Redistribution and the wage-price spirals: Optimal fiscal and monetary policy

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Abstract

When both prices and wages are subject to nominal frictions, an increase in input prices such as energy can initiate a wage-price spiral, as both nominal wages and prices adjust slowly. High inflation in prices and wages reduces welfare as it generates large distributional effects and affects aggregate demand. To analyze optimal policy in this environment, we consider a heterogeneous-agent model, with both wage and price stickiness, where the production sector uses an additional input, which can be subject to aggregate shocks. We derive joint optimal fiscal-monetary policy, considering redistribution over the business cycle. We show that a wage-price spiral can be avoided if labor subsidy, through a reduction in firm social contributions for instance, increases after the shock to the input price. The optimal design of fiscal policy significantly reduces the volatility of the economy, compared to the case where fiscal policy is constant and monetary policy follows a Taylor rule.

Keywords: Heterogeneous agents, wage-price spiral, inflation, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

1 Introduction

Workers usually bargain over nominal wages, with a bargaining power depending on various institutional designs and market properties. In addition, firms facing price-adjustment costs may adjust progressively their prices. As a consequence, price and wage inflation may differ, generating heterogeneous dynamics in real wages and markups. Such a wage-price spiral is likely to be initiated after energy prices shocks, like in the 70s and in the current juncture. The management of wage-price spirals is topical economic question, which can be the source of large inflation swings with adverse economic effects on output and welfare.

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The goal of this paper is both positive and normative. On the one hand, we analyze the distributional effects of wage-price spirals, while on the other hand we determine the optimal fiscal-monetary policy to manage them. We focus on a setup featuring incomplete markets, with nominal frictions for both price and wages. On the positive side, heterogeneous-agent models are an attractive environment for two reasons. The distributive implications can be properly assessed as the model features realistic distributions for wealth and consumption. Second, fiscal tools have quantitatively-sound macroeconomic effects thanks to properly replicated marginal propensities to consume. For instance, a subsidy to households financed by public debt can increase aggregate consumption. On the normative side, we study the joint optimal fiscal-monetary policy in this environment, after both demand and supply-side shock. Indeed, as illustrated by the European responses to the natural gas crisis, fiscal policy may be a powerful instrument during wage-price spirals not only to manage the adverse distributional effects of inflation, but also to dampen the dynamics of inflation by reducing the incentives to increase prices or wages.

We consider an heterogeneous model with capital and aggregate shocks and introduce nominal frictions in both goods and labor markets. The model thus features a price and a wage Phillips curves. Monetary policy consists in setting the nominal interest rate, while fiscal policy includes a realistic set of tools. We consider capital tax, labor tax, labor subsidies, lump-sum transfers and public debt. Capital tax, labor tax and transfers are known to reproduce realistic patterns of the US fiscal systems, such as its progressivity, with a relatively limited set of instruments (Heathcote and Tsujiyama, 2021a or Dyrda and Pedroni, 2022). The new fiscal instrument is a labor subsidy which can affect the labor cost paid by the firm without directly affecting the wage paid to workers. They can be understood as firm social contributions for instance. This economy features two types of nominal frictions – i.e., sticky prices and sticky wages – and a rich fiscal system. Considering two nominal frictions instead of one is not a minor change, as it generates suboptimal real wage rigidity. This has negative adverse implications on the labor market, which is turns affects aggregate demand by changing labor wages. These indirect effects are known to matter for the macroeconomy in these environments (Kaplan et al., 2018). Finally, we introduce energy in the production sector. An increase in the energy price raises the marginal production cost of firms and can possibly initiate a wage-price spiral. In this environment, we derive the optimal Ramsey policy with commitment, considering various assumptions about the instruments that are available to the planner.

Our first result is to identify the distortions due to sticky prices and sticky wages in our environment. To do so, we design the simplest fiscal policy for which the optimal inflation in wages and prices is null after both demand and supply-side shocks. We prove that the combination of a labor subsidy at the firm level, and linear labor and capital taxes allow one to optimally remove any inflation pressure. In other words, if the planner can optimally set these instruments along the business cycle, then it is optimal to have constant wages and constant prices in the dynamics. This theoretical result generalizes the equivalence results of Correia et al. (2008) who consider a representative agent and time-varying VAT tax, and of LeGrand et al. (2022) who consider sticky prices in a HANK model. Combining a nominal friction on prices and on wages makes a labor subsidy a necessary tool to implement price stability as an optimal outcome. This equivalence result has two main implications. First, it allows us to identify the fiscal tools, that can be used to avoid wage-price spirals. Second, when these tools are not available or cannot be optimally set along the business cycle, we can understand the additional roles that monetary policy needs to fulfill.

Our second result involves a quantitative investigation of the role of optimal fiscal policy. First, we simulate the heterogeneous-agent model with sticky prices and sticky wages with a Taylor rule and standard fiscal rules in a quantitatively relevant environment. The model generates a realistic wealth heterogeneity and average short-run marginal propensity to consume (MPC) of 0.30 which is known to be key to assess the role of fiscal policy (Kaplan and Violante, 2014, Ferriere and Navarro, 2020, Auclert et al., 2022 among others). In this exercise, marginal tax rates are assumed to remain constant along the business cycle. We compare the allocation of this economy to the one of an economy all fiscal instruments (capital tax, labor tax, labor subsidies, transfers, and public debt) can be optimally set along the business cycle. We find the fall in aggregate consumption to be twice smaller when the fiscal policy is optimal. The main driver of the reduction in consumption and aggregate volatility comes from a higher wage subsidy, which helps avoid a sharp reduction in labor demand, in labor income and thus in consumption after a negative energy price shock. Aggregate labor supply falls much less with optimal fiscal policy compared to the case with fixed tax rates. Regarding the behavior of the other instruments following an energy shock, the labor tax barely moves, while capital tax increases before rapidly converging back to its steady-state value. Public debt and transfers increase on impact. From this experiment, we conclude using fiscal tools to cut labor costs after an energy price shocks avoids initiating price-inflation spirals.

Literature review. This paper belongs to the literature on optimal policy in heterogeneous agent model on one side, and on wage-price spirals on the other side.

Optimal monetary policy in HANK models is first based on the mechanisms identified by Kaplan et al. (2018) and Auclert (2019) regarding the transmission channels of monetary policy. Deriving optimal policy in heterogeneous-agent models with aggregate shocks is a difficult theoretical and computational task. Some papers consider numerical methods to solve for optimal path of the instruments (Dyrda and Pedroni, 2022). Other papers rely on continuous-time techniques for the theoretical derivation of the first-order conditions of the planner (Nuño and Thomas, 2022 among others). Acharya et al. (2022) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. Bhandari et al. (2021) quantitatively solve for optimal policies in a new-Keynesian model with aggregate shocks. Yang (2022) solves for the optimal monetary policy by optimizing on the coefficients of a Taylor rule. We use the tools of LeGrand and Ragot (2022a) and the improvements of LeGrand and Ragot (2022c) to solve for optimal fiscal and monetary policy with aggregate shocks. This approach allows one to easily solve for optimal policy with many tools (we consider six tools in the benchmark case) and with various nominal frictions. On the theoretical side, we show in the current paper that the Lagrangian approach pioneered in Marcet and Marimon (2019) enables us to derive the first-order conditions of the Ramsey planner in an environment with both wage and price rigidities, .

This paper belongs to the literature on the wage-price spirals. Models with both price and wage stickiness have been studied in representative agent economies (Blanchard, 1986, Blanchard and Gali, 2007 among others). Erceg et al. (2000) study optimal monetary policy in this environment. Recently, Lorenzoni and Werning (2023) analyze more deeply optimal policy and the real wage dynamics in this environment, keeping the complete insurance-market assumption.

2 The environment

We consider a discrete-time economy populated by a continuum of size one of ex-ante identical agents. These agents are assumed to be distributed along a set J, with the non-atomic measure ℓ: $\ell(J) = 1$.[1]

2.1 Risk

We assume that the agents face an idiosyncratic productivity risk. The productivity process, denoted y, is assumed to take value in a finite set $\mathcal{Y}$ and to follow a first-order Markov chain with transition matrix $\boldsymbol{\pi} = (\pi_{yy'})_{y,y'}$. Average productivity level is normalized to one. With wage w, labor supply l, an agent with productivity y earns the labor income wyl. The history of idiosyncratic productivity shocks up to date t for an agent i is denoted by $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\} \in \mathcal{Y}^{t+1}$, where $y_{i,\tau}$ is the date-τ productivity. The transition matrix allows measure of history is denoted by θ_t and can be computed from the initial distribution and matrix transition.

In addition to the previous idiosyncratic risk, agents face an aggregate risk z affecting the economic TFP, denoted by $A(z)$. We show in Section 2.8 that the aggregate risk can be interpreted as a shock on energy price.

2.2 Preferences

Households are expected-utility maximizers endowed with time-separable preferences and a constant discount factor $\beta \in (0, 1)$. In each period, households enjoy utility $U(c, l)$ from the consumption c of the unique consumption good of the economy and suffer from the disutility of providing the labor supply l. We further assume that in each period, the instantaneous utility

[1] We follow Green (1994) and assume that the law of large numbers holds.

is separable in consumption and labor: $U(c,l) = u(c) - v(\ell)$, where $u, v : \mathbb{R}_+ \to \mathbb{R}$ are twice continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

2.3 Production

The specification of the production sector follows the New-Keynesian literature on price stickiness, e.g., LeGrand et al. (2022). The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y^f_{j,t})_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted ε_P:

$$Y_t = \left[\int_0^1 {y^f_{j,t}}^{\frac{\varepsilon_P - 1}{\varepsilon_P}} dj \right]^{\frac{\varepsilon_P}{\varepsilon_P - 1}}.$$

For any intermediate good $j \in [0,1]$, the production $y^f_{j,t}$ is realized by a monopolistic firm and sold at price $p_{j,t}$. The profit maximization for the firm producing the final good implies:

$$y^f_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon_P} Y_t, \text{ where } P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon_P} dj \right)^{\frac{1}{1-\varepsilon_P}}.$$

The quantity P_t is the price index of the consumption good. Intermediary firms are endowed with a Cobb-Douglas production technology and use labor and capital as production factors. The production technology involves that $\tilde{l}_{j,t}$ units of labor and $\tilde{k}_{j,t}$ units of capital are transformed into $y^f_{j,t} = Z_t \tilde{k}^{\alpha}_{j,t} \tilde{l}^{1-\alpha}_{j,t}$ units of intermediate good. Since intermediate firms have market power, the firm's objective is to minimize production costs, including capital depreciation, subject to producing the demand $y^f_{j,t}$. The cost function $C_{j,t}$ of firm j is therefore $C_{j,t} = \min_{\tilde{l}_{j,t}, \tilde{k}_{j,t}} \{(\tilde{r}^K_t + \delta)\tilde{k}_{j,t} + \tilde{w}_t \tilde{l}_{j,t}\}$, subject to $y^f_{j,t} = Z_t \tilde{k}^{\alpha}_{j,t} \tilde{l}^{1-\alpha}_{j,t}$, where $\tilde{w}_t$ is the real before-tax wage and $\tilde{r}^K_t$ the real before-tax net interest rate. The maximization implies the following mark-up:

$$m_t = \frac{1}{Z_t} \left(\frac{\tilde{r}^K_t + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha}. \tag{1}$$

In addition to the production cost, intermediate firms face a quadratic price adjustment cost à la Rotemberg when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\psi_p}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2$. We can thus deduce the real profit, denoted Ω_t at date t of firm j:

$$\Omega_{j.t} = \Big(\frac{p_{j,t}}{P_t} - m_t(1 - \tau^Y_t) \Big) \Big(\frac{p_{j,t}}{P_t} \Big)^{-\epsilon} Y_t - \frac{\psi_p}{2} \left(\frac{p_{j,t}}{p_{j,t-1}} - 1 \right)^2 Y_t - t^Y_t,$$

where t^Y_t is a lump-sum tax financing the subsidy τ^Y. Computing the firm j's intertemporal profit requires to define the firm's pricing kernel. We follow Bhandari et al. (2021) and assume a constant pricing kernel.[2] The firm j's thus stes its price schedule $(p_{j,t})_{t\geq 0}$ to maximize its intertemporal profit at date 0: $\max_{(p_{j,t})_{t\geq 0}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \Omega_{j,t}]$. The solution is independent of the firm type j and all firms in the symmetric equilibrium charge the same price: $p_{j,t} = P_t$. Denoting the price inflation rate as $\pi_t^P = \frac{P_t}{P_{t-1}} - 1$ and setting $\tau^Y = \frac{1}{\varepsilon}$ to obtain an efficient steady state, we obtain the standard equation characterizing the Phillips curve in our environment:

$$\pi_t^P(1+\pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P}\Big(m_t - 1\Big) + \beta\mathbb{E}_t\Big(\pi_{t+1}^P(1+\pi_{t+1}^P)\frac{Y_{t+1}}{Y_t}\Big). \tag{2}$$

The real profit is independent of the firm's type and can be expressed as follows:

$$\Omega_t = \Big(1 - m_t - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Y_t. \tag{3}$$

Furthermore, equation (1) implies:

$$\tilde{r}_t^K + \delta = m_t \alpha Z_t (L_t/K_{t-1})^{1-\alpha} \text{ and } \tilde{w}_t = m_t(1-\alpha)Z_t(K_{t-1}/L_t)^{\alpha}, \tag{4}$$

$$Y_t = F(K_{t-1}, L_t) = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{1}{m_t}((\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t) \tag{5}$$

2.4 Labor market: Labor supply and Union wage decision

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by unions (Auclert et al., 2022). There is a continuum of unions of size 1 indexed by k and each union k supplies at date t L_{kt} hours of labor with nominal wage $\hat{W}_{kt}$. Union-specific labor supplies are then aggregated into aggregate labor supply by a competitive technology featuring a constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk\right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{6}$$

The competitive aggregator demands the union labor supplies $(L_{kt})_k$ that minimize the total labor cost $\int_k \hat{W}_{kt} L_{k,t} dk$ subject to the aggregation constraint (6), where $\tilde{W}_{kt}$ is the nominal wage of the members of union k. We deduce that the demand for labor of union k amounts to:

$$L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t, \tag{7}$$

where $\hat{W}_t = \left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}}$ is the nominal wage index.

Each union k decides its wage $\hat{W}_{kt}$ to maximize the intertemporal welfare of its members subject to fulfilling the demand of equation (7). We assume the presence of quadratic utility

[2] Our own computations also show us that the quantitative impact of the pricing kernel is limited.

costs for adjusting the nominal wage W_{kt}: $\frac{\psi_W}{2}\left(\frac{\hat{W}_{kt}}{\hat{W}_{kt-1}}-1\right)^2 dk$. The objective of union k is thus:

$$\max_{(W_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2}\left(\frac{\hat{W}_{kt}}{\hat{W}_{kt-1}}-1\right)^2\right)\ell(di),$$
$$\text{s.t. } L_{kt} = \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t,$$

where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The first-order condition with respect to W_{kt} thus writes as:

$$\pi_t^W(\pi_t^W+1) = \frac{\hat{W}_{kt}}{\psi_W}\int_i \left(u'(c_{i,t})\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} - v'(l_{i,t})\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}}\right)\ell(di) + \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right],$$

where we denote:

$$\pi_t^W = \frac{\hat{W}_{k,t}}{\hat{W}_{k,t-1}} - 1,$$

the wage inflation rate.

The labor supply l_{it} of agent i is the sum of its hours l_{ikt} to all unions k: $l_{it} = \int_k l_{ikt} dk$, and hence the total individual labor supply. Each union is assumed to ask its members to supply an uniform number of hours, such that: $l_{ikt} = L_{kt}$. We thus deduce from (7):

$$\begin{aligned}\hat{W}_{kt}\frac{\partial l_{i,t}}{\partial \hat{W}_{kt}} &= \hat{W}_{kt}\frac{\partial\left(\int_k \left(\frac{\hat{W}_{kt}}{\hat{W}_t}\right)^{-\varepsilon_W} L_t dk\right)}{\partial \hat{W}_{kt}}\\ &= -\varepsilon_W L_{kt}.\end{aligned}$$

To compute the derivative of consumption $\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}}$, it should observed that it is equal to the derivative of its net total income. The net total income of agent i writes as $(1-\tau_t^L)\hat{W}_t y_{i,t} l_{i,t}/P_t$, where τ_t^L is the labor tax. Formally:

$$\begin{aligned}\hat{W}_{kt}\frac{\partial c_{i,t}}{\partial \hat{W}_{kt}} &= \frac{(1-\tau_t^L)y_{i,t}}{P_t}\hat{W}_{kt}\frac{\partial\left(\left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{1}{1-\varepsilon_W}} l_{i,t}\right)}{\partial \hat{W}_{kt}}\\ &= -\varepsilon_W(1-\tau_t^L)\hat{W}_t y_{i,t} l_{i,t}/P_t + \frac{(1-\tau_t^L)y_{i,t}}{P_t}\hat{W}_{kt}^{1-\varepsilon_W}\left(\int_k \hat{W}_{kt}^{1-\varepsilon_W} dk\right)^{\frac{\varepsilon_W}{1-\varepsilon_W}} l_{i,t}\\ &= (1-\varepsilon_W)(1-\tau_t^L)\hat{W}_t y_{i,t} l_{i,t}/P_t.\end{aligned}$$

We focus on the symmetric equilibrium where all unions choose to set the same wage $\hat{W}_{kt} = \hat{W}_t$, hence all households work the same number of hours, equal to $l_{it} = L_t$.

We deduce the following Phillips curve for wage inflation:

$$\pi_t^W(\pi_t^W+1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t + \mathbb{E}_t\left[\pi_{t+1}^W(\pi_{t+1}^W+1)\right], \quad (8)$$

where $\hat{w}_t = \hat{W}_t/P_t$ is the real pre-tax wage and $w_t = (1-\tau_t^L)\hat{W}_t/P_t$ is the real post-tax wage.

2.5 Assets

Agents have the possibility to trade shares of a mutual fund, which is assumed to pay the before-tax net interest rate $\tilde{r}_t$. The fund share holdings of agent i are denoted $a_{i,t}$ and are constrained to remain above an exogenous threshold $-\underline{a} \leq 0$. The risk-neutral mutual fund collects the two interest-bearing assets that are traded in the economy and issues asset shares. The first asset is nominal public debt, whose supply size is denoted by B_t at date t, and which pays off the pre-determined before-tax gross nominal interest rate $\tilde{R}_{t-1}^N$. The associated real before-tax (gross) interest rate for public debt is $\tilde{R}_{t-1}^N/(1+\pi_t^P)$. Public debt is issued by the government and is assumed to be default free. The second asset consists of capital shares, which pay off a (net and before-tax) real interest rate $\tilde{r}_t^K$ – as introduced above.

The size of the fund A_t is equal to the sum of both asset supplies:

$$A_t = B_t + K_t. \tag{9}$$

The no-profit condition of the fund implies that interests paid out of fund shares are solely financed out of the interests payments of capital and public debt. Formally:

$$\tilde{r}_t A_{t-1} = \tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^N}{1+\pi_t^P} - 1\right) B_{t-1}. \tag{10}$$

Finally, the fund being risk-neutral, a no-arbitrage condition must hold. The two assets must pay the same expected return:

$$\tilde{R}_t^N \mathbb{E}_t \left[\frac{1}{1+\pi_{t+1}^P}\right] = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K\right]. \tag{11}$$

2.6 Government and market clearing

The government has to finance an exogenous public good expenditure G_t, as well as lump-sum transfers $T_t \geq 0$.[3] The latter transfers can be thought of as social transfers, which can contribute to generating progressivity in the overall tax system. The government can use three sets of instruments. First, the government can issue public debt through one-period riskless nominal bonds. Second, the government also fully taxes the firms' profits, which limits the distortions implied by profit distribution.Finally, the government can rely on four different taxes. The first tax is the distorting capital tax, denoted by τ_t^K, levied on asset payoffs. The second tax is the standard labor tax τ_t^L levied on the real wage $\hat{w}_t$ negotiated by unions. The third tax is another labor tax τ_t^S that is levied on the wage $\tilde{w}_t$ paid by firms and internalized by unions. To sum

[3]As is standard in this literature (Aiyagari et al., 2002), we rule out the possibility of lump-sum taxes.

up, fiscal policy is characterized by five instruments $(\tau_t^L, \tau_t^S, \tau_t^K, T_t, B_t)_{t\geq 0}$ given an exogenous public spending $(G_t)_{t\geq 0}$.

After-tax quantities are denoted without a tilde. The real after-tax wage w_t, as well as the real after-tax interest rate r_t (the real interest rates for the capital and public debt do not need to be defined) can therefore be expressed as follows:

$$w_t = (1-\tau_t^L)\hat{w}_t = (1-\tau_t^L)(1-\tau_t^S)\tilde{w}_t, \tag{12}$$

$$r_t = (1-\tau_t^K)\tilde{r}_t. \tag{13}$$

There is a two-stage taxation scheme on labor wage: the first-one is internalized by the unions, while the second one is not.

The government uses its financial resources, made of labor and asset taxes, and public debt issuance, to finance public goods, lump-sum transfers, and debt repayment:

$$G_t + \frac{\tilde{R}_{t-1}^N}{\Pi_t}B_{t-1} + T_t \leq \tau_t^L\hat{w}_tL_t + \tau_t^S\tilde{w}_tL_t + \tau_t^K\tilde{r}_tA_{t-1} + \Omega_t + B_t.$$

The expression of this government budget constraint can be simplified, following Chamley (1986). Using the definition (5) of the final good production, the governmental budget constraint becomes:

$$G_t + \frac{\tilde{R}_{t-1}^N}{\Pi_t}B_{t-1} + \tilde{r}^KK_{t-1} + \tilde{w}_tL_t + T_t \leq \tau_t^L\hat{w}_tL_t + \tau_t^S\tilde{w}_tL_t + \tau_t^K\tilde{r}_tA_{t-1} + m_tF(K_{t-1}, L_t) + \Omega_t + B_t,$$

where the tax bases of the two labor taxes differ from each other. Using the definition of Ω_t, the fund no-profit condition (10) the post-tax rate definitions (12) and (13), we obtain:

$$G_t + r_tA_{t-1} + w_tL_t + T_t \leq \Big(1 - \frac{\psi_P}{2}(\pi_t^P)^2\Big)Y_t - \delta K_{t-1} + B_t. \tag{14}$$

The choice of optimal monetary-fiscal policy is thus the choice of the path of the instruments $(\tau_t^L, \tau_t^S, \tau_t^K, T_t, B_t, \tilde{R}_t^N, \pi_t^P, \pi_t^W)_{t\geq 0}$. These instruments are not independent of each other and are connected through the budget constraint of the government and the two Phillips curves.

2.7 Agents' program, resource constraints, and equilibrium definition

Each agent enters the economy with an initial endowed of fund shares $a_{i,-1}$ and an initial productivity level $y_{i,0}$. The joint initial distribution over fund shares and productivity levels is denoted Λ_0. In later periods, each agent learns her productivity levels, earns her labor supply, her fund share payoffs, the lump-sum payment. The labor supply is decided by the unions and not by the agent – hence . She decides upon her consumption as well as her new saving decisions. The budget constraint of agent i can be written as:

$$c_{i,t} + a_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t + T_t. \tag{15}$$

The agent's program can be written as:

$$\max_{\{c_{i,t}, a_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(L_t) \right), \tag{16}$$

subject to the budget constraint (15), the credit constraint $a_{i,t} \geq -\underline{a}$, and the consumption positivity constraint $c_{i,t} > 0$. The notation $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risks. The solution of the agent's program is a sequence of functions, defined over $([-\bar{a}; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, a_t)_{t\geq 0}$, such that:[4]

$$c_{i,t} = c_t((a_{i,-1}, y_{i,0}), y_i^t, z^t),\ a_{i,t} = a_t((a_{i,-1}, y_{i,0}), y_i^t, z^t). \tag{17}$$

For the sake of simplicity, we will keep using the notation with the i-index. Denoting by $\nu_{i,t}$ the discounted Lagrange multipliers of the credit constraint, the Euler equation corresponding to the agent's program (16) is:

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t}. \tag{18}$$

We finally express the financial market clearing condition and the economy resource constraints:

$$\int_i a_{i,t} \ell(di) = A_t, \tag{19}$$

$$\int_i c_{i,t} \ell(di) + G_t + K_t = \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2 \right) Y_t + K_{t-1} - \delta K_{t-1}. \tag{20}$$

Equilibrium definition.

We can finally provide a definition for our competitive equilibrium.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual functions $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$, of aggregate quantities $(K_t, L_t, A_t, Y_t, \Omega_t, m_t)_{t\geq 0}$, of price processes $(w_t, r_t, \hat{w}_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N)_{t\geq 0}$, of fiscal policies $(\tau_t^L, \tau_t^S, \tau_t^K, \tau_t^B, B_t, T_t)_{t\geq 0}$, and of monetary policies $(\pi_t^W, \pi_t^P)_{t\geq 0}$ such that, for an initial wealth and productivity distribution $(a_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, and for initial values of capital stock and public debt verifying $K_{-1} + B_{-1} = \int_i a_{i,-1} \ell(di)$, and for an initial value of the aggregate shock z_0, we have:*

1. *given prices, the functions $(c_{i,t}, a_{i,t}, \nu_{i,t})_{t\geq 0, i\in\mathcal{I}}$ solve the agent's optimization program (16);*

[4] See e.g. Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018) for a proof of the existence of such functions.

2. *financial, and goods markets clear at all dates: for any $t \geq 0$, equations (19) and (20) hold;*

3. *the government budget is balanced at all dates: equation (14) holds for all $t \geq 0$;*

4. *factor prices $(w_t, \hat{w}_t, \tilde{w}_t, \tilde{r}_t^K)_{t\geq 0}$ are consistent with condition (4), as well as with post-tax definitions (12) and (13);*

5. *interest rates $(r_t, \hat{w}_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N)_{t\geq 0}$ and capital tax $(\tau_t^K)_{t\geq 0}$ are consistent with no-profit and no-arbitrage conditions (10) and (11).*

6. *firms' profits Ω_t and the mark-up m_t are consistent with equations (1) and (3);*

7. *the price inflation path $(\pi_t^P)_{t\geq 0}$ is consistent with the price Phillips curve (2), while the wage inflation path $(\pi_t^W)_{t\geq 0}$ is consistent with the wage Phillips curve (8).*

2.8 Interpretation the TFP shock as an energy price shock

We explain how the TFP shock can be interpreted as an energy price shock. Consider a CRS production function $\tilde{F}$ using capital, labor, and energy. Energy is denoted E and its price is denoted by $\tilde{q}$. We thus have:

$$\tilde{F}(K, L, E) = \tilde{Z} K^{\alpha_K} L^{\alpha_L} E^{1-\alpha_K-\alpha_L},$$

where α_K and α_L are capital and labor shares respectively. We can easily generalize the construction of Section 2.3. The markup of equation (1) is denoted with a tilde and becomes:

$$\tilde{m}_t = \frac{1}{\tilde{Z}_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha_K}\right)^{\alpha_K} \left(\frac{\tilde{w}_t}{\alpha_L}\right)^{\alpha_L} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\alpha_K-\alpha_L},$$

while factor prices are defined as follows:

$$\tilde{r}_t^K + \delta = \tilde{m}_t \alpha_K \tilde{Z}_t K_{t-1}^{\alpha_K-1} L_t^{\alpha_L} E_t^{1-\alpha_K-\alpha_L}, \tag{21}$$

$$\tilde{w}_t = \tilde{m}_t \alpha_L \tilde{Z}_t K_{t-1}^{\alpha_K-1} L^{\alpha_L-1} E_t^{1-\alpha_K-\alpha_L}, \tag{22}$$

$$\tilde{q}_t = \tilde{m}_t (1-\alpha_K-\alpha_L) \tilde{Z}_t K_{t-1}^{\alpha_K} L_t^{\alpha_L} E_t^{-\alpha_K-\alpha_L} \tag{23}$$

Using the expression (23) of $\tilde{q}_t$, we obtain:

$$E_t = \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)\tilde{Z}_t}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}} K_t^{\frac{\alpha_K}{\alpha_K+\alpha_L}} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}}. \tag{24}$$

We introduce the following notation:

$$Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}, \tag{25}$$

$$\alpha = \frac{\alpha_K}{\alpha_K+\alpha_L}, \tag{26}$$

$$m_t = (\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}}. \tag{27}$$

Substituting for the expression (24) of E_t into factor prices (21), we obtain:

$$\begin{aligned}
\tilde{r}_t^K + \delta &= \tilde{m}_t\alpha_K\tilde{Z}_t K_{t-1}^{\alpha_K-1} L^{\alpha_L} \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)\tilde{Z}_t}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\frac{\alpha_K}{\alpha_K+\alpha_L}-\alpha_K} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}-\alpha_L} \\
&= (\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}} \frac{\alpha_K}{\alpha_K+\alpha_L} \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}} K_{t-1}^{\frac{\alpha_K}{\alpha_K+\alpha_L}-1} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}} \\
&= m_t\alpha Z_t K_t^{\alpha-1} L_t^{1-\alpha},
\end{aligned} \tag{28}$$

where the second equality comes from rearrangement and the last from the definitions (25)–(27). Similarly for (22):

$$\begin{aligned}
\tilde{w}_t &= \tilde{m}_t\alpha_L\tilde{Z}_t K_{t-1}^{\alpha_K} L_t^{\alpha_L-1} \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)\tilde{Z}_t}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\frac{\alpha_K}{\alpha_K+\alpha_L}-\alpha_K} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}-\alpha_L} \\
&= (\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}} \frac{\alpha_L}{\alpha_K+\alpha_L} \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}} K_{t-1}^{\frac{\alpha_K}{\alpha_K+\alpha_L}} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}-1} \\
&= m_t(1-\alpha)Z_t K_{t-1}^{\alpha} L_t^{-\alpha}.
\end{aligned} \tag{29}$$

We have been able to rewrite factor prices $\tilde{r}_t$ and $\tilde{w}_t$ consistently with definition (4). We now have to find a consistent definition of the production function. Adapting (5), we have:

$$\tilde{F}(K_{t-1}, L_t, E_t) = \frac{(\tilde{r}_t^K+\delta)K_{t-1} + \tilde{w}_t L_t + \tilde{q}_t E_t}{\tilde{m}_t},$$

or after substituting the expressions of $\tilde{F}$ and E_t and

$$\begin{aligned}
\frac{(\tilde{r}_t^K+\delta)K_{t-1} + \tilde{w}_t L_t}{\tilde{m}_t} &= \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{m}_t(1-\alpha_K-\alpha_L)}{\tilde{q}_t}\right)^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\frac{\alpha_K}{\alpha_K+\alpha_L}} L_t^{\frac{\alpha_L}{\alpha_K+\alpha_L}} (\alpha_K+\alpha_L), \\
&= Z_t(\alpha_K+\alpha_L)\tilde{m}_t^{\frac{1}{\alpha_K+\alpha_L}-1} K_{t-1}^{\alpha} L_t^{1-\alpha},
\end{aligned}$$

where we have used the definitions (25) of $Z_t$and (26) of α. Using the definition (27) of $\tilde{m}_t$, we finally obtain:

$$Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{(\tilde{r}_t^K+\delta)K_{t-1} + \tilde{w}_t L_t}{m_t},$$

which is thus similar to (5). The function $F(K,L) = ZK^{\alpha}L^{1-\alpha}$ with Z and α defined in (25) and (26) is thus consistent with the new definitions of factor prices (28) and (29), the markup (27), as well as with the equation (5) connection output, factor prices and markups.

Interestingly, the TFP expression is $Z_t = \tilde{Z}_t^{\frac{1}{\alpha_K+\alpha_L}} \left(\frac{\tilde{q}_t}{1-\alpha_K-\alpha_L}\right)^{1-\frac{1}{\alpha_K+\alpha_L}}$ with $0 < \alpha_K + \alpha_L < 1$: an increase in energy prices (a higher $\tilde{q}_t$) can thus be interpreted as a drop in TFP Z_t. We will use this analogy in our quantitative exercise of Section 6.

Alternatively to a Cobb Douglas production function, one would consider a production function with Constant Elasticity of Substitution (CES) of the following form:

$$F(K_{t-1}, L_t, E_t) = Z_t\Big[(1-\epsilon)^{\frac{1}{\eta}}\left(K_{t-1}^{\alpha}L_t^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} + \epsilon^{\frac{1}{\eta}}\left(E_t\right)^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}}$$

where ϵ is the energy share and η and the elasticity of substitution between energy inputs and value-added $V_t = K_{t-1}^{\alpha}L_t^{1-\alpha}$. When the elasticity of substitution is close to zero, small fluctuation in quantity of energy supplied can cause large spike in energy prices and marginal costs for firms. This ordering of the nests between energy, labor and capital is suggested by empirical evidence that the energy share in the economy follows closely the fluctuation in energy prices. Such CES expression is well suited for matching such patterns, as suggested in \cite{Hassler et al. (2021)} .

We cover this production function in a forthcoming extension of this work.

3 The Ramsey problem

The Ramsey problem consists in finding the joint optimal fiscal-monetary policy, which yields the competitive equilibrium with the highest aggregate welfare – according to a specific social welfare function. The fiscal-monetary policy consists of eight instruments, $(\tau_t^L, \tau_t^S, \tau_t^K, T_t, B_t, \tilde{R}_t^N, \pi_t^P, \pi_t^W)_{t\geq 0}$, which all have individual and general equilibrium effects. Regarding the aggregate welfare, we opt for a flexible form of social welfare function that is more general than, even though it embeds, the standard utilitarian criterion. The planner is assumed to consider a weighted sum of individual utilities, in which the weight of an agent in a given period depends on agent's productivity level in that period. Formally, weights can be defined as mapping from the set of productivity level realizations, $\mathcal{Y}$, to $\mathbb{R}_+$: $\omega(y_{i,t})$ is the weight of agent i with productivity $y_{i,t}$. Two different agents with the same productivity levels will thus have the same weight. The Utilitarian case corresponds to $\omega(y) = 1$ for all y.[5] The weighted social welfare function is close to approach of Heathcote and Tsujiyama (2021b), and has already been used in an intertemporal setting by LeGrand et al. (2022), Dávila and Schaab (2022) McKay and Wolf (2022), and LeGrand and

[5] A more general specification would consider the weights being a function of the whole history of idiosyncratic shocks for each agent: $\omega_t(y^{i,t})$. A generalization is not needed in the quantitative analysis, we hence follow the simpler formulation.

Ragot (2023). Formally, the planner's aggregate welfare criterion can be expressed as:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega(y_t^i) \left(u(c_t^i) - v(l_t^i) \right) \ell(di) - \frac{\psi_W}{2} (\pi_t^W)^2 \right]. \tag{30}$$

Using this aggregate welfare criterion, the Ramsey program can be expressed as a maximization problem, subject to a number of constraints. These constraints have to guarantee that the planner selects a competitive equilibrium, while the fiscal instruments imply a balanced governmental budget and monetary instruments are consistent with Phillips curves. Formally, the Ramsey program can be expressed as:

$$\max_{\left(\tau_t^L, \tau_t^S, \tau_t^K, B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, \hat{w}_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N, K_t, L_t, Y_t, \Omega_t, m_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} W_0, \tag{31}$$

$$G_t + r_t A_{t-1} + w_t L_t + T_t \leq \left(1 - \frac{\psi_P}{2} (\pi_t^P)^2\right) Y_t - \delta K_{t-1} + B_t, \tag{32}$$

for all $i \in \mathcal{I}$: $c_{i,t} + a_{i,t} = (1 + r_t) a_{i,t-1} + w_t y_{i,t} L_t + T_t,$ (33)

$$a_{i,t} \geq -\bar{a}, \nu_{i,t}(a_{i,t} + \bar{a}) = 0, \ \nu_{i,t} \geq 0, \tag{34}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) u'(c_{i,t+1}) \Big] + \nu_{i,t}, \tag{35}$$

$$\pi_t^W (\pi_t^W + 1) = \frac{\varepsilon_W}{\psi_W} \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di) \right) L_t + \mathbb{E}_t \Big[\pi_{t+1}^W (\pi_{t+1}^W + 1) \Big], \tag{36}$$

$$\pi_t^P (1 + \pi_t^P) = \frac{\varepsilon_P - 1}{\psi_P} (m_t - 1) + \beta \mathbb{E}_t \left(\pi_{t+1}^P (1 + \pi_{t+1}^P) \frac{Y_{t+1}}{Y_t} \right). \tag{37}$$

$$A_t = \int_i a_{i,t} \ell(di) = K_t + B_t, \tag{38}$$

and subject to several other constraints (which are not reported here for space constraints): the definition (1) of m_t, the definition (5) of Y_t, the definition (3) of profits Ω_t, the factor-price relationship (4), the definitions (12)–(13) of post-tax quantities, the no-profit condition (10) of the fund, the no-arbitrage condition (11), the positivity of consumption choices, and initial conditions.

The constraints in the Ramsey program include: the governmental and individual budget constraints (32) and (33), the individual credit constraint (and related constraints on $\nu_{i,t}$) (34), Euler equation (35), the Phillips curves (36) and (37), and financial market clearing conditions (38).

We could further simplify the expression of the previous Ramsey program by following Chamley (1986) and removing taxes and pre-tax quantities, such that the planner directly chooses post-tax quantities. Pre-tax quantities and taxes can be deduced from their definitions and allocation. However, we will also consider economies where some fiscal instruments are set exogenously and not optimal. We have thus chosen to provide the expression of the full-fledged version of the program, where fixing taxes only involves adding a constraint to the program.

To simplify the derivation of first-order conditions, we use some aspects of the methodology of Marcet and Marimon (2019), which is sometimes called the Lagrangian method (Golosov et al., 2016), applied to incomplete-market environments. We denote by $\beta^t\lambda_{i,t}$ the Lagrange multipliers of the Euler equations (35) of agent i at date t. The Lagrange multiplier of the government budget constraint is $\beta^t\mu_t$.

4 An equivalence result

Before presenting our equivalence result, we will discuss the flexible price allocation.

4.1 The flexible-price economy

The flexible-price economy features no price- and no wage-adjustment costs. Inflation rate on wage and price are set to zero and Phillips curves do not constraint the planner's choice and can thus be removed from the program. Because of the null inflation, the firms make no profit and we thus have $m_t = 1$ and $v'(L_t) = w_t \int_i y_{i,t}u'(c_{i,t})\ell(di)$. The Ramsey program therefore actually involves a smaller set of instruments: the post tax rate r_t, the post-tax wage w_t, individual savings $(a_{i,t})_i$, public debt B_t, labor supply L_t, and lump sum tax T_t. Indeed, pre-tax quantities can be deduced from factor price definitions (4), the no-arbitrage condition (11), and $m_t = 1$, while taxes can be computed using post-tax definitions (12) and (13).

Before turning to the FOCs of the Ramsey program, we introduce a useful concept for their formulation and interpretation. We define the *social valuation of liquidity for agent* i, denoted by $\psi_{i,t}$, as:

$$\psi_{i,t} := \underbrace{\omega_t^i u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}} + \underbrace{\lambda_{L,t} y_{i,t} w_t u''(c_{i,t})}_{\text{effect on labor supply}} \tag{39}$$

The valuation $\psi_{i,t}$ measures the benefit – from the planner's perspective – of transferring one extra unit of consumption to agent i. This can be interpreted as the planner's version of the marginal utility of consumption for the agent.[6] As can be seen in equation (39), this valuation consists of three terms. The first is the marginal utility of consumption $\omega_t^i u'(c_{i,t})$, which is the private valuation of liquidity for agent i multiplied by the current weight of agent i. The two other terms can be understood as the internalization, by the planner, of the economy-wide externalities of this extra consumption unit. More precisely, the second term in (39) takes into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between periods t and $t+1$, and thus makes her Euler equation

[6] To simplify the notation, we keep the index i, but the sequential representation can be derived along the lines of equation (17).

(either nominal or real) more "binding". This more "binding" constraint reduces the utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,t}$. The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint of date $t-1$. This is reflected in the quantity $\lambda_{i,t-1}$ $(x = b, k)$. Finally, the last term reflects the wealth effect of the labor supply. Indeed, transferring an extra consumption unit to agent i deters her labor supply incentives through her labor supply $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)$, resulting from union bargaining.

In addition to $\psi_{i,t}$, another key quantity is the Lagrange multiplier, μ_t, on the governmental budget constraint. The quantity μ_t represents the marginal cost in period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_{i,t} - \mu_t$ can be interpreted as the "net" valuation of liquidity: this is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_{i,t} := \psi_{i,t} - \mu_t. \tag{40}$$

The interpretation of first-order conditions is greatly clarified by expressing them using $\hat{\psi}_{i,t}$ rather than the multiplier on Euler equations, $\lambda_{i,t}$.

The Euler-like equation, corresponding to the first-order conditions with respect to individual savings for non-credit-constrained agents, can be written as follows:

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \right]. \tag{41}$$

Constrained agents i have no Euler equation, as $a_{i,t} = -\underline{a}$ and $\lambda_{i,t} = 0$. Equation (41) states that the net social value of liquidity should be smoothed out over time with the post-tax interest rate. It can be interpreted as a Euler-like equation for the planner and generalizes the standard individual Euler equation by taking into account the externalities of saving choices.

The second first-order condition, which regards the post-tax wage rate w_t, is:

$$\int_i \hat{\psi}_{i,t} y_{i,t} l_{i,t} \ell(di) = -\lambda_{L,t} \int_i y_{i,t} u'(c_{i,t}) \ell(di). \tag{42}$$

In the absence of any effect on the labor supply, the planner would choose the labor tax so as to equalize the marginal benefit to its marginal cost. The marginal benefit is equal to $\mu_t \int_i y_{i,t} l_{i,t} \ell(di)$ since the tax base is the efficient labor supply and the extra unit relaxes the governmental budget constraint. The marginal cost is $\int_i \psi_{i,t} y_{i,t} l_{i,t} \ell(di)$ since $\psi_{i,t}$ values the cost of taking one unit away from agent i. Furthermore, the planner also needs to take into account the distortion of the labor tax on the aggregate labor supply, which is proportional to the Lagrange multiplier $\lambda_{L,t}$.

The third first-order condition concerns the post-tax interest rate r_t:

$$\int_i \hat{\psi}_{i,t} a_{i,t-1} \ell(di) = -\int_i \lambda_{i,t-1} u'(c_{i,t}) \ell(di). \tag{43}$$

Similarly to equation (42), in the absence of any saving distortion, the planner would balance the marginal benefit and marginal cost of the capital tax. The marginal benefit would be $\mu_t \int_i a_{i,t-1}\ell(di)$ since the tax base of the capital tax is beginning-of-period asset holdings. The marginal cost would be $\int_i \psi_{i,t} a_{i,t-1}\ell(di)$, since $\psi_{i,t}$ still values the cost of taxing one unit to agent i. The planner additionally to account for the effect of the capital tax on saving incentives. This effect is proportional to the shadow cost of the nominal Euler equation. This shadow cost can be positive or negative, depending on whether the planner perceives excess of or lack of aggregate savings (see LeGrand and Ragot, 2022a, for a lengthier discussion).

The fourth first-order condition regards the public debt:

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + F_{K,t+1} \right) \right], \tag{44}$$

and states that the shadow cost of the governmental budget should be smoothed out over time using the marginal productivity of capital. At the steady state, this equation implies the modified golden rule.

The last first-order equation concerns the labor supply:

$$\left(\int_i \omega_t^i \ell(di)\right) v'(L_t) + \lambda_{L,t} v''(L_t) = w_t \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) + \mu_t F_{L,t}. \tag{45}$$

which equalizes the marginal social cost of labor (left-hand side) to its marginal benefit (right-hand side). Similarly to the expression (39) of $\psi_{i,t}$, the marginal social cost of labor involves the private cost $v'(L_t)$ as well as the planner's internalization of the general-equilibrium effect of modifying labor supply (hence the presence of the multiplier $\lambda_{L,t}$). The marginal benefits of an extra unit of labor supply come from the related increase in individual consumption (through $\hat{\psi}_{i,t}$) and from the higher output and higher labor taxes that relax the governmental budget constraint (through μ_t).

Finally, the last condition regarding the lump-sum transfer T_t is:

$$\int_i \hat{\psi}_{i,t} \ell(di) \leq 0, \tag{46}$$

which holds with equality when $T_t > 0$. The lump-sum transfer implies no distortion and the planner would like to equal its marginal cost (μ_t: one unit taken from the government budget) to its marginal benefit ($\int_i \hat{\psi}_{i,t}\ell(di)$: one unit given to all agents).

4.2 The equivalence result

The monetary economy features two sets of market imperfections. The first set is related to the goods market. Intermediary firms enjoy a monopoly power, which implies a price markup m_t that can differ from one. There is also a Rotemberg cost for price adjustment, which prevents firms from freely setting their price. Note that the good market imperfections are complementary: one vanishes when the other is absent, as can be seen from the price Phillips curve (2). The second set of imperfections is related to the labor market. The union implies that the labor supply of agents is not set optimally, while the Rotemberg cost for wages prevents unions from freely setting wages. Note that in the absence of Rotemberg cost, the labor supply still remains sub-optimal,as as it remains set at the union level. The equation characterizing the choice of the labor supply (common to all agents) is $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di)$, while it would be $v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t})$, if agents were able to choose their individual labor supply $l_{i,t}$. In the flexible price economy of Section 4.1, we therefore focused on the case of a sub-optimal labor supply.

We now explain that the two taxes labor and the capital tax are sufficient tools to offset these two sets of inefficiencies along the business cycle, even when agents are heterogeneous. To do so, we solve for the joint optimal monetary-fiscal policies when the government has access to a full set of fiscal tools. This program can be written as:

$$\max_{\left(B_t, T_t, \pi_t^P, \pi_t^W, w_t, r_t, L_t, (c_{i,t}, a_{i,t}, \nu_{i,t})_i\right)_{t \geq 0}} W_0, \tag{47}$$

subject to the same equations as in the Ramsey program (31)–(38). We can make a number observations. First, we have dropped taxes τ_t^L, τ_t^S, and τ_t^K from the Ramsey program since, as in the flexible-price case, they can be substituted by post-tax quantities w_t, r_t, and $\hat{w}_t$ (and recovered from allocation). Second, the pre-tax nominal rate $\tilde{R}_t^N$ is also dropped, as it does not play any role in the Ramsey program but can be recovered from the no-arbitrage condition (11). Third, the before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ play a role only in the markup coefficient m_t of equation (1) and in the factor price equation (4). In other words, the before-tax rates can be recovered from m_t and from the allocation. Fourth, the markup coefficient m_t only appears in the price Phillips curve and can thus be recovered from the inflation path (π_t^P). Similarly, the profits do not explicitly appear in the Ramsey program and can be deduced from the allocation. The union wage $\hat{w}_t$ can be deduced from the wage inflation path $(\pi_t^W)_t$.

As a consequence, price inflation only appears in the planner's program through the output destruction term (i.e., $-\kappa(\pi_t^P)^2/2$) in the government budget constraint. The planner thus chooses a zero price inflation, as any deviation from zero inflation shrinks the feasible set of the planner. A zero price inflation implies a unit markup: $m_t = 1$ and there is no efficiency left on the goods market. At the end, the planner's faces the same program – and hence chooses the same allocation – as in the real economy. We summarize this first result in the following proposition.

Similarly, wage inflation only appears in the planner's objective as a utility cost (i.e., $-\kappa(\pi_t^W)^2/2$). The planner thus chooses a zero wage inflation, which implies because of the wage Phillips curve that the labor supply is set through $v'(L_t) = w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di)$ – as in the flexible price economy of Section 4.1.

In consequence, all in all, the planner chooses zero price and zero wage inflations and faces the same program as in the flexible price economy of Section 4.1. the planner thus implements the same allocation in both cases.

Proposition 1 (An equivalence result) *When capital and both labor taxes are available, the government exactly reproduces the flexible-price allocation and the inflation on prices and wages is null in all periods.*

Proposition 1 generalizes the equivalence result of LeGrand et al. (2022) to the joint presence of sticky wages and sticky prices.[7] The intuition is the same: in the presence of a sufficiently large fiscal system, monetary policy has no role but price stability. Importantly, the result requires the presence of two labor taxes. The first labor tax τ^S (internalized by the planner) enables the planner to "isolate" the pre-tax rate $\tilde{w}_t$ that is determined by the allocation (with a zero price inflation) from the union wage $\hat{w}_t$ that is determined by the inflation path $(\pi_t^W)_t$. Removing τ^S as an independent instrument would impose a constraint between the factor price $\tilde{w}_t$ and the wage inflation path. In other words, the planner would have to balance the effects of price inflation (determining $\tilde{w}_t$) and of wage inflation (determining $\hat{w}_t$). The result of Proposition 1 would not hold anymore and the economy would feature positive inflation on wages and prices.

5 Simulating the model: The truncation representation

The Ramsey problem of Section cannot be solved with simple simulation techniques. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. The steady-state value of the set of Lagrange multipliers is not easy to find, and the planner's instruments depend on the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of wealth and Lagrange multipliers.[8] The accuracy of optimal policies has been further analyzed in LeGrand and Ragot (2022b) for both the steady state and the dynamics.

The intuition of the method can be summarized as follows. In heterogeneous-agent models, agents differ according to their idiosyncratic history. An agent i has a period-t history $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\}$. Let $h = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$ be a given history of length N. In period t, an agent i is said to have *truncated history* h if the history of this agent for the last N periods is equal to $h = \{y_{i,t-n+1}, \ldots, y_{i,t}\}$. The idea of the truncation method is to aggregate agents having the same truncated history and to express the model using these groups of agents rather than individuals. This generated the so-called *truncated model*, which features a finite state space. In the truncated model, the agents' aggregation assumes full risk-sharing within each truncated

[7]Correia et al. (2008) and Correia et al. (2013) also show an equivalence result in a complete market environment with consumption taxes.

[8]Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement, as the shape of inflation is highly non-linear.

history, while the "true" Bewley model features wealth heterogeneity among the agents having the same truncated history h. This simply comes from the heterogeneity in histories prior to the aggregation period (i.e., more than N periods ago). We capture this within-truncated-history through additional parameters – denoted by "ξs" – which are truncated-history specific. This construction yields a finite state-space representation, which is exogenous to agents' choices and thereby allows one to compute optimal policies.[9]

The previous truncation method is simple to implement, but it has the drawback of considering many histories, some of them being very unlikely to be experienced by agents. By the law of large number, these histories concern a very small number of agents. For instance, for a truncation length of $N = 5$ used below, many histories have a size smaller than 10^{-6}. The idea of LeGrand and Ragot (2022c) is to consider different truncation lengths for different histories. For the sake of clarity, we will call this method the *refined* truncation, while the former one will be called the *uniform* truncation. Histories more likely to be experienced (i.e., with a bigger size) can be "refined", which means that they can be substituted by a set of histories with higher truncation lengths. For instance, the truncated history (y_1, y_1) $(N = 2)$ can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have been in productivity y_1 for two consecutive period is split into $n_y(= Card\mathcal{Y})$ truncated histories.

A benefit of this construction is that the number of histories is a *linear* function of the maximum truncation length, instead of an exponential function. The construction of the refinements is detailed in LeGrand and Ragot (2022c). A difficulty of the construction that the set of refined histories must form a well-defined partition of the set of idiosyncratic histories in each period.

To find the steady-state values of the Lagrange multipliers, we use the following algorithm:

1. Set a truncation length N and guess values for the planner's instruments.

2. Solve the steady-state allocation of the full-fledged Bewley model with the previous instrument values, using standard techniques.

3. Consider the truncated representation of the economy for a truncation length N.

    (a) Solve for the joint distribution of wealth and Lagrange multipliers.

    (b) Analyze whether the values of instruments are above or below their optimal value.

4. Change the instruments' values accordingly (or stop if their value is close enough to the optimal value), and redo the process from Step 2.

5. Increase the truncation length N, and restart from Step 2 until increasing N has no impact on the instruments' values.

[9]Considering wealth bins is not possible, as the savings function and thus the transitions across wealth bins is endogenous to the planner's policy. This would imply a fixed point which would be very hard to solve.

6 Quantitative assessment

6.1 The calibration and steady-state distribution

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$, and the period utility function is: $\log(c) - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$. The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent models. The scaling parameter is $\chi = 0.046$, to obtain an aggregate labor supply of roughly 1/3.

Technology and TFP shock. The production function is Cobb-Douglas: $Y = ZK^{\alpha}L^{1-\alpha}$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $\varepsilon_t^z \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_z^2)$. We use the standard values $\rho_z = 0.8$.

Idiosyncratic risk. We use a standard productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.993$ and a standard deviation of $\sigma_y = 12.1\%$. These values are consistent with empirical estimates (Krueger et al., 2018). This process generates a realistic empirical pattern for wealth. The Gini coefficient of the wealth distribution amounts to 0.79, while the model implies an average annual capital-to-GDP ratio of 2.5. These two values are in line with their empirical counterparts. Finally, the Rouwenhorst (1995) procedure is used to discretize the productivity process into 10 idiosyncratic states with a constant transition matrix.

Steady state taxes. Steady-state parameters of the fiscal rules are calibrated based on the computations of Trabandt and Uhlig (2011), who use the methodology of Mendoza et al. (1994) on public finance data prior to 2008. This approach consists in computing a linear tax on capital and on labor, as well as lump-sum transfers that are consistent with the governmental budget constraint. Their estimations for the US in 2007 yield a capital tax (including both personal and corporate taxes) of 36%, a labor tax of 28% and lump-sum transfers equal to 8% of the GDP. Considering the labor subsidy τ^S, we set it to 0 at the steady state, as there is no direct evidence of any subsidy of this type. We then consider the steady-state values $(\tau_*^L, \tau_*^K, \tau_*^S, T_*/GDP) = (28\%, 36\%, 0\%, 8\%)$. This steady-state fiscal system generates two untargeted outcomes. First, it implies a public debt-to-GDP ratio equal to 63%, which is very close to the value of 63% estimated by Trabandt and Uhlig (2011). Second, it also implies a public spending-to-GDP ratio equal to 12.1%. This value is consistent with other quantitative investigations (Bhandari et al., 2017), even though a little bit low compared to the postwar value, which has decreased to 14.1% in 2017, from 17% in the 1970s.

Monetary parameters. We follow the literature and assume that the elasticity of substitution across goods is $\varepsilon = 6$ and the price adjustment cost is $\kappa = 100$ (see Bilbiie and Ragot, 2021, for a discussion and references). We use the same values for the nominal wage rigidity. Table 1 provides a summary of the model parameters.

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
σ	Curvature utility	1
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.046
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_y	Autocorrelation idio. income	0.993
σ_y	Standard dev. idio. income	12.1%
	Tax system	
τ^K	Capital tax	36%
τ^L	Labor tax	28%
T	Transfer over GDP	8%
τ^S	Labor subsidy	0%
B/Y	Public debt over yearly GDP	63%
G/Y	Public spending over yearly GDP	12.1%
	Monetary parameters	
ψ_p	Price adjustment cost	100
ψ_w	Wage adjustment cost	100
ε_p	Elasticity of sub. between goods	6
ε_w	Elasticity of sub. labor inputs	6

Table 1: Parameter values in the baseline calibration. See text for descriptions and targets.

Steady-state equilibrium distribution. We first simulate the full-fledged Bewley model (i.e., without aggregate shocks) with the steady-state optimal inflation rates $\pi^P = \pi^W = 0$. In Table 2, we report the wealth distribution generated by the model and compare it to the empirical distribution. We compute a number of standard statistics – listed in the first column – including the quintiles, the Gini coefficient.

The empirical wealth distribution reported in the second and third columns of Table 2 is computed using two sources, the PSID for the year 2006 and the SCF for the year 2007. The fourth column reports the wealth distribution generated by our model. The Gini coefficient of the model is 0.79 and is very close to the value of 0.78 in the data. The model reproduces well the distribution.

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	-0.9	-0.2	0.0
Q2	0.8	1.2	0.1
Q3	4.4	4.6	2.7
Q4	13.0	11.9	15.0
Q5	82.7	82.5	82.2
Gini	0.77	0.78	0.79

Table 2: Wealth distribution in the data and in the model.

Truncation period. We now construct the truncated model. We use the refined truncation approach, with a number of length for the refinement equals to $N = 8$. We check that the results do not depend on the choice of the truncation length. As in LeGrand and Ragot (2022a), the truncation provides accurate results, thanks to the introduction of the ξs parameters, as explained in Section 5.

6.2 Dynamics with fiscal and monetary rules

We first simulate the model with ad-hoc fiscal and monetary rules. We will then compare the allocation with ad-hoc fiscal rules with the allocation for the optimal instrument dynamics. Concerning monetary policy, we introduce a standard Taylor rule, which depends on both price and wage inflation:

$$\tilde{R}_t^N = \tilde{R}_*^N + \kappa_p \pi_t^p + \kappa_w \pi_t^w$$

where $\tilde{R}_t^N$ is the nominal interest rate between period t and period $t+1$. The constant $\tilde{R}_*^n = \frac{1}{\beta}$ is the steady-state nominal rate. The coefficients κ_p and κ_w measure the reaction of the interest rate to price inflation and wage inflation, respectively. We consider the standard values of $\kappa_m = \kappa_p = 1.5$. As noted by Erceg et al. (2000), price determinacy is implied by the sum of the two coefficients $\kappa_p + \kappa_m$ being greater than a given threshold. We check that this actually prevails in our setup.

Considering fiscal rules, we assume that tax rates are constant and set to their steady-state values. We only introduce an adjustment in the transfers related to the debt level à la Bohn (1998) to ensure debt sustainability. The fiscal rules thus involve $\tau_t^L = \tau_*^L$, $\tau_t^K = \tau_*^K$, $\tau_t^S = \tau_*^S$

and:

$$T_t = T_* + \rho_B \left(B_t - \bar{B}\right),$$

where we set $\rho_B = 0.08$.

Figure 1 plots the Impulse Response Functions (IRFs) for the main variables in an economy with the previous fiscal rules. We simulate the response of the economy to an input price shock generating an AR(1) dynamics in the scaling parameter A_t. More formally, we assume that A_t follows:

$$A_t = \rho_A A_{t-1} + (1 - \rho_A \grave{a} \bar{A} + \varepsilon_t^A, \tag{48}$$

where ε_t^A is the shock to A_t generated by the input price shock. Figure 1 plot the dynamics for the main variables after a shock ε_0^A normalized to -1%. All variables are reported in percent deviation from their steady-state values.

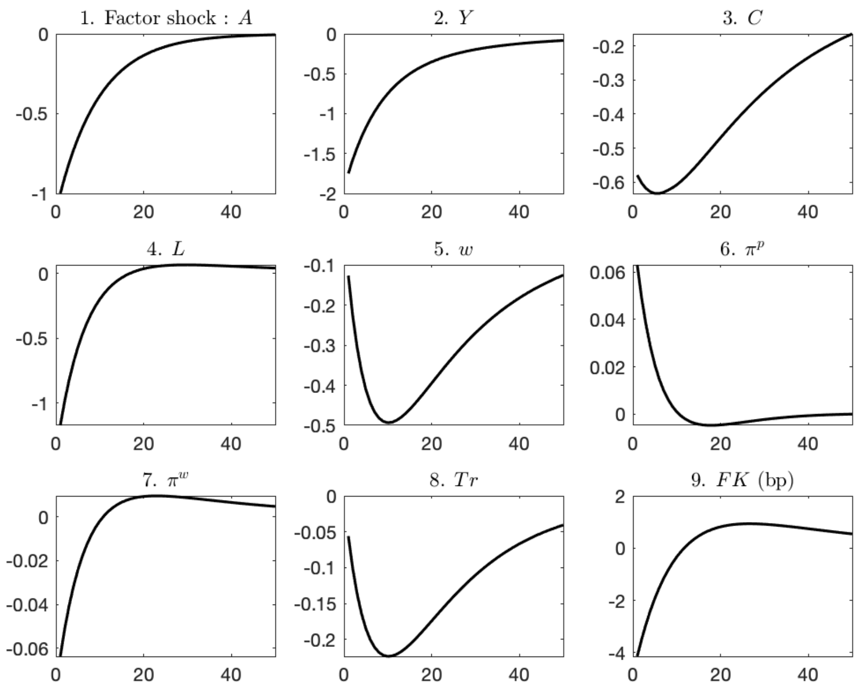

Figure 1: Impulse response functions of main variables after a negative factor shock (A), for the model with a Taylor rule and simple fiscal rules. All variables are reported in proportional percentage deviations from steady state values, except the marginal product of capital (FK), which is a level deviation reported in basis points.

The shock is a negative supply shock. It generates a fall in output (Y), consumption (C) and labor (L). This negative supply shock generated an increase in price inflation and a decrease in wage inflation, as a standard outcome. Transfers are marginally decreasing to ensure debt sustainability, as other tax receipts are decreasing. As the real wage is reported in percentage deviation, and as labor tax and labor subsidy are constant, the dynamics of the real wage is the same for both w, $\hat{w}$ and $\tilde{w}$.

As both wages and prices are sticky in the short run, real wages take time to react to the supply shock. The demand of labor drops significantly, which affect the labor income of household. The aggregate demand channel amplifies the drop in consumption of the households, in particular those with high Marginal Propensity to Consume (MPC) close to the credit constraint. As a result of these two forces, the marginal rate of substitution between labor and consumption $v'(L)/u'(C)$ drops lower than the real wage w which generate nominal wage deflation. Moreover, the spike in firm marginal cost create inflationary pressures that are mildy counteracted by the fall in interest rate and wages. The outcome of these two inflation mechanisms generate a persistent fall in real wage, strenghtening the drop in aggregate demand and consumption. Aggregate output falls almost twice more than the size of the supply shock.

6.3 Optimal policy with the complete set of fiscal tools

We next simulate the dynamics of the economy with the same parameters and after the same shock, but with now optimal monetary and instrument dynamics. We report the IRFs in Figure 2. Consistently with the results of Proposition 1, the optimal inflation response is null: $\pi^p = \pi^W = 0$, because the wage subsidy generates the optimal real labor cost $\tilde{w}_t$ paid by the firm.

After the shock, capital tax increases by 10%,whereas labor taxes barely move. Public debt increases at impact, then decreases to converge back to its steady-state value. Transfer increases sustaining private consumption of constrained agents. The cost of labor paid by the firm $\tilde{w}_t$ decreases on impact, which shows that the government actually implements a wage subsidy to cut labor cost after the shock.

6.4 Comparison between optimal policy and ad-hoc rules

Figure 3 compares the responses corresponding to ad-hoc rules and to optimal policies of Figures 1 and 2. For both economies, we additionally report the after-subsidy labor cost $\tilde{w}_t$ paid by firms.

The fall in output and consumption is much less severe for the optimal allocation than for the one with ad-hoc rules. The fall in consumption is three times bigger with ad-hoc rules compared to the one with the optimal allocation. Total labor falls much more with ad-hoc rules than with optimal fiscal policy.

The difference between the two allocations comes from two effects. First, the optimal wage subsidy decreases much more the labor cost in the optimal allocation compared to the ones with rules, as can be seen in Panel 5. Second, transfers increase in the optimal allocation, which can be seen in Panel 8 of Figure 2. It fosters the consumption of agents – and especially the poorer ones with high MPC. Finally, the increase in public debt is smaller than in the economy with

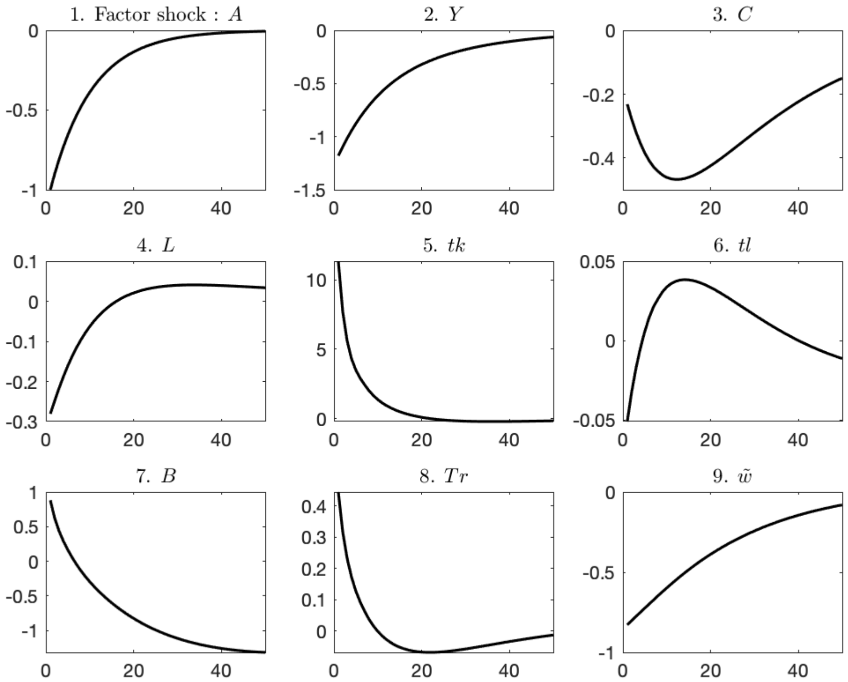

Figure 2: Impulse response functions of main variables after a negative factor shock (A), for the model with optimal monetary and fiscal policy. Output (Y), consumption (C), labor (L), public debt (B) and transfers (Tr) are reported in percentage proportional deviations from steady state. Capital tax tk and labor tax tl are reported in percentage level deviations from steady state. The marginal product of capital (FK) is a level deviation, reported in basis points.

rules, because higher capital taxes in the optimal response contribute to finance the public debt.

6.5 Second-order moments

We report the second-order moments of the main variables in Table 3. We simulate the economies considering the TFP process given in Table 48, for a standard deviation $\sigma^A = 0.01$. We report the unconditional first- and second-order moments for the main variables, in two economies.

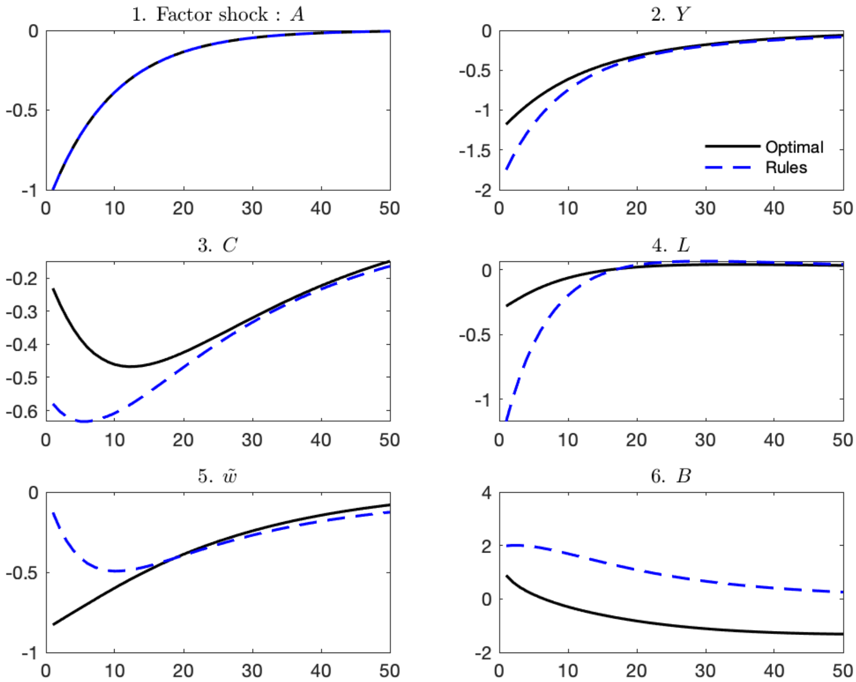

Figure 3: Comparison of the allocation with optimal instruments (black solid line) and with ad-hoc rules (blue dashed line). Variables are reported as percentage deviation from steady state.

		Rules	Optimal
Y	Mean	1.17	1.17
	Std(%)	2.39	1.46
C	Mean	0.74	0.74
	Std(%)	1.65	1.11
K	Mean	12.02	12.02
	Std(%)	25.42	16.59
L	Mean	0.32	0.32
	Std(%)	1.65	0.27
π^p	Mean	0	0
	Std(bp)	6	0
π^w	Mean	0	0
	Std(bp)	8	0

Table 3: First- and second-order moments for key variables, in three economies. the first economy is the one with ad-hoc rules, the second with optimal instruments, the third one with optimal instruments and longer truncation length.

For each variable, we report the steady-state value (labeled "Mean") and the standard deviation in percent. One can check that the economy with ad-hoc rules is much more volatile that the one with optimal policy.

6.6 Comparison with the representative agent economy

In the following exercise, we compare the Heterogeneous agent model described above, with an analogous representative agent economy, where market are complete and credit constraints are absent. We keep the rest of the model identical with price and wage rigidity and moneraty and lump-sum transfer rules. This model is close to the framework of Erceg et al. (2000)with the exception that we include capital and public debt with a large array of fiscal instruments.

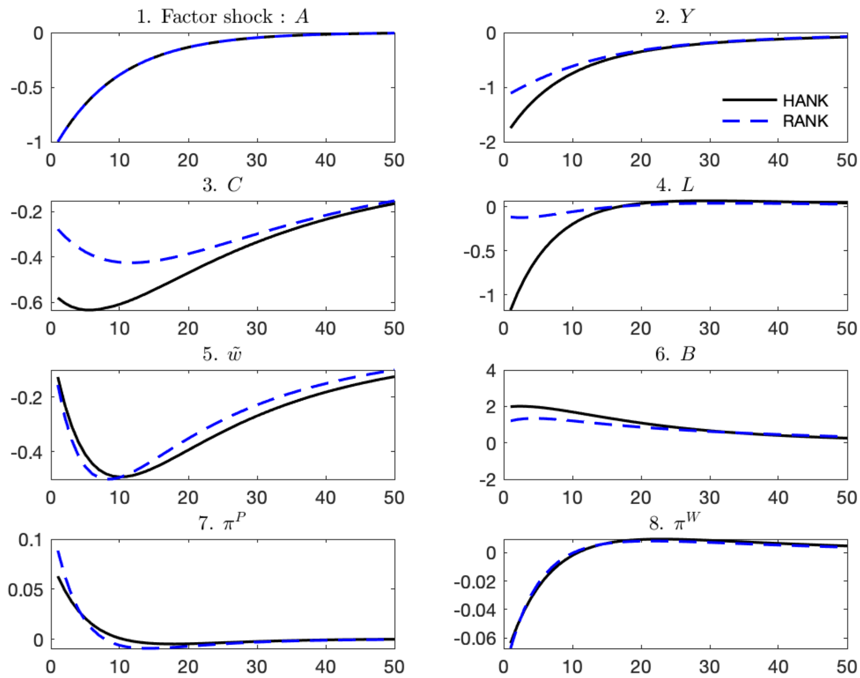

Figure 4: Comparison of the Heterogeneous agent economy (HANK - black solid line) and with the Representative agent model (RANK - blue dashed line). Variables are reported as percentage deviation from steady state.

The aggregate demand channel is amplified almost twofold in the HANK model compared to its representative agent counterpart. The presence of high-MPC households and labor rationing frictions – preventing credit-constrained household to work more – amplifies greatly the drop in consumption and the shortfall in demand from firms. As a result, recession is larger and inflation lower in the heterogeneous agents economy.

7 Conclusion

We derive joint optimal monetary-and-fiscal policy in an heterogeneous agent model with both sticky prices and sticky wages. We consider the response of this economy after input price shocks to study wage-price spirals. We first prove that optimally setting a simple fiscal system featuring capital tax, labor tax and wage subsidy, allows one to implement price and wage stability. The quantitative simulation allows to understand the contributions of the different instruments and hence to identify the distortions implied with two nominal frictions. We also simulate an economy with standard ad-hoc rules (a Taylor rule and fiscal rules). In that economy, recessions are much deeper than with optimal policy.

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Appendix

A Computing the FOCs of the full-fledged Ramsey program with flexible prices

We focus on flexible prices. The governmental budget constraint becomes:

$$G_t + B_{t-1} + r_t A_{t-1} + w_t L_t + T_t = B_t + F(A_{t-1} - B_{t-1}, L_t),$$

while the individual budget constraint is

$$c_{i,t} = -a_{i,t} + (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t + T_t$$

We can thus write the Ramsey program using solely the post tax instrument r_t. The other interest rates can be deduced from the allocation. Formally:

$$\begin{aligned}
\tilde{r}_t^K &= F_K(A_{t-1} - B_{t-1}, L_t), \\
\tilde{R}_t^N &= \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K\right], \\
1 - \tau_t^K &= \frac{r_t A_{t-1}}{\tilde{r}_t^K K_{t-1} + (\tilde{R}_{t-1}^N - 1)B_{t-1}}
\end{aligned}$$

We can thus remove the fund constraint from the Ramsey program. We obtain the following Lagrangian.

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i (u(c_{i,t}) - v(L_t))\ell(di) \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (\lambda_{i,t} - (1 + r_t)\lambda_{i,t-1})u'(c_{i,t})\ell(di) \\
- \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_{L,t} \left(v'(L_t) - w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) \\
+ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(B_t + F(A_{t-1} - B_{t-1}, L_t) - G_t - B_{t-1} - r_t A_{t-1} - w_t L_t - T_t\right)
\end{aligned}$$

with furthermore:

$$\begin{aligned}
c_{i,t} &= -a_{i,t} + (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t + T_t, \\
A_t &= \int_i a_{i,t}\ell(di).
\end{aligned}$$

We use the definitions (39) of $\psi_{i,t}$ and (40) of $\hat{\psi}_{i,t}$.

Derivative with respect to r_t.

$$0 = \int_i \left(\hat{\psi}_{i,t} a_{i,t-1} + \lambda_{i,t-1} u'(c_{i,t}) \right) \ell(di).$$

Derivative with respect to w_t.

$$0 = \int_i \hat{\psi}_{i,t} y_{i,t} L_t \ell(di) + \lambda_{L,t} \int_i y_{i,t} u'(c_{i,t}) \ell(di).$$

Derivative with respect to B_t.

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + F_{K,t+1} \right) \right].$$

Derivative with respect to $a_{i,t}$. For unconstrained agents:

$$\begin{aligned} \psi_{i,t} &= \beta \mathbb{E}_t \left[(1 + r_{t+1}) \psi_{i,t+1} \right] \\ &\quad + \beta \mathbb{E}_t \mu_{t+1} \Big[F_{K,t+1} - r_{t+1} \Big], \end{aligned}$$

or

$$\begin{aligned} \hat{\psi}_{i,t} &= \beta \mathbb{E}_t \Big[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \Big] \\ &\quad - \mu_t + \beta \mathbb{E}_t \mu_{t+1} \Big[F_{K,t+1} + 1 \Big] \end{aligned}$$

Using the FOC on B_t stating that $\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + F_{K,t+1} \right) \right]$ further yields:

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \Big[(1 + r_{t+1}) \hat{\psi}_{i,t+1} \Big].$$

For constrained agents, we have $a_{i,t} = -\bar{a}$ and $\lambda_{i,t} = 0$.

Derivative with respect to L_t.

$$\int_i \omega_t^i \ell(di) v'(L_t) + \lambda_{L,t} v''(L_t) = w_t \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) + \mu_t F_{L,t}.$$

Derivative with respect to T_t.

$$\mu_t \leq \int_j \psi_{j,t} \ell(dj).$$

with equality if $T_t > 0$.

Summary.

$$
\begin{aligned}
0 &= \int_i \Big(\hat{\psi}_{i,t} a_{i,t-1} + \lambda_{i,t-1} u'(c_{i,t})\Big)\, \ell(di), \\
0 &= L_t \int_i \hat{\psi}_{i,t} y_{i,t} \ell(di) + \lambda_{L,t} \int_i y_{i,t} u'(c_{i,t}) \ell(di), \\
0 &= \int_i \hat{\psi}_{i,t} \ell(di), \\
0 &= w_t \int_i y_{i,t} \hat{\psi}_{i,t} \ell(di) + \mu_t F_{L,t} \\
&\quad - (\int_i \omega_t^i \ell(di)) v'(L_t) - \lambda_{L,t} v''(L_t), \\
\mu_t &= \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + F_{K,t+1}\right)\right], \\
\hat{\psi}_{i,t} &= \beta \mathbb{E}_t \Big[(1 + r_{t+1}) \hat{\psi}_{i,t+1}\Big].
\end{aligned}
$$

B Truncated conditions

We have for budget constraints:

$$
\begin{aligned}
c_{t,h} + a_{t,h} &= w_t y_0^h L_t + (1 + r_t) \tilde{a}_{t,h} + T_t, \\
\tilde{a}_{t,h} &= \sum_{\tilde{y}^N \in \mathcal{Y}^N} \pi_{\tilde{h}h} \frac{S_{t-1,\tilde{h}}}{S_{t,h}} a_{t-1,\tilde{h}}, \\
\tilde{\lambda}_{t,h} &= \frac{1}{S_{t,h}} \sum_{\tilde{h} \in \mathcal{H}} S_{t-1,\tilde{h}} \lambda_{t-1,\tilde{h}} \pi_{\tilde{h}h},
\end{aligned}
$$

while Ramsey FOCs are:

$$
\begin{aligned}
\hat{\psi}_{t,h} &= \omega_h \xi_h u'(c_{t,h}) - \Big(\lambda_{t,h} - (1 + r_t) \tilde{\lambda}_{t,h}\Big) \xi_h u''(c_{t,h}) + \lambda_{L,t} y_0^h w_t u''(c_{t,h}) - \mu_t \\
0 &= \sum_{h \in \mathcal{H}} S_{t,h} \hat{\psi}_{t,h} \tilde{a}_{t,h} + \sum_{h \in \mathcal{H}} S_{t,h} \tilde{\lambda}_{t,h} \xi_h u'(c_{t,h}), \\
0 &= L_t \sum_{h \in \mathcal{H}} S_{t,h} y_0^h \hat{\psi}_{t,h} + \lambda_{L,t} \sum_{h \in \mathcal{H}} S_{t,h} y_0^h u'(c_{t,h}), \\
\hat{\psi}_{t,h} &= \beta \mathbb{E}_t \Bigg[(1 + r_{t+1}) \sum_{\tilde{h} \in \mathcal{H}} \pi_{h\tilde{h}} \hat{\psi}_{t+1,\tilde{h}}\Bigg], \\
0 &= w_t \sum_{h \in \mathcal{H}} S_{t,h} y_0^h \hat{\psi}_{t,h} \ell(di) + \mu_t F_{L,t} \\
&\quad - (\sum_h S_h \omega_h) v'(L_t) - \lambda_{L,t} \xi_l v''(L_t), \\
0 &= \sum_h S_{t,h} \hat{\psi}_{t,h}.
\end{aligned}
$$

C Matrix representation

We have for basic definitions:

$$
\begin{aligned}
\boldsymbol{S} &= \boldsymbol{\Pi}^\top \boldsymbol{S}, && (49)\\
\boldsymbol{S} \circ \boldsymbol{c} + \boldsymbol{S} \circ \boldsymbol{a} &= (1+r)\boldsymbol{\Pi}^\top (\boldsymbol{S} \circ \boldsymbol{a}) + wL(\boldsymbol{S} \circ \boldsymbol{y}) + T\boldsymbol{S} && (50)\\
\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}) &= \beta(1+r)\boldsymbol{\Pi}\left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right) + \boldsymbol{\nu}, && (51)\\
\xi_l v'(L) &= w(\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}), && (52)\\
\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_c &= \boldsymbol{\Pi}(\boldsymbol{S} \circ \boldsymbol{\lambda}_c). && (53)
\end{aligned}
$$

which implies that $\boldsymbol{\xi}^{u,E}$ is given by:

$$
u'(\boldsymbol{c}) \circ \boldsymbol{\xi}^{u,E} = (\boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi})^{-1}\boldsymbol{\nu} \tag{54}
$$

The planner's FOCs can be written as follows:

$$
\begin{aligned}
\bar{\boldsymbol{\psi}} &= -\mu\boldsymbol{S} + \bar{\boldsymbol{\omega}} \circ \boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c}) && (55)\\
&\quad - \left(\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - (1+r)\boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - w\lambda_L(\boldsymbol{S} \circ \boldsymbol{y})\right) \circ u''(\boldsymbol{c}) \\
\boldsymbol{P}\bar{\boldsymbol{\psi}} &= \beta(1+r)\boldsymbol{P}\bar{\boldsymbol{\Pi}}\bar{\boldsymbol{\psi}}, && (56)\\
(\boldsymbol{I} - \boldsymbol{P})\bar{\boldsymbol{\lambda}}_c &= 0, && (57)\\
(1^T\bar{\boldsymbol{\omega}})v'(L) + \lambda_L \xi_l v''(L) &= w\boldsymbol{y}^\top\bar{\boldsymbol{\psi}} + \mu F_L && (58)\\
\tilde{\boldsymbol{a}}^\top\bar{\boldsymbol{\psi}} &= -\left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi\bar{\boldsymbol{\lambda}}_c, && (59)\\
\boldsymbol{y}^\top\bar{\boldsymbol{\psi}} &= -(\lambda_L/L)(\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) && (60)\\
1^\top\bar{\boldsymbol{\psi}} &= 0 && (61)
\end{aligned}
$$

Equation (55) implies:

$$
\begin{aligned}
\bar{\boldsymbol{\psi}} &= \bar{\boldsymbol{\omega}} \circ \boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c}) - \left(\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - (1+r)\boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - w\lambda_L\boldsymbol{S} \circ \boldsymbol{y}\right) \circ u''(\boldsymbol{c}) - \mu\boldsymbol{S} \\
\bar{\boldsymbol{\psi}} &= \boldsymbol{D}_{\boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c})}\bar{\boldsymbol{\omega}} - \boldsymbol{D}_{\boldsymbol{\xi}^{u,E} \circ u''(\boldsymbol{c})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi})\bar{\boldsymbol{\lambda}}_c + w\lambda_L\boldsymbol{S} \circ \boldsymbol{y} \circ u''(\boldsymbol{c}) - \mu\boldsymbol{S} \\
&= \hat{\boldsymbol{M}}_0\bar{\boldsymbol{\omega}} + \hat{\boldsymbol{M}}_1\bar{\boldsymbol{\lambda}}_c + \hat{\boldsymbol{V}}_2\lambda_L - \mu\boldsymbol{S} && (62)
\end{aligned}
$$

with:

$$
\begin{aligned}
\hat{\boldsymbol{M}}_0 &= \boldsymbol{D}_{\boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c})}, \\
\hat{\boldsymbol{M}}_1 &= -\boldsymbol{D}_{\boldsymbol{\xi}^{u,E} \circ u''(\boldsymbol{c})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}), \\
\hat{\boldsymbol{V}}_2 &= w\boldsymbol{S} \circ \boldsymbol{y} \circ u''(\boldsymbol{c}).
\end{aligned}
$$

Then, using (55), (56), and (57), we get:

$$\left((\boldsymbol{I}-\boldsymbol{P})+\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\hat{\boldsymbol{M}}_1\right)\bar{\boldsymbol{\lambda}}_c=-\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\hat{\boldsymbol{M}}_0\bar{\boldsymbol{\omega}}$$
$$-\lambda_L\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\hat{\boldsymbol{V}}_2-\mu\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{S}.$$

We define:

$$\begin{aligned}
\tilde{\boldsymbol{R}}_5&=-(\boldsymbol{I}-\boldsymbol{P}+\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}})\hat{\boldsymbol{M}}_1)^{-1}\boldsymbol{P}(I-\beta(1+r)\bar{\boldsymbol{\Pi}}),\\
\boldsymbol{M}_5&=\tilde{\boldsymbol{R}}_5\hat{\boldsymbol{M}}_0,\\
\boldsymbol{V}_2&=\tilde{\boldsymbol{R}}_5\hat{\boldsymbol{V}}_2,\\
\boldsymbol{V}_3&=\tilde{\boldsymbol{R}}_5\boldsymbol{S},
\end{aligned}$$

and obtain:

$$\bar{\boldsymbol{\lambda}}_c=\boldsymbol{M}_5\bar{\boldsymbol{\omega}}+\lambda_L\boldsymbol{V}_2+\mu\boldsymbol{V}_3.$$

We deduce:

$$\begin{aligned}
\bar{\boldsymbol{\psi}}&=\hat{\boldsymbol{M}}_0\bar{\boldsymbol{\omega}}+\hat{\boldsymbol{M}}_1\bar{\boldsymbol{\lambda}}_c+\hat{\boldsymbol{V}}_2\lambda_L-\mu\boldsymbol{S}\\
&=(\hat{\boldsymbol{M}}_0+\hat{\boldsymbol{M}}_1\boldsymbol{M}_5)\bar{\boldsymbol{\omega}}+\lambda_L(\hat{\boldsymbol{V}}_2+\hat{\boldsymbol{M}}_1\boldsymbol{V}_2)+\mu(\hat{\boldsymbol{M}}_1\boldsymbol{V}_3-\boldsymbol{S})
\end{aligned}$$

and:

$$\begin{aligned}
\bar{\boldsymbol{\psi}}&=\boldsymbol{M}_6\bar{\boldsymbol{\omega}}+\lambda_L\boldsymbol{V}_4+\mu\boldsymbol{V}_5 \qquad (63)\\
\boldsymbol{M}_6&=\hat{\boldsymbol{M}}_1\boldsymbol{M}_5+\hat{\boldsymbol{M}}_0\\
\boldsymbol{V}_4&=\hat{\boldsymbol{M}}_1\boldsymbol{V}_2+\hat{\boldsymbol{V}}_2\\
\boldsymbol{V}_5&=\hat{\boldsymbol{M}}_1\boldsymbol{V}_3-\boldsymbol{S}
\end{aligned}$$

We have four constraints left:

$$\begin{aligned}
v'(L)(1^T\bar{\boldsymbol{\omega}})+\lambda_L\xi_l v''(L)&=w\boldsymbol{y}^\top\bar{\boldsymbol{\psi}}+\mu F_L \qquad (64)\\
\tilde{\boldsymbol{a}}^\top\bar{\boldsymbol{\psi}}&=-\left(\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c})\right)^\top\Pi\bar{\boldsymbol{\lambda}}_c, \qquad (65)\\
\boldsymbol{y}^\top\bar{\boldsymbol{\psi}}&=-(\lambda_L/L)(\boldsymbol{S}\circ\boldsymbol{y})^\top\circ u'(\boldsymbol{c}) \qquad (66)\\
1^\top\bar{\boldsymbol{\psi}}&=0 \qquad (67)
\end{aligned}$$

We use the two FOCs. First, we start with (65):

$$
\begin{aligned}
0 &= \tilde{\boldsymbol{a}}^\top \bar{\boldsymbol{\psi}} + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \bar{\boldsymbol{\lambda}}_c, \\
&= \tilde{\boldsymbol{a}}^\top \boldsymbol{M}_6 \bar{\boldsymbol{\omega}} + \lambda_L \tilde{\boldsymbol{a}}^\top \boldsymbol{V}_4 + \mu \tilde{\boldsymbol{a}}^\top \boldsymbol{V}_5 \\
&+ \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi(\boldsymbol{M}_5 \bar{\boldsymbol{\omega}} + \lambda_L \boldsymbol{V}_2 + \mu \boldsymbol{V}_3), \\
&= \left(\tilde{\boldsymbol{a}}^\top \boldsymbol{M}_6 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{M}_5\right) \bar{\boldsymbol{\omega}} \\
&+ \lambda_L \left(\tilde{\boldsymbol{a}}^\top \boldsymbol{V}_4 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{V}_2\right) \\
&+ \mu \left(\tilde{\boldsymbol{a}}^\top \boldsymbol{V}_5 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{V}_3\right),
\end{aligned}
$$

or equivalently:

$$
\begin{aligned}
\lambda_L &= \boldsymbol{V}_6^\top \bar{\boldsymbol{\omega}} + C_5 \mu, \\
\tilde{C}_4 &= \tilde{\boldsymbol{a}}^\top \boldsymbol{V}_4 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{V}_2, \\
\boldsymbol{V}_6 &= -\left(\tilde{\boldsymbol{a}}^\top \boldsymbol{M}_6 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{M}_5\right)^\top / \tilde{C}_4. \\
C_5 &= -\left(\tilde{\boldsymbol{a}}^\top \boldsymbol{V}_5 + \left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \boldsymbol{V}_3\right) / \tilde{C}_4.
\end{aligned}
$$

Using (63), we have for $\bar{\boldsymbol{\psi}}$:

$$
\begin{aligned}
\bar{\boldsymbol{\psi}} &= \boldsymbol{M}_6 \bar{\boldsymbol{\omega}} + \lambda_L \boldsymbol{V}_4 + \mu \boldsymbol{V}_5, \\
&= \boldsymbol{M}_6 \bar{\boldsymbol{\omega}} + \boldsymbol{V}_4 \boldsymbol{V}_6^\top \bar{\boldsymbol{\omega}} + \boldsymbol{V}_4 C_5 \mu + \mu \boldsymbol{V}_5, \\
\bar{\boldsymbol{\psi}} &= \boldsymbol{M}_7 \bar{\boldsymbol{\omega}} + \mu \boldsymbol{V}_7 \qquad (68) \\
\boldsymbol{M}_7 &= \boldsymbol{M}_6 + \boldsymbol{V}_4 \boldsymbol{V}_6^\top \\
\boldsymbol{V}_7 &= C_5 \boldsymbol{V}_4 + \boldsymbol{V}_5
\end{aligned}
$$

We now turn to (66):

$$
\begin{aligned}
0 &= \boldsymbol{y}^\top \bar{\boldsymbol{\psi}} + (\lambda_L / L)(\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) \\
&= \boldsymbol{y}^\top \boldsymbol{M}_7 \bar{\boldsymbol{\omega}} + \mu \boldsymbol{y}^\top \boldsymbol{V}_7 + (\boldsymbol{V}_6^\top \bar{\boldsymbol{\omega}} + C_5 \mu)(\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) / L \\
&= \left(\boldsymbol{y}^\top \boldsymbol{M}_7 + (\boldsymbol{S} \circ \boldsymbol{y})^\top (u'(\boldsymbol{c})) / L \boldsymbol{V}_6^\top\right) \bar{\boldsymbol{\omega}} + \mu \left(\boldsymbol{y}^\top \boldsymbol{V}_7 + C_5 (\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) / L\right) \\
\mu &= \boldsymbol{V}_8^\top \bar{\boldsymbol{\omega}} \\
\boldsymbol{V}_8 &= -\left(\boldsymbol{y}^\top \boldsymbol{M}_7 + \frac{1}{L} (\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) \boldsymbol{V}_6^\top\right)^\top / C_6 \\
C_6 &= \boldsymbol{y}^\top \boldsymbol{V}_7 + C_5 (\boldsymbol{S} \circ \boldsymbol{y})^\top u'(\boldsymbol{c}) / L
\end{aligned}
$$

We deduce from (68):

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_8\bar{\boldsymbol{\omega}}$$
$$\boldsymbol{M}_8 = \boldsymbol{M}_7 + \boldsymbol{V}_7\boldsymbol{V}_8^\top$$

We thus have two constraints left, (64) and (67). FOC (64) becomes:

$$v'(L)(1^T\bar{\boldsymbol{\omega}}) + \xi_l v''(L)\left(\boldsymbol{V}_6^\top + C_5\boldsymbol{V}_8^\top\right)\bar{\boldsymbol{\omega}} = w\boldsymbol{y}^T\boldsymbol{M}_8\bar{\boldsymbol{\omega}} + F_L\boldsymbol{V}_8^\top\bar{\boldsymbol{\omega}}.$$

We finally define

$$\begin{aligned}
&\boldsymbol{L}_1 = 1^\top\boldsymbol{M}_8,\\
&\boldsymbol{L}_2 = v'(\boldsymbol{L})1^T + \xi_l v''(\boldsymbol{L})\left(\boldsymbol{V}_6^\top + C_5\boldsymbol{V}_8^\top\right) - w\boldsymbol{y}^T\boldsymbol{M}_8 - F_L\boldsymbol{V}_8^\top,
\end{aligned}$$

which implies that (64) and (67) become:

$$\boldsymbol{L}_1\bar{\boldsymbol{\omega}} = \boldsymbol{L}_2\bar{\boldsymbol{\omega}} = 0.$$

Finally, defining $\bar{\boldsymbol{\omega}} = \boldsymbol{D_S}\boldsymbol{M}_9\boldsymbol{\omega}^s$ and

$$\begin{aligned}
\boldsymbol{M}_{10} &= \begin{bmatrix} \boldsymbol{L}_1\boldsymbol{D_S}\boldsymbol{M}_9 \\ \boldsymbol{L}_2\boldsymbol{D_S}\boldsymbol{M}_9 \end{bmatrix}\begin{bmatrix} \boldsymbol{L}_2\boldsymbol{D_S}\boldsymbol{M}_9 \\ \boldsymbol{L}_3\boldsymbol{D_S}\boldsymbol{M}_9 \end{bmatrix}^\top,\\
\boldsymbol{V}_{10} &= -\begin{bmatrix} \boldsymbol{L}_1\boldsymbol{D_S}\boldsymbol{M}_9\boldsymbol{1}_K \\ \boldsymbol{L}_2\boldsymbol{D_S}\boldsymbol{M}_9\boldsymbol{1}_K \end{bmatrix},
\end{aligned}$$

we obtain that the weights minimizing the distance to the utilitarian criterion are defined as follows:

$$\boldsymbol{\omega}^s = \boldsymbol{1}_K + \boldsymbol{M}_{10}^{-1}\boldsymbol{V}_{10}\begin{bmatrix} \boldsymbol{L}_1\boldsymbol{D_S}\boldsymbol{M}_9 \\ \boldsymbol{L}_2\boldsymbol{D_S}\boldsymbol{M}_9 \end{bmatrix}^\top.$$