ \\ \chi\left[\alpha + 2\varphi(\omega - \alpha^*\,\gamma_{C^*}(1-\alpha-\alpha^*))\right]\widetilde{s}_t \tag{4}$$

This new version of the NK-Philipps curve[13] is showing the implication aggregate demand – through consumption, public goods and terms-of-trade gaps – on domestic inflation. If consumption is "abnormally high" – when, say, consumer prices are lower than efficient level – this demand generates higher employment and workers request higher real wages. As a result this encourage firms to reset their price at a higher level, leading to inflation. In our setting, an "over"-depreciated terms-of-trade – i.e. $\widetilde{s}_t > 0$ – leads to the same upward inflationary pressure. This feature is absent from most "small-open-economy" models, where each country is infinitesimal. Here, depreciated terms-of-trade have a direct impact on foreign aggregate demand.

Now, we come back to the ***Euler equation*** to see the feedback effect of inflation and gaps on consumption. We can easily rewrite the formula in terms of gaps, subtracting efficient level of consumption on both side:

$$\widetilde{c}_t = \mathbb{E}_t\,\widetilde{c}_{t+1} - (i_t - \mathbb{E}_t\,\pi_{w,t+1} - r^{nat}) - \frac{\gamma_{C^*}(1-\alpha-\alpha^*)}{\gamma_C + \gamma_{C^*}}\ \mathbb{E}_t\,\Delta\widetilde{s}_{t+1} \\ r_t^{nat} = \rho + \frac{1}{\gamma_C + \gamma_{C^*}}\left[\Xi^H\ \mathbb{E}_t(\Delta\bar{a}_t) + \Xi^F\ \mathbb{E}_t(\Delta\bar{a}_t^*)\right] \tag{5}$$

where we express the average inflation $\pi_{w,t} \equiv \frac{1}{\gamma_C + \gamma_{C^*}}(\Xi^H\ \mathbb{E}_t\,\pi_{H,t} + \Xi^F\ \mathbb{E}_t\,\pi_{F,t})$ and obtain the natural interest rate. This corresponds to the rate prevailing in an economy where all gaps are closed: if both consumption gaps and public spending gaps were closed and without inflation, the natural interest is the only equilibrium consistent with an output level constantly equal to natural level[14]

[13] Again, this formula holds for the Foreign country:
$\pi_{F,t} = \beta\ \mathbb{E}_t\ \pi_{F,t+1} + \chi^*\left[1+2\Xi^F\ \varphi\right]\widetilde{c}_t^* + 2\chi^*\varphi^*\left\{(1-\alpha_G^*)\,\gamma_{G^*}\,\widetilde{g}_t^* + \alpha_G\,\gamma_G\,\widetilde{g}_t\right\} - \chi^*\left[\alpha^* + 2\varphi^*(\omega - \alpha\,\gamma_C(1-\alpha-\alpha^*))\right]\widetilde{s}_t$

[14] It does not give households incentives to over-consume (as it reacts one-to-one with productivity) and no firms are willing to adjust their prices. Note that it equals the optimal consumption: $r_t^{nat} = \rho + \frac{1}{\gamma_C + \gamma_{C^*}}\,\hat{\bar{c}}_{w,t}$.

Policy trade-offs at union-level

We now turn to Union-level trade offs, especially relevant for union-wide monetary authorities. To obtain the Union-level ***NK-Philipps Curve***, we just have to average the two inflation rates:

$$\pi_{w,t} = \beta\, \mathbb{E}_t\, \pi_{w,t+1} + \Xi^H \big(\tfrac{\chi}{\gamma_C + \gamma_{C^*}} + \xi_h\big)\, \widetilde{c}_t + \Xi^F \big(\tfrac{\chi^*}{\gamma_C + \gamma_{C^*}} + \xi_f\big)\, \widetilde{c}_t^* \qquad (6)$$

$$+ \gamma_G \left\{(1-\alpha_G)\xi_h + \alpha_G\, \xi_f\right\} \widetilde{g}_t + \gamma_{G^*} \left\{(1-\alpha_G^*)\xi_f + \alpha_G^*\, \xi_h\right\} \widetilde{g}_t^*$$

$$+ \Big[\tfrac{\alpha\, \gamma_C}{\gamma_C + \gamma_{C^*}}(\chi - \chi^*) + (\xi_h - \xi_f)(\omega - \alpha\, \gamma_C(1 - \alpha - \alpha^*))\Big]\widetilde{s}_t$$

where we defines the new parameters: $\xi_h \equiv \frac{2\Xi^H \chi\varphi}{\gamma_C + \gamma_{C^*}}$ and $\xi_f \equiv \frac{2\Xi^F \chi^*\varphi^*}{\gamma_C + \gamma_{C^*}}$ to make the formulas (slightly) lighter. It represents "supply-side" factors : labor disutility, price flexibility and the relative weight of the good produced in the union-economy.

This equation is one of the most important of our model[15]. The lessons to be learned are the following: a surge in demand from public spending, consumption or terms-of-trade, have a greater impact on union-wide inflation through the country which has the "most *inflationary*" supply-side factors i.e. the *highest parameter* ξ. Said differently, the greater the price-flexibility, the stickier the labor markets and the higher the share of domestic good in consumer baskets, the stronger will be the inflationary pressure on the union. Moreover, in our version of the model, the last term is new. When countries are heterogeneous, the terms-of-trade *differential* may affect the inflation *average* of the union: additional heterogeneity (e.g. higher ξ_h) implies a new transmission channel from terms-of-trade depreciation ($\widetilde{s}_t > 0$) to union inflation (higher π_w).

Turning back to the dynamic *IS-equation*, we express average consumption gaps is terms of interest rate:

$$\widetilde{c}_{w,t} = \mathbb{E}_t\, \widetilde{c}_{w,t+1} - \{(\gamma_C + \gamma_{C^*})[i_t - r^{nat}] - (\Xi^H\ \mathbb{E}_t\, \pi_{H,t+1} + \Xi^F\ \mathbb{E}_t\, \pi_{F,t+1})\} \qquad (7)$$

This relation is not really new and is only the aggregation of the consumption of both country. However, even though we kept heterogeneity in most of the parameters, there are no relative terms in this relation, only aggregate terms. As a result, aggregate productivity shock will not have any asymmetric transmission effect, even in presence of structural heterogeneity.

[15]Even though it is the longest!

In conclusion, it has to be highlighted that government spending can only play a role through inflation. Consumption/saving decisions are not related to government policies – as there is no distortive taxes in this setting. However, as shown in the Philipps Curve, government purchases imposes an extra demand, boosting employment, wage bargaining and raising marginal costs and inflation. In a second step, this inflation have an influence on consumption and terms-of-trade, initiating differential between the two countries with the presence of heterogeneity.

In contrast, Central Bank can have a direct impact on consumption of agents, by manipulating the union interest rate. Households interiorize the monetary reaction: they expect the monetary policy to be more or less aggressive vis-a-vis inflation. As a result, they are influenced in their consumption, even though interest rate does not change! This effect will be at the center of policy spillovers, i.e. the responses of Foreign variables after shocks to Home variables. This analysis will be the topic of the following section.

5 Optimal fiscal and monetary policy with nominal rigidities

In this section and the next, we derive and compute the optimal policy, i.e. the monetary-fiscal policy mix that would maximize the Union household welfare. For policy evaluation, we derive a *welfare criterion*, i.e. the second-order Taylor approximation of Household utility function in each country. We then consider Utilitarian welfare function aggregating equally the two utilities. However, if the welfare function provides some insights, the full characterization of the *optimal policy under commitment* is relatively heavy[16] and provides almost no relevant economic intuitions. Therefore, we choose not the expose it here. Instead, we choose to expose how fiscal and monetary policy can replicate the optimal policy using *simple monetary and fiscal rules.*

Welfare loss function

The second order approximation of the utility function of both Households yields the following synthetic expression, with *t.i.p.* the terms independent of policy (namely productivity shocks) and $\mathcal{O}(\|\xi\|^3)$ terms of third or higher order:

[16]Seven additional (and really long...) equations, including five non-instructive Lagrange multipliers, for each equilibrium relations of the model

$$\mathbb{W} = \sum_{t=0}^{\infty} \beta^t \; \mathbb{E}_t(\mathbb{L}_t^w) \; + \; t.i.p. \; + \; \mathcal{O}(\|\xi\|^3) \tag{8}$$

$$\mathbb{L}_t^w = \tfrac{\epsilon}{2\,\chi}\,\pi_{H,t}^2 + \tfrac{\epsilon}{2\,\chi^*}\,\pi_{F,t}^2 \; + \; \gamma_{\mathrm{C}}\;\widetilde{c}_t^{\,2} \; + \; \gamma_{\mathrm{C}^*}\;\widetilde{c}_t^{*\,2} + \gamma_{\mathrm{G}}\;\widetilde{g}_t^{\,2} + \gamma_{\mathrm{G}^*}\;\widetilde{g}_t^{*\,2} + \omega\,\widetilde{s}_t^{\,2}$$
$$(\widetilde{c}_t^{w,pol})^2 \; + \; (\widetilde{g}_t^{w,pol})^2 \; + \; (\widetilde{s}_t^{w,pol})^2 \; + \; \widetilde{c}_t^{w,pol} \cdot \widetilde{g}_t^{w,pol} \; + \; \widetilde{c}_t^{w,pol} \cdot \widetilde{s}_t^{w,pol} \; + \; \widetilde{g}_t^{w,pol} \cdot \widetilde{s}_t^{w,pol}$$

where the terms denoted by "*w,pol*" are defined as weighted average of variables at Home and in the Foreign country. They are represented using vectors (and quadratic terms are thus product of two vectors) and account for the countries heterogeneity, in particular agents tastes and labor disutility. As a result, we defined "*w,pol*", the "policy average" as follow:

$$\widetilde{c}_t^{w,pol} \equiv M_l \begin{pmatrix} \widetilde{c}_t^{w,H} \\ \widetilde{c}_t^{w,F} \end{pmatrix} \qquad \widetilde{g}_t^{w,pol} \equiv M_l \begin{pmatrix} \widetilde{g}_t^{w,H} \\ \widetilde{g}_t^{w,F} \end{pmatrix} \qquad \widetilde{s}_t^{w,pol} \equiv M_l \begin{pmatrix} 2\,\omega\,\widetilde{s}_t \\ -2\,\omega\,\widetilde{s}_t \end{pmatrix}$$

where $\widetilde{c}_t^{w,H/F}$ (resp. $\widetilde{g}_t^{w,H/F}$ and $\pm 2\,\omega\,\widetilde{s}_t$) are the gaps in union-consumption (resp. public spending and terms-of-trade) that affects supply of Home/Foreign goods. Therefore, differences in labor supply elasticity are included in M_l. More formally:

$$M_l \equiv \begin{pmatrix} \sqrt{2\,\varphi} & 0 \\ 0 & \sqrt{2\,\varphi^*} \end{pmatrix} \qquad \begin{cases} \widetilde{c}_t^{w,H} \equiv \gamma_{\mathrm{C}}\,(1-\alpha)\,\widetilde{c}_t + \gamma_{\mathrm{C}^*}\,\alpha^*\,\widetilde{c}_t^* \\ \widetilde{c}_t^{w,F} \equiv \gamma_{\mathrm{C}^*}(1-\alpha^*)\,\widetilde{c}_t^* + \gamma_{\mathrm{C}}\,\alpha\,\widetilde{c}_t \end{cases} \qquad \begin{cases} \widetilde{g}_t^{w,H} \equiv \gamma_{\mathrm{C}}\,(1-\alpha_{\mathrm{G}})\,\widetilde{g}_t + \gamma_{\mathrm{G}^*}\,\alpha_{\mathrm{G}}^*\,\widetilde{g}_t^* \\ \widetilde{g}_t^{w,F} \equiv \gamma_{\mathrm{C}^*}(1-\alpha_{\mathrm{G}}^*)\,\widetilde{g}_t^* + \gamma_{\mathrm{G}}\,\alpha_{\mathrm{G}}\,\widetilde{g}_t \end{cases}$$

Welfare loss features quadratic terms and cross products, in inflation (i), in consumption and public spending (ii), in weighted average of consumption and government spending (iii), and in terms-of-trade (iv). We use a notation using matrices, slightly more convenient and synthetic[17].

Inflations (i) appears in the loss function, as usual in the New-Keynesian framework because price dispersion distorts households consumption. The factor for inflation is increasing in substitutability across goods (ϵ) and in price rigidity (i.e. decreasing in χ).

All the remaining terms show aggregate demand effects: "gaps" increase welfare loss through labor demand, implying workers' disutility. (ii) The first terms in consumption, government spending and terms-of-trade gaps affect labor supply of both countries equally, according to their weights in the economy (c.f. first line terms, with factors $\gamma_{\mathrm{C}}, ..., \omega$). These terms can be gathered in simple "output gaps", rather standard in the literature.

However, another set of terms (iii), that we define as "policy-weighted average" (e.g. $\widetilde{c}_t^{w,pol}$)

[17] In particular, with vectors, it is easier to express cross-products in terms of union-level variable. However, we misused the notation and we should more rigorously note that quadratic terms $\widetilde{c}_t^{w,pol}$ are in fact product of matrices, i.e. $(\widetilde{c}_t^{w,pol})^2 = (\widetilde{c}_t^{w,pol})^T\,(\widetilde{c}_t^{w,pol})$

shows more intricate linkages between demand of the two countries: they depends on demand effects direction toward Home or Foreign (i.e. $\widetilde{c}_t^{w,H}$ vs. $\widetilde{c}_t^{w,F}$), private sector size or government size (i.e. $\gamma_{\mathrm{C}}, \gamma_{\mathrm{G}}$), openness (i.e. $\alpha, \alpha_{\mathrm{G}}$), and labor supply elasticity (i.e. φ, φ^*), included in M_l. These variables are quadratic in consumption, in government spending gaps (the first two terms denoted by "*w,pol*") or feature cross-product interactions between the two sets of aggregate demand (the fourth term of the second line). In one way or another, these terms appear in most of the welfare objectives highlighted in the open-economy literature.

However – and this is our last point – terms-of-trade gap (iv) should deserve a closer look. In small-open-economy models, terms-of-trade gap does not appear in the welfare objective of the Union due to the infinitesimal size of each country. The terms-of-trade distortions cancel each other when aggregating, as described in Galí and Monacelli (2008). However, in a two countries model, spillovers and feedback effects have a significant impact, as shown throughout this model, and the difference in our welfare function (due to the more general setting) lies in these interaction terms. In Beetsma and Jensen (2005), there is an interaction between government spending gap and terms-of-trade, but not between consumption gap and terms-of-trade. Why? Mainly because they set the model without Home-bias (cf. the literature review). Perfect openness and risk-sharing lead to full equalization of consumption, canceling out such terms. Therefore, we show that Home-bias can bring about union welfare loss due to this an interaction term between consumption gap and terms-of-trade gap.

In conclusion, our formulation of the Union Welfare Loss is more general that in the rest of the literature. It includes country heterogeneity. Therefore, if the policymaker consider accurately Household utility – and especially when it comes about labor supply – what matters is not the simple average of aggregate demand. The policy-relevant welfare criteria is an average *weighted by Home-bias and by labor supply elasticity.* The benevolent social planner will choose the appropriate policy minimizing this loss function.

Optimal policy - characterization and simple rules

To provide an analytical formulation for optimal policy in this environment, we should solve the linear-quadratic optimization problem, i.e. minimizing the second-order Loss function, eq. (8), subject to the first-order structural equations (domestically: two NK Phillips Curves in eq. (4), two IS/Euler equations in eq. (5), and the two at Union level, in eq. (6) and in eq. (7)). This optimization problem over the full set of policy instruments determines the optimal path of central bank interest rate and both country government spending. However, the mathematical characterization is relatively heavy and without important insights. Therefore, we choose not to show it here. Following the simulation (cf. below), there are several main lessons to remember.

First, *Central Bank* considers the union as a whole when setting the interest rate, but weighted for countries heterogeneity. This result is similar to Benigno (2004) – focusing on price flexibility (here χ) – but another important parameter in our model is the degree of labor disutility φ, representing labor supply responsiveness to demand shocks. The Taylor principle still holds and central bank still reacts "more than one-to-one" to union inflationary pressure.

Second, the *two fiscal policies* should stabilize "relative" prices and output distortions. In particular, the main goal of government spending in this setting is to counteract aggregate demand changes that induce an "overloading" of the labor market of one country. Therefore, when a country faces output gaps, fiscal policy should act in order to close it, but without causing inflation at the Union level.

Third, from the previous comments, we could replace the optimal characterization by three simple rules: for central bank and for the two fiscal authorities. This "fiscal rule" replication is accurate (less than 10^{-10} differences), and the path of the main variables is perfectly replicated. The monetary rule is a simple rule to follow the natural interest rate, react to inflation – standard in the New Keynesian literature. The fiscal policy rule, however, is aimed at reacting to terms-of-trade gaps (here each country should react to half of it) and consumption gaps:

$$
\begin{aligned}
i_t &= r_t^{nat} + \phi_\pi \, \pi_{w,t} \\
\widetilde{g}_t &= -{}^1\!/_2 \, \psi_s \, \widetilde{s}_t - \psi_c \, \widetilde{c}_t \\
\widetilde{g}_t^* &= +{}^1\!/_2 \, \psi_s \, \widetilde{s}_t - \psi_c \, \widetilde{c}_t^*
\end{aligned}
$$

However, country structural heterogeneity, as we will see in section 8 makes the separation between aggregate and relative stabilization much less clear: fiscal policy act against a negative asymmetric supply shock and such policy can be transmitted to the other country through relative-prices and trade, triggering a rising interest rate from Central Bank. As a consequence, these two policies should work hand-in-hand.

6 Technology shocks, optimal policy and simple rules

To illustrate the model dynamics, we first run Impulse Response Functions (IRF), using Dynare software, (cf. Adjemian et al. (2011)). The economy is hit by an exogenous ***productivity (TFP) shock*** ε_t^a affecting an AR(1) technological process a_t and this shock is asymmetric, *hitting only the Home country.* We simulate the model with three type of policy responses: (1) a simple reaction of monetary policy to inflation and a *passive* fiscal policy (i.e. $\widetilde{g}_t = \widetilde{g}_t^* = 0$), (2) the optimal policy of central bank and fiscal policy, under commitment, and (3) the replication of optimal policy by three simple rules, as described above. The result is shown in fig. 1.

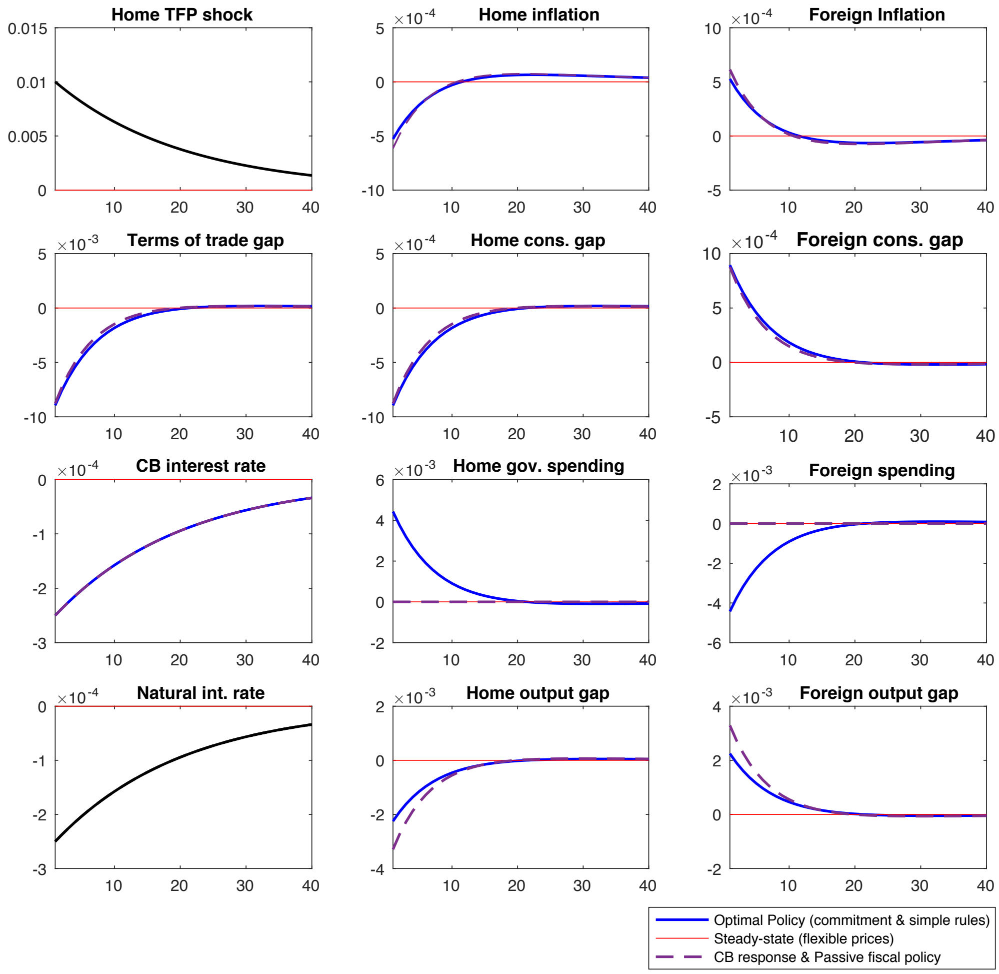
Before that, a word on the model ***calibration***. We use standard value for structural parameters. For now, we consider a *symmetric* equilibrium, where all the parameters are equal between Home and the Foreign country. We choose a *strong rigidity in price setting*, i.e. $\theta = 0.91$. Only 9 % of firms reset their prices (average price duration: 11 quarters). This is equivalent to a very flat slope for the Philipps Curve, i.e. $\chi = 0.009$. This low level is consistent with structural estimations made in Euro-area – cf. Smets and Wouters (2007). Such a slope for NKPC allows us to highlight the demand effect implied by sluggish price adjustment. We can differentiate between autarkic and "open" government purchases, and both calibration yield an import-to-GDP ratio of around 30%, which is consistent with the data of most large Euro Area economies.

Table 1: Baseline – *symmetric* – structural parameters value

β	0.995025	Discount factor	*Autarkic government*		
ρ	0.005	Riskless return per period	α	0.4	Openness in private consumption
φ	3	Inverse Frisch elasticity	α_G	0	Openness in public consumption
χ	0.009	Slope of NKPC	ω	0.18	Output–terms-of-trade elasticity
θ	0.91165	Price stickiness			
ρ_a, ρ_f	0.95	Persistence of shocks			
μ	1.2	Firm's markup	*"Open" government*		
ϵ	6	Elasticity of substitution across goods	α	0.35	Openness in private consumption
Γ, Γ^*	0.25	Preference for public goods	α_G	0.25	Openness in public consumption
γ_C, γ_{C^*}	0.375	Private consumption share	ω	0.2175	Output–terms-of-trade elasticity
Ξ^H, Ξ^F	0.375	Union demand for national good			
γ_G, γ_{G^*}	0.125	Government size			

The result of the simulation is shown in the figure below. A productivity shock, increasing the output of the (Home) economy is not followed by a rise in consumption. In our setting, the *terms-of-trade gap is also falling*, because relative prices do not adjust as fast as productivity difference: the Home terms-of-trade is over-appreciated compared to efficient level (i.e. $\widetilde{s}_t < 0$). As Home economy accounts for a large weight of the Union, central bank reacts to the change in productivity, mimicking the decreasing natural interest rate. This drop, added to the terms-of-trade gap, *boosts Foreign consumption* above the efficient level and therefore causes Foreign inflation. Note that the mechanisms described above do not appear in small-open economy models: union-level variables would not change due to the infinitesimal weight for each country.

Figure 1: Response to a 1% productivity shock



Technology shock follows a AR(1) process, Central bank reacts to inflation under the rules detailed in table 2, and fiscal policy is either passive or follows optimal policy/fiscal rule

From the insights detailed in the previous section, what are the gains from optimal policy? The monetary policy is similar in the two situations, and therefore, fiscal policy can constitute another tool for stabilization. The role of government spending is to "reroute" aggregate demand from the Foreign to the Home country. This prevents Foreign labor markets to be "overloaded" and it reduces the output gap and the welfare loss of the union. Moreover, we compute the welfare gains and the fiscal multiplier from this experiment. The fiscal policy stabilization can reduce the welfare loss four-fold, from 8% consumption-equivalent to 2%. These results are computed in the table 2: we compute the union welfare loss for each experiment in terms of consumption equivalent. Moreover, we compute the fiscal multiplier for the fiscal policy as the integral of output gaps[18] over the integral of fiscal gap.

$$\frac{\Delta Y_t}{\Delta G_t} \approx \frac{\frac{Y_h}{Y_w} \sum_{t=1}^{T} \widetilde{y}_t}{\frac{G}{Y_w} \sum_{t=1}^{T} \widetilde{g}_t} \qquad \textit{Fiscal multiplier} \tag{9}$$

In this experiment, the fiscal multiplier is rather small – around 0.68 – meaning that for 1 unit of public spending, output rises by 0.68 on average. This ratio is smaller than 1 mainly because consumption is slightly crowded-out after the shock hits (more on this in the next section). We agree that the quantitative result (in terms of welfare and fiscal multipliers) is highly dependent on the parameter calibration. Therefore, in fig. 6 in appendix, we provide numerical computation of those values in function of three key parameters: openness α, labor supply elasticity φ and price flexibility χ. Even though a complete analysis is impossible here, we could note three simple facts: (i) the size of the fiscal multiplier does change much with openness, as would conclude "small open economy" models, but welfare loss is much greater in autarky, (ii) complete price rigidity ($\theta > 0.97$) raises the spending multiplier much closer to one, (iii) labor inelasticity can undermine greatly the effectiveness of government spending. We now turn to the main results of this model: what would be transmission mechanisms from a government spending shock?

Table 2: Results: Spillovers of shocks, Welfare loss and Fiscal multipliers

Type of shock	Monetary response	Fiscal policy response	Welfare loss	Fiscal multiplier
TFP shock	*Inflation rule*: $\phi_\pi = 1.5$	Passive	0.083	NA
TFP shock	Optimal policy (commitment)	Optimal policy (commitment)	0.026	0.678
TFP shock	*Inflation rule*: $\phi_\pi = 1.5$	*Fiscal rule*: $\psi_s = 0.5$ & $\psi_c = 2.5$	0.026	0.678
Fiscal shock	*Inflation rule*: $\phi_\pi = 2$	Passive (after the shock)	0.342	0.414
Fiscal shock	*Taylor-rule*: $\phi_\pi = 1.1$ & $\phi_c = 0.5$	Passive (after the shock)	0.347	0.676
Fiscal shock	*Inertial-Taylor-rule*: $\phi_\pi = 1.1$, $\phi_c = 0.5$	Passive (after the shock)	0.360	0.650

∘ Central Bank follows a simple policy rule: $i_t^* = r_t^{nat} + \phi_\pi \, \pi_{w,t} + \phi_c \, \widetilde{c}_{w,t}$
Or it may react with inertia: $i_t = \rho_i \, i_{t-1} + (1 - \rho_i) \, i_t^*$ (with $\rho_i = 0.95$)
∘ Fiscal policy is either passive: $\widetilde{g}_t = 0$ or follows a simple policy rule: $\widetilde{g}_t = -1/2 \, \psi_s \, \widetilde{s}_t - \psi_c \, \widetilde{c}_t$ (and $\widetilde{g}_t^* = +1/2 \, \psi_s \, \widetilde{s}_t - \psi_c \, \widetilde{c}_t^*$)
∘ Welfare loss is expressed as percent of consumption
∘ Government spending multiplier is measured as the ratio of integrals of output gaps over spending gaps
∘ Baseline calibration: countries Home and Foreign are symmetric.

[18] More precisely, we compute the *difference* in output gaps with or without fiscal policy intervention: the output gap is negative in the two cases but greatly reduced when government spending is active.

7 Transmission of fiscal shocks

We should now analyze the transmission mechanisms from one country's fiscal policy. We again run the model, using the *symmetric* calibration using above. Now, the economy is hit by a ***fiscal spending shock*** ε_t^f to the AR(1) "fiscal disturbance" process f_t^ε that shifts up the national level of public spending above its efficient level, i.e. $\widetilde{g}_t > 0$.

The economy response follows the model dynamics, but the spillover can be very different depending on the policy response. First, we suppose that beside the fiscal spending shock, the *fiscal policy is passive*: In the two countries, government are only concerned by closing the public spending gap, i.e. $\widetilde{g}_t = \widetilde{g}_t^* = 0$ and securing the efficient level of public goods.

Second, we mainly consider the case where government purchase is "autarkic", i.e. purchase only good from domestic production, with a complete Home-bias ($\alpha_G = \alpha_G^* = 0$). At the end of this section, we briefly consider the case of "open" government purchase to explain the main differences in terms of spillovers.

Third, the central monetary authority follows a simple *monetary rule* for setting the interest rate. We will differentiate the response depending on the rules:

$$i_t = r_t^{nat} + \phi_\pi\, \pi_{w,t} \qquad \textit{Inflation rule}$$

$$i_t = r_t^{nat} + \phi_\pi\, \pi_{w,t} + \phi_c\, \widetilde{c}_{w,t} \qquad \textit{Taylor rule}$$

$$i_t = \rho_i\, i_{t-1} + (1-\rho_i)\,(r_t^{nat} + \phi_\pi\, \pi_{w,t} + \phi_c\, \widetilde{c}_{w,t}) \qquad \textit{Inertial (Taylor)-rule}$$

We will differentiate alternative cases: Central Bank can react only to inflation ($\phi_\pi = 2$, following the Taylor principle and $\phi_c = 0$) – we call it *inflation averse* central bank. Monetary policy can also react to both inflation and consumption gap (less strongly: $\phi_\pi = 1.1$ and $\phi_c = 0.5$), i.e. the well-known *Taylor rule*. We lastly consider the case of inertia in monetary policy, i.e. $\rho_i = 0.95$, and we call it *inertial response*. We highlight the differences between these three cases in the following impulse response functions and the results in terms of welfare loss and fiscal multiplier, as defined in eq. (9), are summarized in table 2 above.

The 1% increase in government spending raises aggregate demand and there is a direct impact on Home inflation through labor and wage bargaining, pushing marginal cost higher (cf. the NKPC). This simply triggers the Union-wide inflation and central bank reaction. Depending on the central bank mandate, the dynamics are the following:

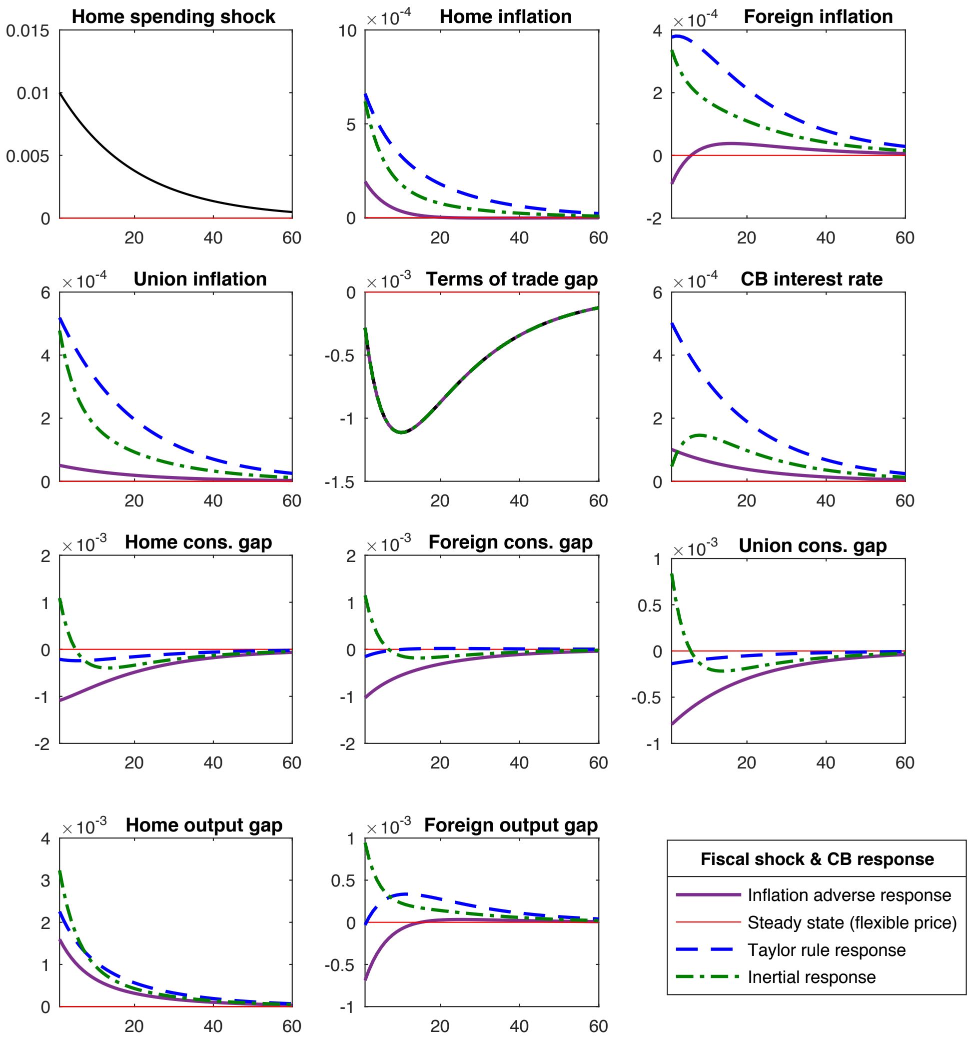
(i) If the central-bank is *inflation-adverse*: More than the effective interest rate hike, the key mechanism is the "threat" of interest rate rise, i.e. the strong hawkish central bank behavior (high ϕ_π) which is at the root of the Taylor principle. Therefore, on the short-run, inflationary government spending results in a sharp drop of consumption – a "crowding-out" effect – common in New-Keynesian models. However, the main difference here is that union-wide inflation – which stays low at the union level around 0.05 % – and the strong monetary reaction affects *both* Households, inducing a large (1%) union-wide negative consumption gap. The Foreign household is hurt by the central bank response: highly depressed demand (consumption and output) and deflation are the short-run spillovers of this fiscal policy shock. On the medium run, after the price adjustment, Home inflation appreciates terms-of-trade and Foreign goods are relatively cheaper. This shifts aggregate demand from Home to Foreign goods and this trade rebalancing creates inflationary pressures on the Foreign country, leading to a term-of-trade reversal. To summarize, under this setting, the spillovers are mainly negative due to monetary reaction and it is difficult to conclude that Foreign country can "benefit" from Home government spending.

(ii) When the monetary policy follows a *Taylor-rule*: The Central Bank mandate now includes both inflation and consumption gap, and even though the Union Inflation is ten time higher, the union consumption is seven time smaller. The crowding-out effect in consumption is significantly smaller, and it does not dampen Foreign consumption as much as before, nor does it imply deflation. Note that all the differences appear at the Union-level. In particular, terms-of-trade dynamics are identical. Interestingly, the higher inflation and smaller consumption gap are implied by the "Taylor-rule" mandate, but the Central Bank effectively implement a sharp increase in interest rate – five times higher than before. The key point is the internalization of the central bank mandate by consumers. Again, as prices adjust – Home inflation and terms-of-trade appreciation – Home consumption gap widen even more, because of Home-bias, and aggregate demand is redirected toward Foreign goods, leading to the same terms-of-trade reversal.

To conclude, the change in the central bank mandate is key to understand how the Foreign country benefit from Home fiscal shock. A more "balanced" mandate between consumption and inflation prevents deflationary episode and implies positive output gap for the Foreign country.

(iii) When central bank policy reacts with *inertia*: The slow pace of interest rate adjustment on the short-run changes the dynamics, and this time, there *is no consumption drop* at the Union level. Even more, the inflationary pressure and the slow pace of terms-of-trade adjustment lead them to increase their consumption: as aggregate demand shift toward Foreign goods, inflation rise. After several periods, with the adjustment of interest rate, both consumption and inflation drop, especially for Home. As a result, the Foreign Household benefit again from the spending shock when Central Bank takes time to adjust: for a similar level of inflation, the Foreign consumption and output gaps are positive.

Figure 2: Response to a 1% spending shock



Public spending shock follows a AR(1) process, Central bank reacts to inflation under the rules detailed in table 2, and fiscal policy is passive after the shock, i.e. $\widetilde{g}_t = \widetilde{g}_t^* = 0$

In this type of model, the transmission of fiscal policy across the currency union depends exceedingly on central bank mandate. This is also the case when we *differentiate between **"autarkic"** and **"open"** government spending.* Previously, government spending was "autarkic", purchasing only domestic goods, with a complete Home-bias ($\alpha_G = \alpha_G^* = 0$). When considering an "open" government ($\alpha_G = \alpha_G^* = 0.25$, almost as open as private consumption), the main difference is the extended transmission of inflation. Consequently, government demand affects directly the Foreign NK-Philipps Curve. However, beside this minor difference, the dynamics are very much alike. Concerning the role of central bank reaction for spillover effects, the main insight is analogous.

8 Structural heterogeneity: a rationale for policy cooperation

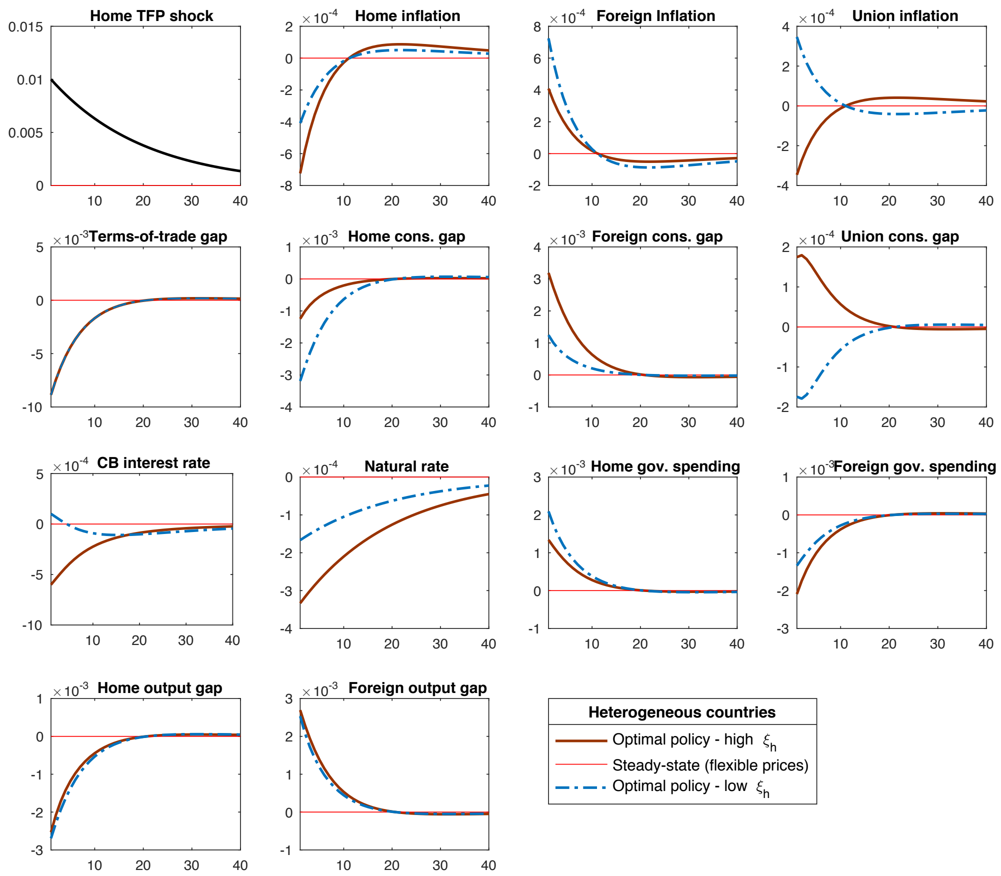
Our model presents heterogeneity in most parameters. Studying how it would affect spillovers of Productivity or Government spending shocks is a long and monotonous task. Each parameter may change slightly one particular channel, but a meticulous review would be mostly redundant. However, some of the structural parameters – what we can call "supply-side factors" – are acting in the same direction. In union-NKPC eq. (6), we collected them in a new parameter ξ_h for Home and ξ_f for Foreign, defining it as $\xi_h \equiv \frac{2\Xi^H \chi \varphi}{\gamma_C + \gamma_{C^*}}$ (and $\xi_f \equiv \frac{2\Xi^F \chi^* \varphi^*}{\gamma_C + \gamma_{C^*}}$). It represents how structural factors, such as labor supply elasticity, price rigidity and relative taste for Home/Foreign good, affects inflation through the New-Keynesian Philipps Curve. More rigorously, low Frisch elasticity (high φ), more flexible prices (low θ, high χ) and higher tastes for private goods (high Ξ^H), all increases ξ_h, and, as a result, inflation at Home is relatively more reactive than in the Foreign country. Such difference will have important implications for policy, and we try to provide a succinct view on this issue. We use the following calibration[19].

Table 3: Comparison and *heterogeneity* in structural parameters value

Parameter	High value of ξ	Low value of ξ	Signification
φ	4	2	Inverse Frisch elasticity
χ	0.012	0.007	Slope of NKPC
θ	0.898	0.922	Price stickiness (measure of firms who can't reset prices)
Ξ^i	0.5	0.25	Union demand for Home good
α	0.2	0.3	Openness in private consumption
ξ_i	0.064	0.0093	Inflationary effect of supply-side factors $\xi_i \equiv \frac{2\Xi^i \chi \varphi}{\gamma_C + \gamma_{C^*}}$

[19]This calibration is motivated by empirical estimates: Frisch elasticity $1/\varphi$ vary between 0.25 and 0.5, while estimate of the NKPC slope χ vary between 0.008 and 0.014. Our range of parameter value for calibration (0.007-0.012) can be justified by the absence of wages rigidities in this model

Figure 3: Response to a 1% productivity shock, comparison with two heterogeneous countries



TFP follows a AR(1) process. Countries have heterogeneous structural parameters, summarized by $\xi_h \equiv \frac{2\Xi^H \chi \varphi}{\gamma_C + \gamma_{C^*}}$. Monetary and fiscal policies are optimal (commitment). Results are shown in table 4

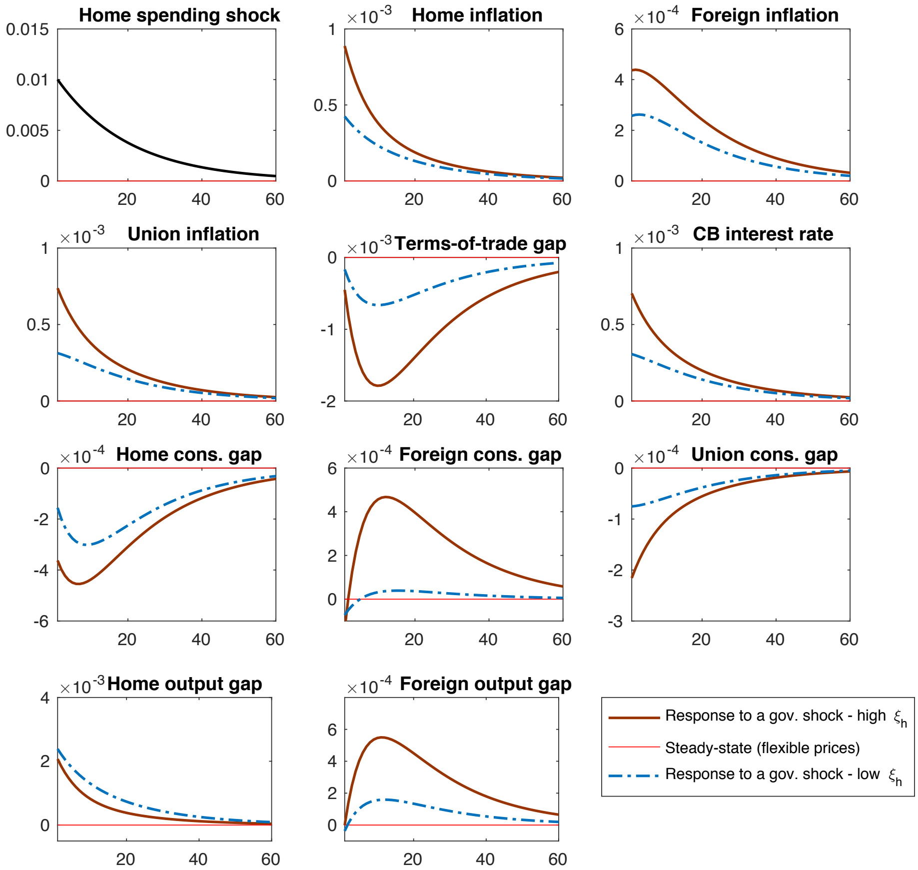
How heterogeneity affects the transmission of an asymmetric shock

As we discussed above, an asymmetric shock can have a very different impact on aggregate variables depending on structural parameters. As we observe, an asymmetric productivity shock have spillover effects on Union inflation and output gap and prevent central bank from following Natural interest rate. We show two cases: in the first (a), Home economy has a high ξ_h and is more affected by price changes: the deflationary episode it experiences after the supply-shock have a faster influence on Union prices, which also adjust downward. Facing deflation at Union-level, monetary policy reacts by cutting interest rate close to two time below the natural rate. A contrario, in (b) when the Foreign country is more subject to inflationary pressure, the Union inflation is pushed upward following the Home supply shock. This is because the terms-of-trade gap worsen in favor of the Foreign country, which attracts the union aggregate demand.

Therefore, the central bank is forced to raise policy rate, which is highly damaging for Home consumption gap – three time lower than in the previous case. As we see, the central bank response has changed with country heterogeneity, even though the main dynamics are similar to the ones in the previous section.

As a second experiment, we study what are the spillovers of an asymmetric fiscal spending shock. Similarly, we see the two cases in the next graph. Here the difference is mostly quantitative. Rather intuitively, the inflationary effect of the government spending increases with ξ, i.e. it is stronger in the first case (a) than in (b). More, the terms-of-trade gap in favor of the Foreign country is much largely emphasized with high ξ, which benefit more or less the Foreign country in terms of positive consumption gap and inflation.

Figure 4: Response to a 1% asymmetric gvt. spending shock, comparison with two heterogeneous countries



Public spending shock follows a AR(1) process. Countries are heterogeneous on structural parameters, summarized in $\xi_h \equiv \frac{2\Xi^H \chi \varphi}{\gamma_C + \gamma_{C^*}}$. Central bank policy follows a Taylor rule and fiscal policy is passive. Results are shown in table 4

Table 4: Results: Spillovers of shocks, Welfare loss, Fiscal multipliers and country heterogeneity

Type of shock	Parameter ξ		Monetary	Fiscal policy	Welfare loss	Fiscal multiplier	
	Home	Foreign	response	response		Home	Foreign
Home TFP shock	High (0.064)	Low (0.0093)	Inflation rule	Passive	0.012	NA	NA
Home TFP shock	High (0.064)	Low (0.0093)	*Optimal policy*	*Optimal policy*	0.010	0.956	0.3734
Home TFP shock	Low (0.0093)	High (0.064)	*Optimal policy*	*Optimal policy*	0.010	0.3734	0.956
Home fiscal shock	High (0.064)	Low (0.0093)	Taylor rule	Passive	0.038	0.643	NA
Home fiscal shock	Low (0.0093)	High (0.064)	Taylor rule	Passive	0.034	0.617	NA
Union fiscal shock	High (0.064)	Low (0.0093)	Taylor rule	Passive	0.080	0.770	0.883

Notes: ∘ Central Bank follows a simple policy rule: $i_t^* = r_t^{nat} + \phi_\pi \, \pi_{w,t} + \phi_c \, \widetilde{c}_{w,t}$
Or it may react with inertia: $i_t = \rho_i \, i_{t-1} + (1-\rho_i) \, i_t^*$ (with $\rho_i = 0.95$)
∘ Fiscal policy is either passive: $\widetilde{g}_t = 0$ or follows the optimal policy under commitment.
∘ Welfare loss is expressed as percent of consumption
∘ Government spending multiplier is measured as the ratio of integrals of output gaps over spending gaps
∘ Calibration: Home (resp. Foreign) have either "low" (resp. high) or "high" (resp. low) parameter value ξ_h (resp. ξ_f)

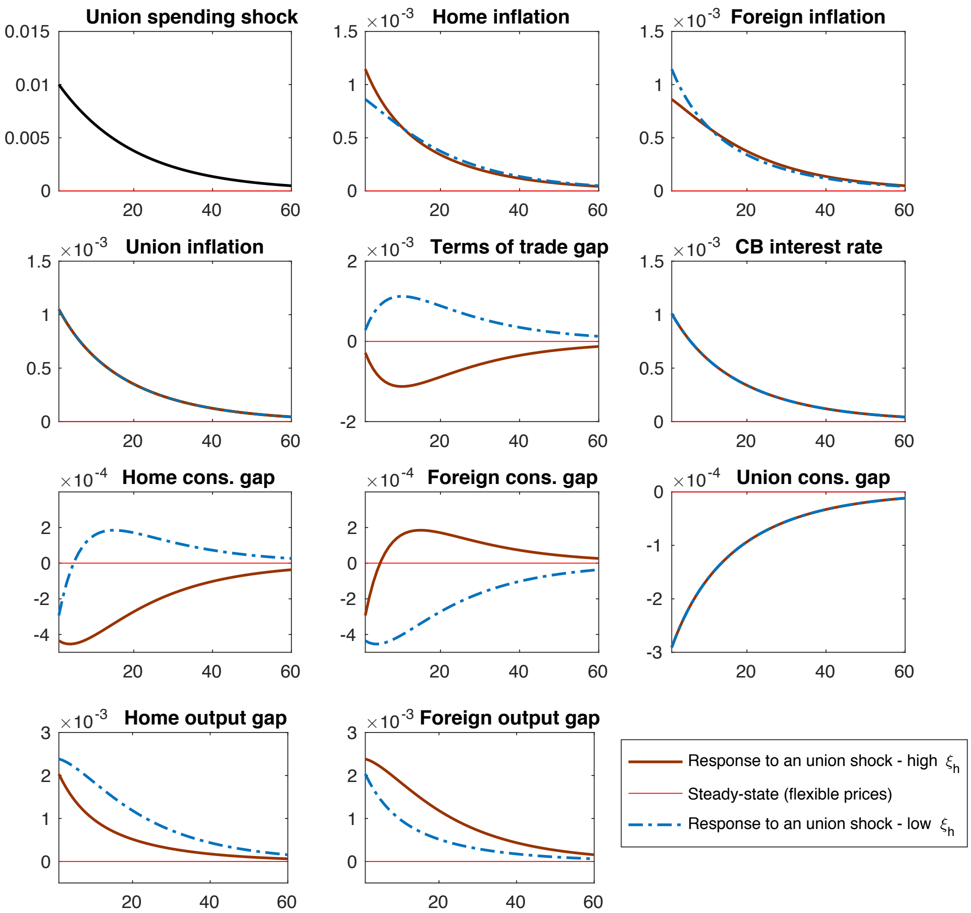
How heterogeneity affects the transmission of an aggregate shock

This model was well suited to study asymmetric shock. However, heterogeneity in structural parameters provides the two economies with different resilience to a common aggregate shock, which, in turn, results in asymmetric spillovers. We should again study two types of common shock: a productivity shock and spending shock.

Even though we do not show the graph, we argue that a common aggregate shock does not produce asymmetric spillovers. As we described in eq. (7), the response to an aggregate shock is simply a drop in natural interest rate and Central Bank nominal rate. This has an impact on the Union-level Dynamic-IS equation, and this affects consumption at Home and Abroad equally. Therefore, there is no spillover on inflation, neither in the individual countries, nor at the Union-level. This is once more rooted in the Taylor principle, (and the "treat" of Central Bank anti-inflation mandate). This is analogous to the simple "one country" NK model.

However, concerning a common (demand) fiscal common shock, the situation is very different. We simulate a common fiscal shock, or equivalently a rise in government purchases affecting both countries equally: the size of government is the same, and both fiscal shock are "autarkic": AR(1) processes out of a passive fiscal mandate. This kind of experiment could be relevant, for example, if the Currency Union is involved in a political conflict and should rise military spending in all its region. The consequence is an asymmetric diffusion of inflation, higher in the region with the highest parameter ξ_i. As we note in the following graph, when ξ_h is high, inflation is higher at Home, even though government spending is the same, and this distorts terms-of-trade for the benefit of the Foreign country, with a positive consumption gap.

Figure 5: Response to a 1% aggregate government spending shock, analysis with heterogeneity



Spending shock follows a AR(1) process and hits both economies. Countries are heterogeneous on structural parameters, summarized in $\xi_h \equiv \frac{2\Xi^H \chi \varphi}{\gamma_C + \gamma_{C^*}}$. CB follows a Taylor rule and fiscal policy is passive.

Fiscal policy experiments – conclusion

There are plenty of policy experiments to be implemented. Concerning asymmetric fiscal shocks, beside various types of spillovers, common characteristics remain. The general story is the following: a fiscal shock brings about inflation and central bank reaction, worsening consumption gap in the two countries. In different cases, the spillovers on Foreign consumption gap can be positive: in such situation, Central Bank necessarily follows a rule that accounts for consumption gap. When countries are heterogeneous, labor elasticity, openness and price flexibility change quantitatively the transmission of inflation through terms-of-trade: Foreign country' inflation and consumption are much severely affected when structural parameter ξ_f is high. All these experiments, as well as fig. 6 in appendix provide a clearer picture of the different effect on welfare and fiscal multipliers and cross-country spillovers.

9 Conclusion

In this master thesis, we provided a two country DSGE model with "large economies" in a monetary union. Our New-Keynesian framework emphasized the transmission channels due to interactions between the two economies, their fiscal policies and a common central bank. In particular, we allow for Home bias in consumption of private and public goods as well as for heterogeneity in preferences and structural "supply-side" factors (share of goods production, labor elasticity, price flexibility).

From this very general setting, we extracted many different results: we first determined the cooperating optimal allocation at the first-best: private consumption and public spending should follow the productivity of the two countries, weighted by structural preferences. Second, when nominal rigidities exists, there exists policy trade-off, in terms of consumption gaps, inflation and terms-of-trade gap. We determined how to achieve optimal policy by the mean of simple rules: inflation rule for central bank policy and reaction to terms-of-trade and consumption gap for fiscal authorities. Third, this environment was suitable to study many different experiments. We analyzed the various implications of productivity and government spending shocks, and spillover effects from one large country to the other. The reaction of the central bank is key in this mechanism. An asymmetric positive supply shock (at Home), followed by a cut in the union interest rate boost consumption and inflation in the Foreign country. An asymmetric spending shock, a contrario, can create depression and deflation abroad if the central bank is strongly inflation. Taylor-rule mandate, inertia in interest rate, "open"-government spending and heterogeneity in country reliance to inflation can all induce more inflationary spillover in the Foreign country. Finally, our model presented heterogeneity in most parameters and we could study different policy implications. We could briefly note that fiscal policy effectiveness – through government spending multipliers – can be greater when one country is more sensitive to inflationary pressures.

This last observation brings about a key question: *when one country experiences a supply or demand shock, what are the gains from policy cooperation?* We studied welfare gains when fiscal policy was optimal – substantial gains due to labor inelasticity and trade imbalances between the two countries. However, we did not study non-cooperative situations. For example, if Home country experiences a supply shock, is the Foreign country willing to "help" the Home country for stabilizing this shock? or would it "deviates" and let Home fiscal policy hold all the burden of the fiscal adjustment? If we are able to compare the diverse outcomes, the answer to this question would require an extended game-theoretical framework and in particular dynamic games and recursive contracts. We consider this question relevant and would like to pursue on such questions in the future.

There are plenty of different directions for future research: if the model included rather heavy notations and cumbersome equilibrium relations (for the optimal policy) our framework still abstracted from many features: realistic labor markets ; incomplete financial markets where fiscal union could act as an insurance tool ; public sector with government debt and distortive taxation, emphasizing the costs of policy intervention, and finally credit constraint and imperfect consumption/saving decisions by households or firms. We plan to study some of these topics in the future, in the context of monetary unions.

However, with our simple framework, we could describe many policy implications: we showed the various transmission channels of supply and public demand shock, through central bank reaction and trade spillovers, and how heterogeneity between the two countries could transform an "asymmetric" issue – e.g. terms-of-trade or relative consumption gap – into an "aggregate" one. This is a strong case for macroeconomic coordination between the national fiscal authorities and the union central bank. We believe this article provide a clearer view on these topics and can shed light on several policy issues in the context of European Union.

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Appendices

Structural parameters – heterogeneity and restrictions

In this model, we refer to a number of structural parameters. The definitions and how the diverse hypothesis constrained the range of their value is referenced in the following list:

- Demand parameters:

  α Share of foreign goods in Household consumption bundle, indice of "openness" $[\underline{\alpha \neq \alpha^*}]$; α_G Share of foreign goods in government spending, indice of "government openness" $[\underline{\alpha_G \neq \alpha_G^*}]$; Γ Preference of Household for public goods: $[\underline{\Gamma \neq \Gamma^*}]$; ϵ Elasticity of substitution across varieties of goods

  $\gamma_C, \gamma_{C^*}, \gamma_G, \gamma_{G^*}$ respectively, share of Home consumption, Foreign consumption, Home government spending, Foreign government spending, in union aggregate demand ;

  $\omega \equiv (1-\alpha)\alpha\,\gamma_C + (1-\alpha^*)\alpha^*\,\gamma_{C^*} + (1-\alpha_G)\,\alpha_G\,\gamma_G + (1-\alpha_G^*)\,\alpha_G^*\,\gamma_{G^*}$. It represents how demand for Home-goods increases with a terms-of-trade depreciation ($s_t > 0$), and by symmetry it also shows how demand for Foreign goods is dampened following the same depreciation. $[\omega = \omega^*]$ It increases in openness of the different agents, and equals zero in complete autarky ;

  $\Xi^H \equiv (1-\alpha)\,\gamma_C + \alpha^*\,\gamma_{C^*}$ and $\Xi^F \equiv (1-\alpha^*)\,\gamma_{C^*} + \alpha\,\gamma_C$ Share of Home/Foreign-goods in union consumption

- Supply parameters:

  φ Disutility of labor, inverse of the Frisch Elasticity $[\underline{\varphi \neq \varphi^*}]$; θ Degree of price stickiness $[\underline{\theta \neq \theta^*}]$; χ How inflation reacts to a rise in marginal costs: $\chi = (1-\theta)(1-\beta\theta)/\theta$, decreasing in price stickiness and infinite in flexible prices $[\underline{\chi \neq \chi^*}]$

  $\eta = (1-\Gamma)(1-\alpha) + (1-\Gamma^*)\alpha^* + \Gamma(1-\alpha_G) + \Gamma^*\,\alpha_G^*$ represents agent preferences for Home goods, and as a result, is related to the structural labor supply at optimum $N = \eta^{\frac{\varphi}{1+\varphi}}$ and similarly for the foreign country: $\eta^* \equiv (1-\Gamma^*)(1-\alpha^*) + (1-\Gamma)\alpha + \Gamma^*(1-\alpha_G^*) + \Gamma\,\alpha_G$

  $\xi_h \equiv \frac{2\Xi^H \chi\,\varphi}{\gamma_C + \gamma_{C^*}}$ and similarly $\xi_f \equiv \frac{2\Xi^F \chi^*\,\varphi^*}{\gamma_C + \gamma_{C^*}}$ represents "supply-side" factors influencing the resilience of a country to inflationary demand shocks.

- Parameters restrictions – assumptions

  1. Equal size of the two countries, $Y = Y^* = 1/2\, Y_w$, therefore shares of the different part of the economy at steady-state are constrained as follow:

  $$\begin{aligned} 1/2 = \gamma_C(1-\alpha) + \gamma_{C^*}\,\alpha^* + \gamma_G\,(1-\alpha_G) + \gamma_{G^*}\,\alpha_G^* &= \Xi^H + \gamma_G\,(1-\alpha_G) + \gamma_{G^*}\,\alpha_G^* \\ = \gamma_{C^*}(1-\alpha^*) + \gamma_C\,\alpha + \gamma_{G^*}\,(1-\alpha_G^*) + \gamma_G\,\alpha_G &= \Xi^F + \gamma_{G^*}\,(1-\alpha_G^*) + \gamma_G\,\alpha_G \end{aligned}$$

  2. Risk-sharing condition implies that consumption is equalized, modulo terms-of-trade differences. Holding at steady-state we therefore have this null long-run trade balance.

  $$\gamma_C = \Xi^H \quad \& \quad \gamma_{C^*} = \Xi^F \qquad \text{this implies} \qquad \alpha^*\,\gamma_{C^*} = \alpha\,\gamma_C$$

  3. Structural preferences yield an unitary labor supply η, we thus have a necessary condition for preference homogeneity of the form:

  $$(\alpha^* - \alpha) + \Gamma(\alpha - \alpha_G) = \Gamma^*(\alpha^* - \alpha_G^*)$$

Equilibrium relations with policy trade-offs

- Definition of terms of trade gaps:

$$\Delta\widetilde{s}_t = \pi_{F,t} - \pi_{H,t} - (\Delta a_t - \Delta a_t^*)$$

- Evolution of exogenous productivity shocks as AR(1) process

$$a_{t+1} = \rho_a\, a_t + \varepsilon_t^a \qquad \& \qquad a_{t+1}^* = \rho_a\, a_t^* + \varepsilon_t^{a^*}$$

- Evolution of exogenous fiscal shocks as AR(1) process

$$f_{t+1}^{\varepsilon} = \rho_f\, f_t^{\varepsilon} + \varepsilon_t^f \qquad \& \qquad f_{t+1}^{*\varepsilon} = \rho_f\, f_t^{*\varepsilon} + \varepsilon_t^{f*}$$

- Evolution of monetary policy and natural interest rates:

$$i_t = \rho_i\, i_{t-1} + (1-\rho_i)\,(r_t^{nat} + \phi_\pi\, \pi_{w,t} + \phi_c\, \widetilde{c}_{w,t})$$

$$r_t^{nat} = \rho + \tfrac{1}{\gamma_C + \gamma_{C^*}}\big[\Xi^H\ \mathbb{E}_t(\Delta\bar{a}_{t+1}) + \Xi^F\ \mathbb{E}_t(\Delta\bar{a}_{t+1}^*)\big]$$

- Fiscal policy rules

$$\widetilde{g}_t = -{}^1\!/\!{}_2\psi_s\, \widetilde{s}_t - \psi_c\, \widetilde{c}_t + f_t^{\varepsilon} \qquad \& \qquad \widetilde{g}_t^* = +{}^1\!/\!{}_2\psi_s^*\, \widetilde{s}_t - \psi_c^*\, \widetilde{c}_t^* + f_t^{*\varepsilon}$$

- Domestic New Keynesian Philipps Curve equation, at Home and in the Foreign country:

$$\pi_{H,t} = \beta\ \mathbb{E}_t\, \pi_{H,t+1} + \chi\left[1 + 2\Xi^H\varphi\right]\widetilde{c}_t + 2\chi\varphi\big\{(1-\alpha_G)\,\gamma_G\,\widetilde{g}_t\ +\ \alpha_G^*\,\gamma_{G^*}\,\widetilde{g}_t^*\big\} + \chi\left[\alpha + 2\varphi(\omega - \alpha^*\,\gamma_{C^*}(1-\alpha-\alpha^*))\right]\widetilde{s}_t$$

$$\pi_{F,t} = \beta\ \mathbb{E}_t\, \pi_{F,t+1} + \chi^*\left[1 + 2\Xi^F\varphi^*\right]\widetilde{c}_t^* + 2\chi^*\varphi^*\big\{(1-\alpha_G^*)\,\gamma_{G^*}\,\widetilde{g}_t^*\ +\ \alpha_G\,\gamma_G\,\widetilde{g}_t\big\} - \chi^*\left[\alpha^* + 2\varphi^*(\omega - \alpha\,\gamma_C(1-\alpha-\alpha^*))\right]\widetilde{s}_t$$

- Union-level Philipps Curve

$$\pi_{w,t} = \beta\ \mathbb{E}_t\, \pi_{w,t+1} + \Xi^H\big(\tfrac{\chi}{\gamma_C+\gamma_{C^*}} + \xi_h\big)\,\widetilde{c}_t + \Xi^F\big(\tfrac{\chi^*}{\gamma_C+\gamma_{C^*}} + \xi_f\big)\,\widetilde{c}_t^* + \gamma_G\,\big\{(1-\alpha_G)\xi_h + \alpha_G\,\xi_f\big\}\ \widetilde{g}_t + \gamma_{G^*}\,\big\{(1-\alpha_G^*)\xi_f + \alpha_G^*\,\xi_h\big\}\ \widetilde{g}_t^* + \Big[\tfrac{\alpha\,\gamma_C}{\gamma_C+\gamma_{C^*}}(\chi-\chi^*) + (\xi_h - \xi_f)\Big]\widetilde{s}_{t+1}$$

- Domestic Euler equation, at Home and in the Foreign country:

$$\widetilde{c}_t = \mathbb{E}_t\, \widetilde{c}_{t+1} - (i_t - r^{nat}) + \mathbb{E}_t\, \pi_{w,t+1} - \frac{\gamma_{C^*}\,(1-\alpha-\alpha^*)}{\gamma_C+\gamma_{C^*}}\ \mathbb{E}_t\, \Delta\widetilde{s}_{t+1}$$

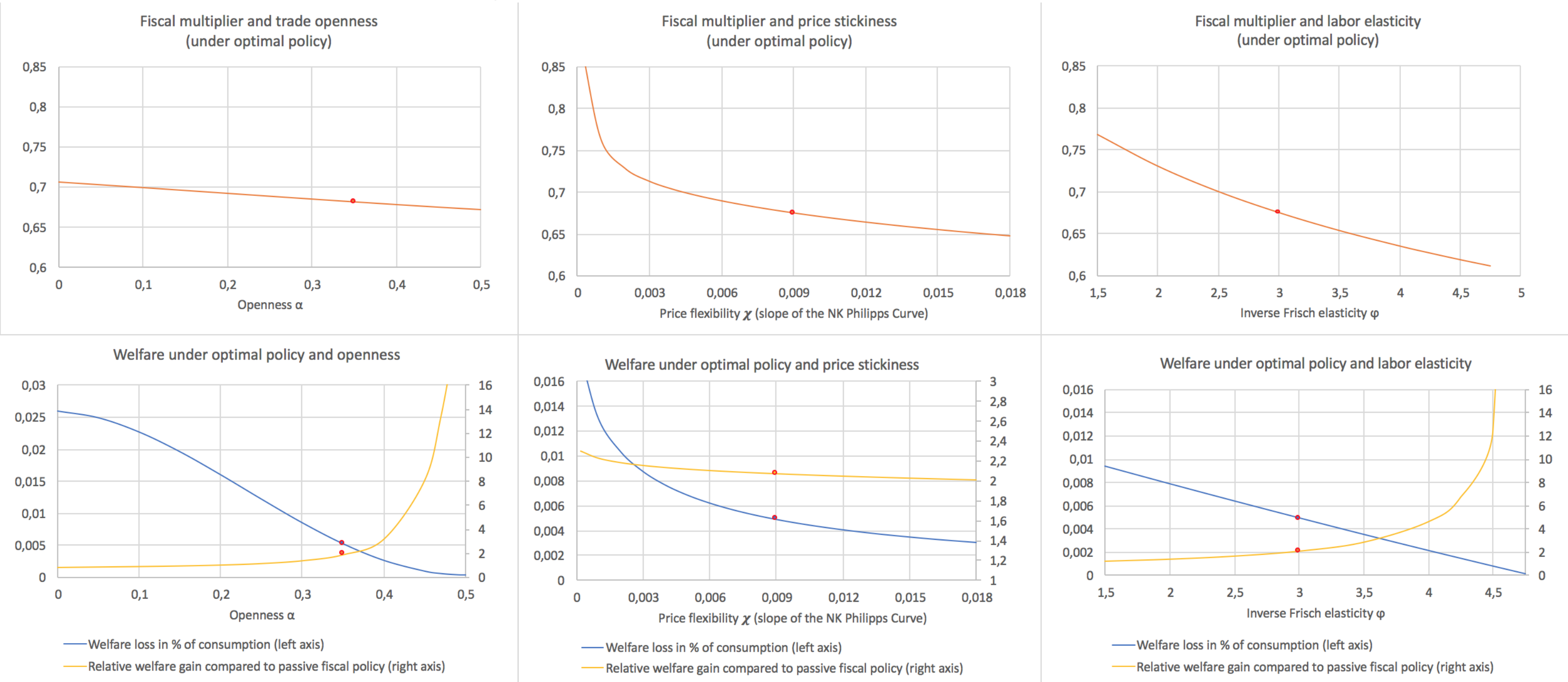
$$\widetilde{c}_t^* = \mathbb{E}_t\, \widetilde{c}_{t+1}^* - (i_t - r^{nat}) + \mathbb{E}_t\, \pi_{w,t+1} + \frac{\gamma_C\,(1-\alpha-\alpha^*)}{\gamma_C+\gamma_{C^*}}\ \mathbb{E}_t\, \Delta\widetilde{s}_{t+1}$$

- Union-level Euler equation

$$\widetilde{c}_{w,t} = \mathbb{E}_t\, \widetilde{c}_{w,t+1} - \{(\gamma_C+\gamma_{C^*})[i_t - r^{nat}] - (\Xi^H\ \mathbb{E}_t\, \pi_{H,t+1} + \Xi^F\ \mathbb{E}_t\, \pi_{F,t+1})\}$$

Welfare, fiscal multiplier and effectiveness of optimal policy in function of structural parameters

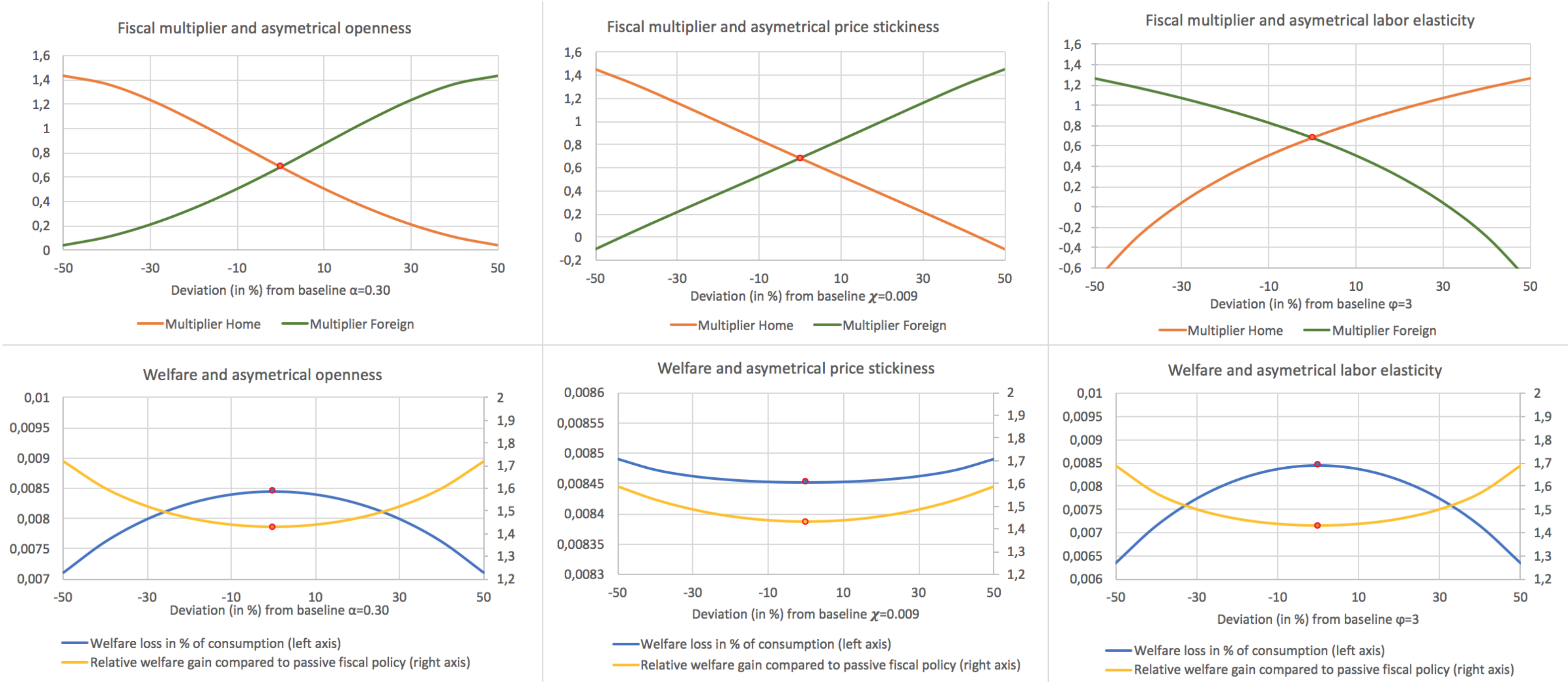
Figure 6: Results in terms of structural parameters



This graph displays the effectiveness of fiscal of policy in optimal policy – compared to the passive situation – using two criterions: first the fiscal multiplier (output response to spending - orange) and second the relative loss (yellow) of the welfare loss in the non-intervention scenario compared to optimum – and the welfare loss as the optimum (blue). These graph examine the sensitivity of these variables to change in (i) openness, (ii) price stickiness, (iii) labor supply elasticity.

For instance, the left graph displays that at the benchmark calibration (red dot) the fiscal multiplier is 0.7, the welfare loss (blue) at optimum corresponds to 0.005% of consumption (very small!) and that the loss under passive policy is 2 times higher. When country is fully open $\alpha = 0.5$, with more flexible prices (high χ) or labor supply is rigid (small Frisch elasticity), there is a smaller welfare loss at optimum and the fiscal multiplier is smaller (0.6 instead of 0.7-0.8). Fiscal policy is particularly effective (in terms of welfare), when labor supply is rigid as it balances the labor demand between the two countries.

Figure 7: Results in terms of structural parameters



This graph displays the effectiveness of fiscal of policy in optimal policy – compared to the passive situation – using two criterions: first the fiscal multiplier (output response to spending - orange for Home country, green for Foreign) and second the relative loss (yellow) of the non-intervention scenario compared to optimum – and the welfare loss as the optimum (blue). These graph examine the sensitivity of these variables to change in (i) openness, (ii) price stickiness, (iii) labor supply elasticity.
The multipliers of the two countries are inversely related: for instance when Home is less open than Foreign, its multipliers is substantially greater. In all graphs, the fiscal policy is *more effective* in terms of welfare when the two countries are *asymmetric*, i.e. the welfare loss is 70% higher when the policy is absent when the differential between the two countries is high.

Derivation of the Welfare-function

In this section, we state the Household utility function, approximated at the second order. The formula is heavy due parameters heterogeneity. The various multiplicative factors are induced by interactions between the two countries (structural parameters γ_i representing the weights of the aggregates in the "Union economy"). Also, variable z_t represents the usual New-Keynesian's style price dispersion i.e. $z_t \equiv \ln \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} di.$

$$
\begin{aligned}
U(C_t, G_t, N_t) &= (1-\Gamma)\ \ln(C_t) + \Gamma\ \ln(G_t) - \tfrac{N_t^{1+\varphi}}{1+\varphi} \\
&\simeq (1-\Gamma)\,\widetilde{c}_t + \Gamma\,\widetilde{g}_t - z_t - 2[(1-\alpha)\,\gamma_{\mathrm{C}}\,\widetilde{c}_t + \alpha^*\,\gamma_{\mathrm{C}^*}\,\widetilde{c}_t^* + (1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}}\,\widetilde{g}_t + (1-\alpha_{\mathrm{G}}^*)\,\gamma_{\mathrm{G}^*}\,\widetilde{g}_t^* + \omega\widetilde{s}_t] \\
&\qquad - \left((1-\alpha)\,\gamma_{\mathrm{C}} + 2\varphi(1-\alpha)^2\,\gamma_{\mathrm{C}}{}^2\right)(\widetilde{c}_t)^2 - \left(\alpha^*\,\gamma_{\mathrm{C}^*} + 2\varphi\,\alpha^{*2}\,\gamma_{\mathrm{C}^*}{}^2\right)(\widetilde{c}_t^*)^2 \\
&\qquad - \left((1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}} + 2\varphi(1-\alpha_{\mathrm{G}})^2\,\gamma_{\mathrm{G}}{}^2\right)(\widetilde{g}_t)^2 - \left(\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*} + 2\varphi\,\alpha_{\mathrm{G}}^{*2}\,\gamma_{\mathrm{G}^*}{}^2\right)(\widetilde{g}_t^*)^2 \\
&\qquad - \left(\nu+2\varphi\omega^2\right)\widetilde{s}_t^2 - \left((1-\alpha)\,\gamma_{\mathrm{C}}\,(2\alpha+4\omega\varphi)\right)\widetilde{c}_t\,\widetilde{s}_t - \left(\alpha^*\,\gamma_{\mathrm{C}^*}(2(1-\alpha^*)+4\omega\varphi)\right)\widetilde{c}_t^*\,\widetilde{s}_t \\
&\qquad - \left((1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}}\,(2\,\alpha_{\mathrm{G}} + 4\omega\varphi)\right)\widetilde{g}_t\,\widetilde{s}_t - \left(\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}(2(1-\alpha_{\mathrm{G}}^*)+4\omega\varphi)\right)\widetilde{g}_t^*\,\widetilde{s}_t \\
&\qquad - 4\varphi\Big[\left((1-\alpha)\,\gamma_{\mathrm{C}}\,\alpha^*\,\gamma_{\mathrm{C}^*}\right)\widetilde{c}_t\,\widetilde{c}_t^* - \left((1-\alpha)\,\gamma_{\mathrm{C}}\,(1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}}\right)\widetilde{c}_t\,\widetilde{g}_t - \left((1-\alpha)\,\gamma_{\mathrm{C}}\,\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}\right)\widetilde{c}_t\,\widetilde{g}_t^* \\
&\qquad - \left(\alpha^*\,\gamma_{\mathrm{C}^*}(1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{G}}\right)\widetilde{c}_t^*\,\widetilde{g}_t - \left(\alpha^*\,\gamma_{\mathrm{C}^*}\,\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}\right)\widetilde{c}_t^*\,\widetilde{g}_t^* - \left((1-\alpha_{\mathrm{G}})\,\gamma_{\mathrm{C}^*}\,\alpha_{\mathrm{G}}^*\,\gamma_{\mathrm{G}^*}\right)\widetilde{g}_t\,\widetilde{g}_t^*\Big]
\end{aligned}
$$

We obtain a similar formula for the Foreign Household, replacing parameters by their "$*$" equivalent, and $\widetilde{s}_t$ by $-\widetilde{s}_t$. We observe that this second order approximation includes first-order terms. Hopefully, these terms cancel out when we consider the Union as a whole. This is because, as discussed in first section of appendices (parameters restrictions), we have $\Gamma = 2\ \gamma_{\mathrm{G}}$ and $\Gamma^* = 2\ \gamma_{\mathrm{G}^*}$. Therefore, we rewrite the loss function as the negative utility:

$$
\begin{aligned}
\mathbb{L}_t^w &= -U_t(C_t, G_t, N_t) - U_t^*(C_t^*, G_t^*, N_t^*) \\
&= -(1-\Gamma)\ \ln(C_t) - \Gamma\ \ln(G_t) + \tfrac{N_t^{1+\varphi}}{1+\varphi} - (1-\Gamma^*)\ \ln(C_t^*) - \Gamma^*\ \ln(G_t^*) + \tfrac{N_t^{*(1+\varphi^*)}}{1+\varphi^*} \\
&\simeq z_t + z_t^* + [\gamma_{\mathrm{C}}\,\widetilde{c}_t^2 + \gamma_{\mathrm{C}^*}\,\widetilde{c}_t^{*\,2} + \gamma_{\mathrm{G}}\,\widetilde{g}_t^2 + \gamma_{\mathrm{G}^*}\,\widetilde{g}_t^{*\,2} + \omega\widetilde{s}_t^2] \\
&\qquad + 4\,\gamma_{\mathrm{C}}\,\omega((1-\alpha)\varphi-\alpha\varphi^*)\,\widetilde{c}_t\widetilde{s}_t + 4\,\gamma_{\mathrm{C}^*}\,\omega(\alpha^*\,\varphi-(1-\alpha^*)\varphi^*)\,\widetilde{c}_t^*\widetilde{s}_t \\
&\qquad + 4\,\gamma_{\mathrm{G}}\,\omega((1-\alpha_{\mathrm{G}})\varphi-\alpha_{\mathrm{G}}\,\varphi^*)\,\widetilde{g}_t\widetilde{s}_t + 4\,\gamma_{\mathrm{G}^*}\,\omega(\alpha_{\mathrm{G}}\,\varphi-(1-\alpha_{\mathrm{G}}^*)\varphi^*)\,\widetilde{g}_t^*\widetilde{s}_t \\
&\qquad + 2\,\gamma_{\mathrm{C}}{}^2((1-\alpha)^2\varphi+\alpha^2\varphi^*)\,\widetilde{c}_t^2 + 2\,\gamma_{\mathrm{C}^*}{}^2(\alpha^{*2}\,\varphi+(1-\alpha^*)^2\varphi^*)\,\widetilde{c}_t^{*\,2} \\
&\qquad + 2\,\gamma_{\mathrm{G}}{}^2((1-\alpha_{\mathrm{G}})^2\varphi+\alpha_{\mathrm{G}}{}^2\,\varphi^*)\,\widetilde{g}_t^2 + 2\,\gamma_{\mathrm{G}}{}^2(\alpha_{\mathrm{G}}^{*2}\,\varphi+(1-\alpha_{\mathrm{G}}^*)^2\varphi^*)\,\widetilde{g}_t^{*\,2} + \\
&\qquad + 2\,\gamma_{\mathrm{C}}\,\gamma_{\mathrm{C}^*}((1-\alpha)\,\alpha^*\,\varphi+\alpha(1-\alpha^*)\varphi^*)\,\widetilde{c}_t\,\widetilde{c}_t^* + 2\,\gamma_{\mathrm{C}}\,\gamma_{\mathrm{G}}((1-\alpha)(1-\alpha_{\mathrm{G}})\varphi+\alpha\,\alpha_{\mathrm{G}}\,\varphi^*)\,\widetilde{c}_t\,\widetilde{g}_t \\
&\qquad + 2\,\gamma_{\mathrm{C}}\,\gamma_{\mathrm{G}^*}((1-\alpha)\,\alpha_{\mathrm{G}}\,\varphi+\alpha(1-\alpha_{\mathrm{G}})\varphi^*)\,\widetilde{c}_t\,\widetilde{g}_t^* + 2\,\gamma_{\mathrm{C}^*}\,\gamma_{\mathrm{G}}(\alpha^*(1-\alpha_{\mathrm{G}})\varphi+(1-\alpha)\,\alpha_{\mathrm{G}}\,\varphi^*)\,\widetilde{c}_t^*\,\widetilde{g}_t \\
&\qquad + 2\,\gamma_{\mathrm{C}^*}\,\gamma_{\mathrm{G}^*}(\alpha^*\,\alpha_{\mathrm{G}}\,\varphi+(1-\alpha^*)(1-\alpha_{\mathrm{G}}^*)\varphi^*)\,\widetilde{c}_t^*\,\widetilde{g}_t^* + 2\,\gamma_{\mathrm{G}}\,\gamma_{\mathrm{G}^*}((1-\alpha_{\mathrm{G}})\,\alpha_{\mathrm{G}}^*\,\varphi+\alpha_{\mathrm{G}}(1-\alpha_{\mathrm{G}}^*)\varphi^*)\,\widetilde{g}_t\,\widetilde{g}_t^*
\end{aligned}
$$

As we see, this formula is rather cumbersome. Rearranging the terms, it is possible to express the Loss function with vectors, and obtain a more compact formula, as shown in eq. (8). More, as shown in Galí and Monacelli (2008), the term representing the price dispersion z_t can be expressed as the square of inflation.