Advanced Macroeconomics – Part 2 – Theme 1 (Lectures 1–2)

Problem set: Business Cycle Accounting and the New Keynesian Model

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Scope. This problem set covers Theme 1 and has two parts.

- **Part I – Business cycle accounting** (Exercises 1–3). Each exercise takes an economy with a specific friction and shows that its aggregate allocations are those of a prototype growth model with time-varying wedges.
- **Part II – The New Keynesian model** (Exercises 4–7). The exercises cover a two-period model with a fixed price, the derivation of the Phillips curve under Calvo pricing, the welfare effects of demand under monopolistic competition, and the time inconsistency of monetary policy.

The parts are linked in two places: the markup shock of Exercise 3 is the cost-push shock of Part II, and Exercises 4 and 6 use the wedges of Part I.

How to use it. The sheet is long and is not meant to be done in one sitting. Each exercise can be done on its own; when one uses a result from another, the reference is given. Each part starts with a reminder of its notation.

Contents.

1. Input-financing frictions → efficiency wedge — *CKM (2007), Prop. 1*
2. Sticky nominal wages → labor wedge — *Bordo, Erceg & Evans (2000); CKM, Prop. 2*
3. Markup shocks → labor and investment wedges
4. Sticky prices in a two-period model — *Mankiw & Weinzierl (2011)*
5. From the Calvo system to the New Keynesian Phillips curve — *Calvo (1983); Galí (2015), ch. 3*
6. Monopolistic competition and the aggregate demand externality — *Blanchard & Kiyotaki (1987)*
7. Time inconsistency and the inflation bias — *Kydland & Prescott (1977)*

Part I – Business cycle accounting

Objective. Chari, Kehoe & McGrattan (2007, *Econometrica*, henceforth CKM) show in their Section 1.2 that economies with specific frictions are *equivalent* to the prototype growth model with time-varying wedges. The three exercises of this part derive such results step by step. Input-financing frictions show up as an *efficiency wedge* (Exercise 1), sticky nominal wages as a *labor wedge* (Exercise 2), and markup shocks as *labor and investment wedges* (Exercise 3).

Reminder: the prototype economy. Abstracting from growth, the equilibrium of the prototype economy is summarized by

$$c_t + x_t + g_t = y_t, \qquad y_t = A_t F(k_t, l_t),$$
$$-\frac{U_{lt}}{U_{ct}} = (1-\tau_{lt})\, A_t F_{lt}, \qquad U_{ct}(1+\tau_{xt}) = \beta\, \mathbb{E}_t\Big[U_{ct+1}\big\{A_{t+1}F_{kt+1} + (1-\delta)(1+\tau_{xt+1})\big\}\Big],$$

with efficiency wedge A_t, labor wedge $1-\tau_{lt}$, investment wedge $1/(1+\tau_{xt})$ and government consumption wedge g_t. Distortionary "tax" revenues are rebated lump sum. *Equivalence* means: for a given detailed economy, we can choose wedge processes such that the aggregate allocations of the detailed economy are an equilibrium of the prototype economy.

Exercise 1 – Input-financing frictions → efficiency wedge

The detailed economy. Time is discrete and there is no aggregate uncertainty within the period. There are two sectors $i \in \{1, 2\}$.

- A competitive final-good producer combines sectoral outputs into gross output $q_t = q_{1t}^{\phi}\, q_{2t}^{1-\phi}$, $\phi \in (0,1)$. The price of q_t is normalized to 1; p_{it} is the price of sector-i output.
- Sector i produces $q_{it} = m_{it}^{\theta}\, z_{it}^{1-\theta}$, $\theta \in (0,1)$, using intermediate goods m_{it} (one unit of q_t each) and a composite value-added good z_{it} with price v_t. Intermediate goods must be *financed in advance* within the period, at gross rate $R_{it} = 1 + \tau_{it}$, where $\tau_{it} \geq 0$ is a spread induced by financing constraints (the within-period rate paid to savers is 1). Sector-i producers solve

$$\max_{m_{it}, z_{it}} \; p_{it} q_{it} - v_t z_{it} - (1+\tau_{it})\, m_{it}.$$

- A competitive producer makes the composite good from capital and labor, $z_{1t} + z_{2t} = z_t = F(k_t, l_t)$, with F constant returns to scale, and solves max $v_t z_t - w_t l_t - r_t k_t$.
- Consumers maximize $\sum_t \beta^t U(c_t, l_t)$ subject to $c_t + k_{t+1} = r_t k_t + w_t l_t + (1-\delta)k_t + T_t$, where $T_t = \sum_i \tau_{it} m_{it}$ rebates the spreads lump sum.
- Resource constraint: $c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1-\delta)k_t$. Final output (value added) is $y_t = q_t - m_{1t} - m_{2t}$.

Throughout, define $a_{1t} = \dfrac{\phi}{1+\tau_{1t}}$ and $a_{2t} = \dfrac{1-\phi}{1+\tau_{2t}}$.

Part A – Solving the detailed economy

1. **Final-good aggregator.** Solve the problem of the final-good producer, max $q_t - p_{1t}q_{1t} - p_{2t}q_{2t}$, and show that expenditure shares are constant: $p_{1t}q_{1t} = \phi\, q_t$ and $p_{2t}q_{2t} = (1-\phi)\, q_t$.

2. **Sectoral demands.** Write down the first-order conditions of the sector-i producer. Using Question 1, show that

$$m_{it} = \theta\, a_{it}\, q_t, \qquad z_{1t} = \phi\, z_t, \qquad z_{2t} = (1-\phi)\, z_t, \qquad v_t = (1-\theta)\frac{q_t}{z_t}.$$

3. **Final output.** Show that final output is

$$y_t = q_t\big[1 - \theta(a_{1t} + a_{2t})\big].$$

   What is y_t/q_t without frictions ($\tau_{1t} = \tau_{2t} = 0$)? Why do frictions *raise* y_t/q_t?

4. **Aggregate production function.** Substitute the demands of Question 2 into $q_t = q_{1t}^{\phi} q_{2t}^{1-\phi}$ and solve for q_t as a function of z_t. Deduce that

$$y_t = A_t\, F(k_t, l_t), \qquad A_t = \kappa\left(a_{1t}^{\phi}\, a_{2t}^{1-\phi}\right)^{\frac{\theta}{1-\theta}}\big[1 - \theta(a_{1t} + a_{2t})\big], \qquad \kappa = \theta^{\frac{\theta}{1-\theta}}\phi^{\phi}(1-\phi)^{1-\phi}.$$

5. **Factor prices and the labor wedge.** Using the composite producer's problem and Question 2, show that

$$w_t = v_t F_{lt}, \qquad r_t = v_t F_{kt}, \qquad v_t = \frac{1-\theta}{1-\theta(a_{1t}+a_{2t})}\, A_t.$$

   Consider a prototype economy in which consumers face taxes on labor income τ_{lt} and on capital income τ_{kt}, and firms pay factors their marginal products $A_t F_{lt}$ and $A_t F_{kt}$. Show that the consumers' first-order conditions coincide in the two economies if and only if

$$\frac{1}{1-\tau_{lt}} = \frac{1}{1-\theta}\big[1 - \theta(a_{1t} + a_{2t})\big], \qquad \tau_{kt} = \tau_{lt}.$$

   Why is the distortion the *same* for labor and capital? Show that $\tau_{lt} \geq 0$ whenever spreads are non-negative.

6. **Budget and resource constraints.** Show that, in the detailed economy, consumer income $r_t k_t + w_t l_t + T_t$ equals y_t (all producers make zero profits). Show that the resource constraint can be written $c_t + k_{t+1} = y_t + (1-\delta)k_t$. Conclude that the aggregate allocations (c_t, k_{t+1}, l_t, y_t) of the detailed economy are an equilibrium of the prototype economy with A_t from Question 4, $\tau_{lt} = \tau_{kt}$ from Question 5, $g_t = 0$ and tax revenues rebated lump sum. This is CKM's **Proposition 1**.

Part B – Relative distortions look like technological regress

Suppose now that the spreads fluctuate but the weighted average $S \equiv a_{1t} + a_{2t}$ stays constant.

7. **Which wedges move?** Using Questions 4–5, show that the labor and capital wedges are constant, while A_t moves only through $G_t \equiv a_{1t}^{\phi} a_{2t}^{1-\phi}$.

8. **Dispersion lowers measured TFP.** Holding S fixed, show that G_t is maximized when $\tau_{1t} = \tau_{2t}$. Interpret this result in terms of the allocation of intermediate goods across sectors.

9. Second-order effect. Let $a_{1t} = \phi S + \varepsilon$ and $a_{2t} = (1-\phi)S - \varepsilon$. Show that

$$\ln A_t - \ln A^* \approx -\frac{\theta}{1-\theta}\,\frac{\varepsilon^2}{2S^2\phi(1-\phi)},$$

where A^* is the value with equal spreads. Why is the first-order term zero?

10. A numerical example. Let $\phi = \theta = 1/2$. In period 0, $\tau_{10} = \tau_{20} = 20\%$.

(a) Compute S and the labor wedge τ_l in period 0.

(b) In period 1, small firms are hit by a credit crunch: $\tau_{11} = 40\%$. Find τ_{21} such that S is unchanged. Compute the percentage change in A_t and in τ_{lt}. Compare with the approximation of Question 9.

(c) Suppose instead that both spreads rise to 40%. Compute the change in A_t and in τ_{lt}.

11. Discussion. An econometrician observes data generated by the economy in Question 10(b) and measures the wedges of the prototype economy.

(a) What would she conclude about the source of the recession?

(b) What additional data would allow her to distinguish a financial shock from a genuine technology shock?

(c) What does this example imply for the interpretation of the Solow residual in the RBC literature?

Exercise 2 – Sticky wages → labor wedge

The detailed economy (Bordo, Erceg & Evans, 2000). Let s^t denote the history of events and $\pi_t(s^t)$ its probability.

- A competitive final-good producer produces $y(s^t) = F\big(k(s^{t-1}), l(s^t)\big)$ with a labor aggregate of differentiated labor types $j \in [0,1]$:

$$l(s^t) = \left[\int_0^1 l(j,s^t)^v\, dj\right]^{1/v}, \qquad v \in (0,1).$$

  It accumulates capital, $k(s^t) = (1-\delta)k(s^{t-1}) + x(s^t)$, and maximizes $\sum_t \sum_{s^t} Q(s^t)\big[P(s^t)y(s^t) - P(s^t)x(s^t) - W(s^{t-1})l(s^t)\big]$, where $Q(s^t)$ is the price of a dollar at s^t, $P(s^t)$ the price level and $W(s^{t-1})$ the aggregate nominal wage.

- Consumers are organized in unions; union j supplies labor of type j. It sets its nominal wage $W(j, s^{t-1})$ *one period in advance*, before the realization of s_t, and then supplies whatever labor is demanded. Members have preferences

$$\sum_t \sum_{s^t} \beta^t \pi_t(s^t)\Big[U\big(c(j,s^t), l(j,s^t)\big) + V\big(M(j,s^t)/P(s^t)\big)\Big],$$

  trade a complete set of state-contingent nominal bonds, and receive dividends and lump-sum money transfers $P(s^t)T(s^t) = M(s^t) - M(s^{t-1})$.

- Money supply: $M(s^t) = \mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic monetary shock.

Part A – Labor demand and wage setting

1. **Demand for each labor type.** For a given aggregate $l(s^t)$, the producer minimizes $\int_0^1 W(j, s^{t-1})\, l(j, s^t)\, dj$ subject to the definition of $l(s^t)$. Show that

$$l^d(j, s^t) = \left[\frac{W(s^{t-1})}{W(j, s^{t-1})}\right]^{\frac{1}{1-v}} l(s^t), \qquad W(s^{t-1}) = \left[\int_0^1 W(j, s^{t-1})^{\frac{v}{v-1}} dj\right]^{\frac{v-1}{v}},$$

and that the minimized cost is $W(s^{t-1})\, l(s^t)$. What is the elasticity of demand for labor type j with respect to its wage?

2. **The producer's first-order conditions.** Show that the producer's choice of aggregate labor and capital implies

$$P(s^t)\, F_l(s^t) = W(s^{t-1}), \qquad Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})\big[F_k(s^{t+1}) + 1 - \delta\big].$$

Explain why, given $W(s^{t-1})$, a surprise increase in $P(s^t)$ raises employment.

3. **Wage setting.** From the bond FOC, $Q(s^t) \propto \beta^t \pi_t(s^t)\, U_c(s^t)/P(s^t)$. Use this to show that, when choosing $W(j, s^{t-1})$, union j maximizes

$$\sum_{s^t | s^{t-1}} Q(s^t) \left[W(j, s^{t-1})\, l^d(j, s^t) + P(s^t)\frac{U_l(s^t)}{U_c(s^t)}\, l^d(j, s^t)\right],$$

taking aggregates as given. Derive the optimal wage

$$W(j, s^{t-1}) = -\frac{\sum_{s^t} Q(s^t)P(s^t)\, \frac{U_l(j,s^t)}{U_c(j,s^t)}\, l^d(j, s^t)}{v \sum_{s^t} Q(s^t)\, l^d(j, s^t)}.$$

4. **Steady state.** Show that in a deterministic steady state $\dfrac{W}{P} = \dfrac{1}{v}\left(-\dfrac{U_l}{U_c}\right)$. Combine this with Question 2 to compute the ratio of the marginal rate of substitution (MRS) to the marginal product of labor (MPL) in steady state. Give a value for $v = 0.9$.

Part B – The equivalent real economy

Consider a *real* prototype economy with the same technology and producer problem, in which a representative consumer maximizes $\sum_t \sum_{s^t} \beta^t \pi_t(s^t) U(c(s^t), l(s^t))$ subject to

$$c(s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t)\, b(s^{t+1}) \le \big[1 - \tau_l(s^t)\big] w(s^t) l(s^t) + b(s^t) + \text{transfers} + \text{dividends},$$

where lowercase letters are real variables.

5. **Proposition 2.** Write down the consumer's labor FOC in the prototype economy. Let starred variables denote the equilibrium of the sticky-wage economy for a given money supply process $M^*(s^t)$. Show that if

$$1 - \tau_l(s^t) = -\frac{U_l^*(s^t)}{U_c^*(s^t)}\, \frac{1}{F_l^*(s^t)},$$

and real prices are set to $w(s^t) = W^*(s^{t-1})/P^*(s^t)$ and $q(s^{t+1}|s^t) = Q^*(s^{t+1})P^*(s^{t+1})/[Q^*(s^t)P^*(s^t)]$, then the starred allocations satisfy all equilibrium conditions of the prototype economy.

6. **The steady-state labor wedge.** Using Question 4, what is the value of τ_l in the steady state? Is the labor wedge always positive in this economy?

Part C – Monetary shocks as labor wedges

Let $F(k,l) = k^{\alpha} l^{1-\alpha}$ and $U(c,l) = \ln c + \psi \ln(1-l)$. Consider a period-$t$ monetary surprise that raises the price level by $d\ln P_t > 0$. Capital $k(s^{t-1})$ and the nominal wage $W(s^{t-1})$ are pre-determined.

7. **Employment and output.** Using $P(s^t)F_l(s^t) = W(s^{t-1})$, show that $d\ln l_t = d\ln P_t/\alpha$ and compute $d\ln y_t$. Give numbers for $\alpha = 1/3$ and $d\ln P_t = 1\%$.

8. **Response of the measured labor wedge.** Show that

$$d\ln(1-\tau_{lt}) = d\ln c_t + \left(\frac{l}{1-l} + \alpha\right) d\ln l_t.$$

   Evaluate it with $l = 1/3$, $\alpha = 1/3$, $d\ln l_t = 3\%$ and $d\ln c_t = 1\%$. Does the labor wedge rise or fall after a monetary expansion? Explain intuitively using the gap between MRS and MPL.

9. **The Great Depression.** Bordo, Erceg & Evans (2000) argue that the monetary contraction of 1929–33 caused the Depression through sticky wages. What does the model predict for the measured labor wedge in that episode? CKM find that the labor wedge worsened sharply in 1930–33 but that the efficiency wedge accounts for most of the fall in output. What does this suggest about the sticky-wage mechanism as the *only* explanation?

10. **Persistence.** In this model, wages are set only one period in advance. How long does the effect of a one-time monetary surprise on the labor wedge last? Compare with the persistence of measured labor wedges in the data. What modifications of the model would be needed?

11. **Lucas critique.** Suppose the central bank changes its money supply rule $\mu(s^t)$. Is the stochastic process for $\tau_l(s^t)$ in the equivalent prototype economy unchanged? What does this imply for using BCA to evaluate counterfactual policies?

Exercise 3 – Markup shocks → labor and investment wedges

The detailed economy. Time is discrete, s^t is the history of shocks and $\Pi_t(s^t)$ its probability. There is no rental market for capital; each firm owns its own.

- A representative household solves

$$\max_{\{C_t(s^t),H_t(s^t)\}} \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \Pi_t(s^t)\, U\big(C_t(s^t), H_t(s^t)\big)$$

$$\text{subject to} \qquad A_0 = \sum_{t=0}^{\infty} \sum_{s^t} Q_0^t(s^t)\big[P_t(s^t)C_t(s^t) - W_t(s^t)H_t(s^t)\big],$$

  taking as given the date-0 nominal prices $Q_0^t(s^t)$, the wage $W_t(s^t)$ and the price level $P_t(s^t)$.

- A competitive final-good producer bundles a continuum of intermediate varieties $j \in [0,1]$,

$$Y_t(s^t) = \left[\int_0^1 y_{jt}(s^t)^{\frac{\varepsilon_t(s^t)-1}{\varepsilon_t(s^t)}}\, dj\right]^{\frac{\varepsilon_t(s^t)}{\varepsilon_t(s^t)-1}}, \qquad \varepsilon_t(s^t) > 1,$$

  and maximizes $\sum_t \sum_{s^t} Q_0^t\big[P_t Y_t - \int_0^1 p_{jt}\, y_{jt}\, dj\big]$ taking P_t and $\{p_{jt}\}$ as given. The elasticity of substitution ε_t is stochastic and is the only source of uncertainty.

- Intermediate producer j chooses its price, labor and capital to maximize

$$V_{j0} = \max_{\{p_{jt},h_{jt},k_{j,t+1}\}} \sum_{t=0}^{\infty} \sum_{s^t} Q_0^t(s^t)\big[p_{jt}\, y_{jt} - W_t\, h_{jt} - P_t\, x_{jt}\big], \qquad y_{jt} = k_{jt}^{\alpha}\big(z_t\, h_{jt}\big)^{1-\alpha},$$

  with $k_{j,t+1} = (1-\delta)k_{jt} + x_{jt}$, $k_{j0} = k_0$ given, taking as given Q_0^t, W_t, P_t and its own demand curve $y_{jt} = \Phi_t(p_{jt}, s^t)$.

- Market clearing: $H_t = \int_0^1 h_{jt}\, dj$ and $Y_t = C_t + \int_0^1 x_{jt}\, dj$.

Throughout, define the gross markup

$$\mu_t \equiv \frac{\varepsilon_t}{\varepsilon_t - 1} > 1.$$

Part A – Solving the detailed economy

1. Household. Derive the first-order conditions. Show that they imply a labor-supply condition and an expression for the nominal stochastic discount factor:

$$-\frac{U_{Ht}}{U_{Ct}} = \frac{W_t}{P_t}, \qquad \frac{Q_0^{t+1}(s^{t+1})\, P_{t+1}(s^{t+1})}{Q_0^t(s^t)\, P_t(s^t)} = \beta\, \frac{\Pi_{t+1}(s^{t+1})}{\Pi_t(s^t)}\, \frac{U_{Ct+1}(s^{t+1})}{U_{Ct}(s^t)}.$$

2. Final-good producer. Derive the first-order condition. Show that the problem has a solution only if

$$P_t = \left[\int_0^1 p_{jt}^{1-\varepsilon_t}\, dj\right]^{\frac{1}{1-\varepsilon_t}},$$

that profits are then zero, and that the demand curve faced by firm j is

$$\Phi_t(p_{jt}) = \left(\frac{p_{jt}}{P_t}\right)^{-\varepsilon_t} Y_t.$$

3. Intermediate producer. Write the firm's problem with the demand curve substituted in and a multiplier λ_{jt} on the production function. Show that

$$\underbrace{\lambda_{jt} = \frac{W_t\, h_{jt}^{\alpha}}{(1-\alpha)\, k_{jt}^{\alpha}\, z_t^{1-\alpha}}}_{\text{nominal marginal cost}}, \qquad p_{jt} = \mu_t\, \lambda_{jt},$$

$$Q_0^t P_t = \sum_{s^{t+1} \succ s^t} Q_0^{t+1}\left[P_{t+1}(1-\delta) + \lambda_{j,t+1}\, \alpha\, k_{j,t+1}^{\alpha-1}\big(z_{t+1} h_{j,t+1}\big)^{1-\alpha}\right].$$

4. Aggregate equilibrium conditions. Impose symmetry ($p_{jt} = P_t$, $h_{jt} = H_t$, $k_{jt} = K_t$) and combine Questions 1–3 to show that the equilibrium is characterized by the resource constraint, $Y_t = K_t^{\alpha}(z_t H_t)^{1-\alpha}$, and

$$-\frac{U_{Ht}}{U_{Ct}} = \frac{1}{\mu_t}\, \underbrace{(1-\alpha)\frac{Y_t}{H_t}}_{=\partial Y_t/\partial H_t}, \qquad U_{Ct} = \beta\, \mathbb{E}_t\left[U_{Ct+1}\left\{\frac{1}{\mu_{t+1}}\, \alpha \left(\frac{z_{t+1}H_{t+1}}{K_{t+1}}\right)^{1-\alpha} + 1 - \delta\right\}\right].$$

Part B – Which wedges do markup shocks generate?

5. Efficiency wedge. Show that the aggregate production function of the detailed economy is that of the prototype economy with $A_t = z_t^{1-\alpha}$ and $F(K,H) = K^{\alpha}H^{1-\alpha}$. Does a markup shock move the efficiency wedge? Compare with Exercise 1.

6. Labor and capital wedges. Consider a prototype economy in which the household faces a tax τ_{lt} on labor income and τ_{kt} on capital income, and firms pay factors their marginal products. Show that the two economies have the same first-order conditions if and only if

$$1-\tau_{lt} = 1-\tau_{kt} = \frac{1}{\mu_t}.$$

Compare with Question 5 of Exercise 1. Why is the distortion the same for labor and capital?

7. The investment wedge. CKM's prototype economy has an investment wedge $1+\tau_{xt}$ instead of a tax on capital income:

$$U_{Ct}\,(1+\tau_{xt}) = \beta\,\mathbb{E}_t\Big[U_{Ct+1}\big\{A_{t+1}F_{Kt+1} + (1-\delta)(1+\tau_{xt+1})\big\}\Big].$$

(a) Show that with full depreciation ($\delta = 1$) and μ_{t+1} known at t, the markup economy is a prototype economy with $1+\tau_{xt} = \mu_{t+1}$.

(b) Show that with $\delta < 1$ the two economies are equivalent only if the markup is constant. Where does the difference come from?

Part C – Calibrating the balanced growth path

Assume $z_t = (1+g)^t$ and $\varepsilon_t = \varepsilon$ constant, and take

$$U(C,H) = \ln C - \gamma\,\frac{H^{1+\varphi}}{1+\varphi},$$

where φ is the inverse Frisch elasticity of labor supply, as in Lecture 2. Look for a balanced growth path with $C_t = (1+g)^t c^*$, $K_t = (1+g)^t k^*$, $H_t = h^*$.

8. Factor shares. Show that on the balanced growth path

$$\underbrace{s_L \equiv \frac{Wh^*}{Py^*} = \frac{1-\alpha}{\mu}}_{\text{labor share}}, \qquad \underbrace{s_K = \frac{\alpha}{\mu}}_{\text{capital income share}}, \qquad \underbrace{s_\pi = 1-\frac{1}{\mu}}_{\text{pure profit share}},$$

and that these three shares add up to one.

9. Balanced growth path. Show that (c^*, k^*, h^*) solve

$$\gamma\,c^*\,(h^*)^{\varphi} = \frac{1}{\mu}\frac{(1-\alpha)(k^*)^{\alpha}}{(h^*)^{\alpha}}, \qquad 1+g = \beta\left[1-\delta+\frac{\alpha}{\mu}\Big(\frac{h^*}{k^*}\Big)^{1-\alpha}\right], \qquad (k^*)^{\alpha}(h^*)^{1-\alpha} = c^* + (g+\delta)k^*.$$

10. Calibration. Take as quarterly targets $s_L = 0.60$, $K/Y = 12.8$, $X/Y = 0.26$ and $g = 0.005$.

(a) Compute δ and c^*/y^*. Do they depend on μ?

(b) Show that $\alpha = 1-\mu\,s_L$ and that the gross return on capital is

$$\frac{\alpha}{\mu}\frac{Y}{K} = \Big(\frac{1}{\mu} - s_L\Big)\frac{Y}{K}.$$

Compute α, s_π and β for $\varepsilon \to \infty$, $\varepsilon = 11$ and $\varepsilon = 6$. Comment.

(c) Normalizing $h^* = 1$, show that $\gamma = s_L/(c^*/y^*)$ and compute it. Does it depend on μ?

(d) How does the markup change the calibration of the balanced growth path? Which additional data would identify μ?

Part D – Markup shocks and the business cycle

Assume now $\log \varepsilon_t$ follows a first-order Markov process with mean $\log \bar{\varepsilon}$ and autocorrelation ρ. Write $\widehat{v}_t \equiv \log(v_t/v^*)$ for log deviations of detrended variables.

11. From ε to μ. Show that $\widehat{\mu}_t = -\dfrac{\widehat{\varepsilon}_t}{\varepsilon - 1}$ and that $\widehat{\mu}_t$ inherits the autocorrelation ρ. For $\varepsilon = 11$, by how much does a 1% rise in ε move the markup and the measured labor wedge?

12. Log-linearized system. Set $g = 0$ for simplicity and let $R \equiv 1-\delta+\rho_k$ with $\rho_k \equiv (\alpha/\mu)(h^*/k^*)^{1-\alpha}$. Show that the equilibrium is characterized by

$$\widehat{c}_t + (\varphi + \alpha)\widehat{h}_t = \alpha \widehat{k}_t - \widehat{\mu}_t \qquad \text{(labor wedge)}$$

$$\mathbb{E}_t \widehat{c}_{t+1} - \widehat{c}_t = \frac{\rho_k}{R}\,\mathbb{E}_t\big[(1-\alpha)(\widehat{h}_{t+1} - \widehat{k}_{t+1}) - \widehat{\mu}_{t+1}\big] \qquad \text{(Euler)}$$

$$\alpha \widehat{k}_t + (1-\alpha)\widehat{h}_t = \frac{c^*}{y^*}\widehat{c}_t + \frac{k^*}{y^*}\big[\widehat{k}_{t+1} - (1-\delta)\widehat{k}_t\big] \qquad \text{(resource)}$$

13. A closed-form special case. Let $\delta = 1$ and $g = 0$, so that $y_t = k_t^{\alpha} h_t^{1-\alpha}$ and $k_{t+1} = y_t - c_t$.

(a) Guess that the household saves a fraction s_t of output, $k_{t+1} = s_t y_t$ and $c_t = (1-s_t)y_t$, and show that the Euler equation becomes the linear recursion

$$q_t = 1 + \alpha\beta\,\mathbb{E}_t\Big[\frac{q_{t+1}}{\mu_{t+1}}\Big], \qquad q_t \equiv \frac{1}{1-s_t}.$$

Deduce that with a constant markup $s = \alpha\beta/\mu$.

(b) Show that hours do not depend on the capital stock,

$$h_t^{1+\varphi} = \frac{1-\alpha}{\gamma\,\mu_t\,(1-s_t)},$$

and that if $\widehat{\mu}_t$ follows an AR(1) with persistence ρ, then to first order

$$\widehat{h}_t = -\frac{1}{(1+\varphi)\,(1-s\rho)}\,\widehat{\mu}_t, \qquad \widehat{y}_t = \alpha\widehat{k}_t + (1-\alpha)\widehat{h}_t.$$

(c) Interpret the factor $1/(1-s\rho)$ and check the cases $\rho = 0$ and $\rho = 1$. Compute $\widehat{h}_t$ and $\widehat{y}_t$ on impact for $\alpha = 1/3$, $\beta = 0.95$, $\mu = 1.1$, $\varphi = 1$, $\rho = 0.95$ and $\widehat{\mu}_t = 1\%$.

14. (Optional, computational) Calibrate the model of Part C, feed in a markup process with $\rho = 0.95$ and σ_ε chosen to match the volatility of the measured labor wedge, simulate 50 years of quarterly data, HP-filter with $\lambda = 1600$ and report the variance-covariance matrix of $(C_t, H_t, K_t, X_t, Y_t, \mu_t)$. Which moments does the model match, and which does it miss?

15. Discussion.

(a) An econometrician applies business cycle accounting to data from this economy. Which wedges does she find, and what would she conclude?

(b) In the New Keynesian model of Lecture 2, a shock to ε_t appears as the cost-push shock u_t in the Phillips curve $\pi_t = \beta\mathbb{E}_t\pi_{t+1} + \kappa x_t + u_t$. How is u_t related to the markup shock of this exercise? Why does the shock create a trade-off for the central bank in the NK model but not here?

Part II – The New Keynesian model

Objective. The four exercises of this part follow Lecture 2. Exercise 4 solves a two-period model in which the first-period price is fixed, so that output is determined by demand, and then measures the wedges of Part I in that economy. Exercise 5 derives the Phillips curve from the non-linear equilibrium conditions of the Calvo model. Exercise 6 compares the welfare gain from higher demand, which is of first order under monopolistic competition, with a firm's loss from not adjusting its price, which is of second order. Exercise 7 shows that the optimal monetary policy is then not time consistent.

Reminder: the three-equation model. With preferences $C^{1-\sigma}/(1-\sigma) - N^{1+\varphi}/(1+\varphi)$, a continuum of varieties with elasticity of substitution ε, technology $Y_t(i) = Z_t N_t(i)$ and Calvo pricing with reset probability $1-\theta$, the log-linearized equilibrium is

$$\begin{aligned} &\text{(DIS)} && x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big) \\ &\text{(NKPC)} && \pi_t = \beta\, \mathbb{E}_t \pi_{t+1} + \kappa\, x_t \ \big[+\ u_t\big] \\ &\text{(rule)} && i_t = \rho + \phi_\pi\, \pi_t + \phi_y\, x_t + v_t \end{aligned}$$

where $x_t = y_t - y_t^n$ is the *output gap* with respect to the natural (flexible-price) level of output, σ is the inverse elasticity of intertemporal substitution, φ the inverse Frisch elasticity, $\mu \equiv \varepsilon/(\varepsilon-1)$ the desired gross markup, mc_t real marginal cost, $\rho \equiv -\log\beta$, and

$$\big(Y_t^n\big)^{\sigma+\varphi} = \frac{Z_t^{1+\varphi}}{\mu}, \qquad \widehat{mc}_t = (\sigma+\varphi)\, x_t, \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \qquad \kappa \equiv \lambda\,(\sigma+\varphi).$$

Exercises 5 and 6 derive these expressions. Exercise 7 takes them as given.

Exercise 4 – Sticky prices in a two-period model

The economy (Mankiw & Weinzierl, 2011; Moll, EC2B1 Lecture 7). There are two periods, $t = 1, 2$, and a preliminary period 0 in which a reference price P_0 is set.

- A representative household owns the firms and solves

$$\max_{C_1, C_2}\ U(C_1) + \beta\, U(C_2) \quad \text{s.t.} \quad P_1 C_1 + \frac{P_2 C_2}{1+i_1} = \Omega, \qquad U(C) = \frac{C^{1-\sigma}}{1-\sigma},$$

  where i_1 is the nominal interest rate and Ω the present value of nominal profits. As in Lecture 2, $1/\sigma$ is the elasticity of intertemporal substitution (EIS).

- Technology is linear in capital: $Y_1^S = A_1 K_1$ with K_1 given, $Y_2 = A_2 K_2$ and $K_2 = I_1$ (capital fully depreciates). Firms maximize $\Omega = P_1(Y_1 - I_1) + P_2 A_2 K_2/(1+i_1)$.

- Money demand follows the quantity theory, $M_t = P_t C_t$.

- Monetary policy sets (i_1, M_2), and M_1 adjusts to implement i_1.

- Inflation and the real rate: $1 + \pi_2 = P_2/P_1$ and $1 + r_1 = (1+i_1)P_1/P_2$.

Throughout define

$$\Psi \equiv A_2\,(\beta A_2)^{-1/\sigma}.$$

Part A – Flexible prices: monetary neutrality

1. **Household and firm.** Derive the Euler equation and show that it can be written $C_1 = C_2\big[\beta(1+r_1)\big]^{-1/\sigma}$. Show that the firm's problem has an interior solution only if
$$1 + r_1 = A_2,$$
and that in that case the real value of the firm is $W \equiv \Omega/P_1 = A_1 K_1$.

2. **Real allocation.** Combining the Euler equation, the budget constraint and goods-market clearing ($C_1 + I_1 = Y_1$, $C_2 = Y_2$), show that
$$C_1 = \frac{\Psi}{1+\Psi} A_1 K_1, \quad I_1 = K_2 = \frac{1}{1+\Psi} A_1 K_1, \quad C_2 = Y_2 = \frac{A_2}{1+\Psi} A_1 K_1, \quad Y_1 = A_1 K_1, \quad 1+r_1 = A_2.$$

3. **Nominal variables and neutrality.** Show that
$$P_1 = \frac{1+\Psi}{A_1 K_1}\frac{M_2}{1+i_1}, \qquad P_2 = \frac{1+\Psi}{A_2 A_1 K_1} M_2, \qquad M_1 = \Psi \frac{M_2}{1+i_1}, \qquad 1+\pi_2 = \frac{1+i_1}{A_2}.$$
In what sense is money neutral? In what sense does the classical dichotomy hold?

Part B – Sticky prices: monetary non-neutrality

Assume now that P_1 is fixed at P_0: firms set it before they know A_1, A_2 and β. The price P_2 remains flexible. Output in period 1 is determined by the short side of the market,
$$Y_1 = \min\{Y_1^D, Y_1^S\}, \qquad Y_1^D = C_1 + I_1, \qquad Y_1^S = A_1 K_1,$$
and parameters are such that $Y_1^D \le Y_1^S$.

4. **The equilibrium allocation.** Show that
$$C_1 = \Psi \frac{M_2}{(1+i_1)P_1}, \quad I_1 = \frac{M_2}{(1+i_1)P_1}, \quad Y_1 = (1+\Psi)\frac{M_2}{(1+i_1)P_1}, \quad C_2 = Y_2 = A_2 \frac{M_2}{(1+i_1)P_1},$$
and $P_2 = (1+i_1)P_1/A_2$. [Hint: start from the firm's investment condition, then use the period-2 money market, then the Euler equation.]

5. **Non-neutrality and the output gap.** Show that monetary policy now moves real variables, and that the output gap is
$$\frac{Y_1}{Y_1^S} = \frac{(1+\Psi)\,M_2}{(1+i_1)P_1 A_1 K_1} \le 1.$$
What value of i_1 closes the gap? Interpret it.

6. **Demand shocks.** Consider two demand shocks: households expect lower future productivity ($A_2 \downarrow$), or they become more patient ($\beta \uparrow$).
   (a) Show that $\beta \uparrow$ always lowers Y_1, and that $A_2 \downarrow$ lowers Y_1 if and only if the EIS is below one, $1/\sigma < 1$.
   (b) Show that with flexible prices neither shock affects Y_1. Why does a fixed price change the answer?

7. **The paradox of thrift.** Suppose $\beta \uparrow$. Compare the responses of C_1, I_1, Y_1, C_2 and Y_2 under flexible and sticky prices, and explain the difference. What is the role of the condition $\Omega = P_1 Y_1$?

8. **The real interest rate.** Show that $1 + r_1 = A_2$ also holds when P_1 is fixed, so that a change in i_1 does not move the real rate. Which assumption is responsible? Through which channel does monetary policy affect output here? Compare with the dynamic IS curve $x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma}\big(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n\big)$ of the three-equation model.

Part C – Back to business cycle accounting

9. Measured wedges in a demand recession. An econometrician observes the economy of Part B in a recession ($Y_1 < A_1 K_1$) and measures the wedges of the prototype economy of Part I, with $y_t = A_t k_t$, full depreciation and no labor.

(a) Show that the measured efficiency wedge is

$$A_1^{\text{meas}} = \frac{Y_1}{K_1} = \frac{(1+\Psi)M_2}{(1+i_1)P_1 K_1} < A_1.$$

(b) Show that the measured investment wedge is zero, $\tau_{x1} = 0$.

(c) What would she conclude? Compare with Question 9 of Exercise 2 and with CKM's finding that the efficiency wedge accounts for most of the fall in output in the Great Depression.

10. Partial stickiness. Suppose instead that P_1 is partially flexible, $P_1 = P_0^{1-\omega}\tilde{P}_1^{\omega}$ with $\omega \in [0,1]$, where $\tilde{P}_1$ is the flexible-price level from Question 3. How does the output gap of Question 5 depend on ω? What do the cases $\omega = 1$ and $\omega = 0$ correspond to?

Exercise 5 – From the Calvo system to the New Keynesian Phillips curve

Objective. The Calvo model is usually presented in log-linear form. This exercise starts from its non-linear equilibrium conditions, reduces them to three equations, finds the zero-inflation steady state, and log-linearizes last. The derivation shows where $\lambda = (1-\theta)(1-\beta\theta)/\theta$ comes from and why price dispersion can be ignored to first order.

The system. Labor is the only input, $y_{jt} = z_t h_{jt}$, utility is $U(C,H) = \ln C - v(H)$ with $v' > 0$, $v'' > 0$, varieties have elasticity of substitution $\varepsilon > 1$ and a firm may reset its price each period with probability $1-\theta$. Writing P_t for the ideal (consumption) price index, $\tilde{P}_t$ for the "production" price index, m_t for real marginal cost summed over the expected life of a price and d_t for the corresponding discount factor, an equilibrium is a collection of processes for $C_t(s^t), H_t(s^t), W_t(s^t), Q_0^t(s^t), P_t(s^t), \tilde{P}_t(s^t), m_t(s^t), d_t(s^t)$ such that

$$C_t\, v'(H_t) = \frac{W_t}{P_t} \tag{1}$$

$$C_t = z_t H_t \Big(\frac{\tilde{P}_t}{P_t}\Big)^{\varepsilon} \tag{2}$$

$$P_{t+1}^{1-\varepsilon} = \theta\, P_t^{1-\varepsilon} + (1-\theta)\Big(\frac{\varepsilon\, m_{t+1} P_{t+1}}{(\varepsilon-1)d_{t+1}}\Big)^{1-\varepsilon} \tag{3}$$

$$\frac{1}{C_t} = \beta\, \frac{\Pi_0^{t+1}(s^{t+1})\, Q_0^t(s^t)\, P_t(s^t)}{\Pi_0^t(s^t)\, Q_0^{t+1}(s^{t+1})\, P_{t+1}(s^{t+1})\, C_{t+1}(s^{t+1})} \tag{4}$$

$$\tilde{P}_{t+1}^{-\varepsilon} = \theta\, \tilde{P}_t^{-\varepsilon} + (1-\theta)\Big(\frac{\varepsilon\, m_{t+1} P_{t+1}}{(\varepsilon-1)d_{t+1}}\Big)^{-\varepsilon} \tag{5}$$

$$m_t = \frac{W_t}{z_t P_t} + \theta \sum_{s^{t+1} \succ s^t} \frac{Q_0^{t+1}\, C_{t+1}\, P_{t+1}^{\varepsilon+1}}{Q_0^t\, C_t\, P_t^{\varepsilon+1}}\, m_{t+1} \tag{6}$$

$$d_t = 1 + \theta \sum_{s^{t+1} \succ s^t} \frac{Q_0^{t+1}\, C_{t+1}\, P_{t+1}^{\varepsilon}}{Q_0^t\, C_t\, P_t^{\varepsilon}}\, d_{t+1} \tag{7}$$

The reset price is $P_t^* = \dfrac{\varepsilon}{\varepsilon-1}\dfrac{m_t P_t}{d_t} = \mu \dfrac{m_t P_t}{d_t}$ with $\mu \equiv \varepsilon/(\varepsilon-1)$, a markup over a weighted average of current and expected future marginal costs.

Part A – Reducing the system

Define $\pi_t \equiv P_t/P_{t-1}$ and $\chi_t \equiv \tilde{P}_t/P_t$.

1. Eliminating the intertemporal price. Use (4) to show that (6) and (7) become

$$m_t = \frac{W_t}{z_t P_t} + \theta\beta\,\mathbb{E}_t\big[\pi_{t+1}^{\varepsilon}\, m_{t+1}\big], \qquad d_t = 1 + \theta\beta\,\mathbb{E}_t\big[\pi_{t+1}^{\varepsilon-1}\, d_{t+1}\big].$$

2. An explicit solution for d_t. Divide (3) by $P_t^{1-\varepsilon}$ and solve for d to show that

$$d_t = \Big[\frac{1-\theta}{\pi_t^{1-\varepsilon}-\theta}\Big]^{\frac{1}{1-\varepsilon}} \frac{\varepsilon\, m_t\, \pi_t}{\varepsilon-1}, \qquad \text{equivalently} \qquad \frac{P_t^*}{P_t} = \frac{1}{\pi_t}\Big[\frac{\pi_t^{1-\varepsilon}-\theta}{1-\theta}\Big]^{\frac{1}{1-\varepsilon}}.$$

Interpret the second expression. What is P_t^*/P_t when $\pi_t = 1$?

3. A relation between m and π. Substitute Question 2 into the recursion for d_t to obtain

$$\Big[\frac{1-\theta}{\pi_t^{1-\varepsilon}-\theta}\Big]^{\frac{1}{1-\varepsilon}} \frac{\varepsilon\, m_t\, \pi_t}{\varepsilon-1} = 1 + \theta\beta\,\mathbb{E}_t\left[\pi_{t+1}^{\varepsilon}\Big[\frac{1-\theta}{\pi_{t+1}^{1-\varepsilon}-\theta}\Big]^{\frac{1}{1-\varepsilon}} \frac{\varepsilon\, m_{t+1}}{\varepsilon-1}\right].$$

4. A relation between χ and π. Divide (5) by $P_{t+1}^{-\varepsilon}$ and use Question 2 to show that

$$\chi_{t+1}^{-\varepsilon} = \theta\Big(\frac{\chi_t}{\pi_{t+1}}\Big)^{-\varepsilon} + (1-\theta)\left(\Big[\frac{1-\theta}{\pi_{t+1}^{1-\varepsilon}-\theta}\Big]^{\frac{1}{1-\varepsilon}}\pi_{t+1}\right)^{\varepsilon}.$$

This equation relates χ and π only.

5. A relation between m, H and χ. Eliminate C_t between (1) and (2), then eliminate the real wage in the equation for m_t from Question 1, to get

$$m_t = H_t\, \chi_t^{\varepsilon}\, v'(H_t) + \theta\beta\,\mathbb{E}_t\big[\pi_{t+1}^{\varepsilon}\, m_{t+1}\big].$$

Why does z_t not appear?

Part B – Steady state and log-linearization

The system is now three equations (Questions 3–5) in $(m_t, \pi_t, \chi_t, H_t)$.

6. The zero-inflation steady state. Look for a steady state with $\pi_t = 1$ for all t and show that

$$\chi = 1, \qquad m = \frac{1}{\mu\,(1-\theta\beta)} = \frac{\varepsilon-1}{\varepsilon(1-\theta\beta)}, \qquad H\,v'(H) = \frac{1}{\mu} = \frac{\varepsilon-1}{\varepsilon}.$$

Interpret the last condition, and compare it with Exercise 2 (Question 4) and with the natural level of output of Lecture 2.

7. **Price dispersion is transitory.** Log-linearize the equation of Question 4 around $(\pi, \chi) = (1, 1)$ and show that

$$\widehat{\chi}_{t+1} = \theta\, \widehat{\chi}_t.$$

Conclude that $\widehat{\chi}_t \to 0$ monotonically from any initial condition, and explain why this justifies setting $\widehat{\chi}_t = 0$ in what follows. [Hint: show first that the function $\Lambda(\pi) \equiv \left[\frac{1-\theta}{\pi^{1-\varepsilon}-\theta}\right]^{\frac{1}{1-\varepsilon}} \pi$ satisfies $\Lambda(1) = 1$ and $d\log\Lambda/d\log\pi = -\theta/(1-\theta)$ at $\pi = 1$; note that $P_t^*/P_t = 1/\Lambda(\pi_t)$.]

8. **The Phillips curve.** Log-linearize the equations of Questions 3 and 5 around the zero-inflation steady state, set $\widehat{\chi}_t = 0$, and show that $\widehat{m}_t$ drops out, leaving

$$\boxed{\widehat{\pi}_t = \beta\, \mathbb{E}_t \widehat{\pi}_{t+1} + \lambda\,(1+\omega)\,\widehat{H}_t, \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \qquad \omega \equiv \frac{H\, v''(H)}{v'(H)}.}$$

9. **Mapping to the lecture.** Take $v(H) = \gamma H^{1+\varphi}/(1+\varphi)$.

   (a) Show that $\omega = \varphi$ and that the slope is $\lambda(1+\varphi)$.

   (b) Suppose z_t is constant. Show that $\widehat{H}_t$ equals the output gap $x_t = y_t - y_t^n$ of Lecture 2, so that the boxed equation is $\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa\, x_t$ with $\kappa = \lambda(\sigma + \varphi)$ and $\sigma = 1$.

   (c) What changes if z_t is stochastic?

10. **Discussion.** The parameter θ enters λ twice: through the frequency of price changes, $1-\theta$, and through the weight $\beta\theta$ that a firm puts on next period's marginal cost when it sets its price.

    (a) Show that $\lambda \to \infty$ as $\theta \to 0$ and $\lambda \to 0$ as $\theta \to 1$, and interpret.

    (b) For $\beta = 0.99$ and $\theta = 2/3$ (prices reset every three quarters on average), compute λ and κ for $\varphi = 1$. Compare with the estimates discussed in Lecture 2 ($\kappa \approx 0.006$–0.03).

    (c) Which parameters could reconcile the two? Would a different ε help (see Question 8)?

Exercise 6 – Monopolistic competition and the aggregate demand externality

Objective. This exercise proves two results of Blanchard & Kiyotaki (1987) in the New Keynesian model of Lecture 2. First, under monopolistic competition an increase in aggregate demand raises welfare at first order, while a firm that does not adjust its price loses only at second order. Small costs of changing prices can then have large real effects. Second, no firm has a private incentive to bring this increase about, which is why it is called an aggregate demand *externality*.

The environment (Lecture 2). A representative household has preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right], \qquad \text{so} \qquad \frac{W_t}{P_t} = N_t^{\varphi}\, C_t^{\sigma},$$

with σ the inverse EIS and φ the inverse Frisch elasticity. Firm $i \in [0,1]$ produces $Y_t(i) = Z_t N_t(i)$ and faces demand $Y_t(i) = \big(P_t(i)/P_t\big)^{-\varepsilon} Y_t$ with $\varepsilon > 1$. Real marginal cost is $mc_t \equiv W_t/(P_t Z_t)$, common across firms, and the desired gross markup is $\mu \equiv \varepsilon/(\varepsilon - 1)$. In equilibrium $C_t = Y_t = Z_t N_t$ (ignoring price dispersion, which is of second order by Exercise 5).

Part A – Natural output is inefficiently low

1. **The flexible-price allocation.** Show that with flexible prices every firm sets $P_t(i) = \mu\, W_t/Z_t$, so that $mc_t = 1/\mu$, and deduce the *natural* level of output

$$\left(Y_t^n\right)^{\sigma+\varphi} = \frac{Z_t^{1+\varphi}}{\mu}.$$

2. **The efficient allocation.** Show that a planner who maximizes period utility subject to $Y = ZN$, $C = Y$ equates $-U_N/U_C = Z_t$ and obtains $\left(Y_t^e\right)^{\sigma+\varphi} = Z_t^{1+\varphi}$. Deduce

$$\frac{Y_t^n}{Y_t^e} = \mu^{-\frac{1}{\sigma+\varphi}} < 1, \qquad \text{i.e.} \qquad y_t^n - y_t^e = -\frac{\log \mu}{\sigma+\varphi},$$

and compute the gap for $\varepsilon = 6$, $\sigma = \varphi = 1$.

Part B – The first-order welfare gain of a demand expansion

With sticky prices, output is determined by demand. Since $P_t(i) > W_t/Z_t$, firms are willing to serve extra demand at the posted price.

3. **Welfare effect of a demand expansion.** Consider raising output by dY_t at given prices. Show that

$$dU_t = C_t^{-\sigma}\left(1 - mc_t\right) dY_t, \qquad \text{equivalently} \qquad \frac{dU_t}{U_{Ct}\, C_t} = \left(1 - mc_t\right) x_t,$$

where $x_t \equiv y_t - y_t^n$ is the output gap in logs. Evaluate at $mc_t = 1/\mu$ and show that the coefficient on x_t is

$$\Phi \equiv 1 - \frac{1}{\mu} = \frac{1}{\varepsilon}.$$

Why is this gain of first order, when it would be of second order under perfect competition?

4. **The externality.** All firms and the household would gain if every firm cut its price. Why does no firm do so on its own? Which two assumptions does the argument rely on?

5. **The second-order private loss.** Consider a firm that faces real marginal cost mc_t and aggregate demand Y_t and charges the relative price $p \equiv P_t(i)/P_t$, so that its real profit is $\Pi(p) = (p - mc_t)\, p^{-\varepsilon} Y_t$.

   (a) Show that $\Pi'(p^*) = 0$ at $p^* = \mu\, mc_t$, that $\Pi(p^*) = (p^*)^{1-\varepsilon} Y_t/\varepsilon$, and that $\Pi''(p^*) = -(\varepsilon - 1)(p^*)^{-\varepsilon-1} Y_t$.

   (b) Deduce that a firm charging $p = p^*(1+\xi)$ loses, as a fraction of its revenue,

$$\frac{\Pi(p^*) - \Pi(p)}{p^*\, y(p^*)} \approx \frac{\varepsilon - 1}{2}\, \xi^2.$$

   (c) Now suppose all prices are fixed and the output gap is x_t. Show that the desired relative price moves by $\widehat{p_t^*} = \widehat{mc}_t = (\sigma + \varphi) x_t$, so that a firm that does not adjust loses

$$\frac{\varepsilon - 1}{2}\, (\sigma + \varphi)^2\, x_t^2$$

   of its revenue. Compute this for $\varepsilon = 6$, $\sigma = \varphi = 1$ and $x_t = 5\%$, and compare with the welfare gain of Question 3.

Part C – Policy implications

6. Correcting the level distortion. Suppose the government pays firms an employment subsidy at rate τ, financed by lump-sum taxes, so that the effective marginal cost is $(1-\tau)W_t/(P_t Z_t)$. Show that

$$\tau = \frac{1}{\varepsilon} \qquad \Longleftrightarrow \qquad (1-\tau)\mu = 1 \qquad \Longrightarrow \qquad Y_t^n = Y_t^e, \quad \Phi = 0.$$

7. Welfare and the loss function. Lecture 2 states that, *with the subsidy in place*, a second-order approximation to welfare is

$$\mathbb{W} \approx -\frac{1}{2}\frac{\varepsilon}{\lambda}\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\pi_t^2 + \alpha_x\, x_t^2\Big], \qquad \alpha_x = \frac{\kappa}{\varepsilon}.$$

(a) Why does this expression contain no linear term? What changes without the subsidy?

(b) Why does inflation enter, given that price dispersion is of second order (Exercise 5)?

(c) Why is the relative weight on the output gap so small ($\alpha_x = \kappa/\varepsilon \approx 0.05$ for $\kappa = 0.3$, $\varepsilon = 6$)?

8. Discussion. What do the results of this exercise imply for

(a) the use of Calvo pricing as a modeling device?

(b) the output gap that a benevolent central bank would choose without the subsidy?

(c) the slope of the Phillips curve?

Exercise 7 – Time inconsistency and the inflation bias

Objective. By Exercise 6, without the employment subsidy welfare increases with the output gap at first order, so a benevolent policymaker wants a positive gap. Kydland & Prescott (1977) and Barro & Gordon (1983) show what follows. The optimal policy is not time consistent, and under discretion inflation is positive while output stays at its natural level.

Part A – The classic formulation

Consider an economy with an expectations-augmented Phillips curve

$$u = u_n - \alpha\,(\pi - \pi^e), \qquad \alpha > 0,$$

where u is unemployment, u_n its natural rate and π^e expected inflation. The government minimizes

$$L(u,\pi) = \frac{1}{2}\big(u - k\,u_n\big)^2 + \frac{1}{2}\gamma\,\pi^2, \qquad k < 1,\ \gamma > 0,$$

The government would like zero inflation and unemployment below the natural rate; γ is the weight on inflation relative to unemployment. Assume that the government controls π directly and that agents form expectations before the government moves.

1. Policy when agents expect zero inflation. Suppose agents expect $\pi^e = 0$. Show that the government chooses

$$\tilde{\pi} = \frac{\alpha}{\alpha^2 + \gamma}(1-k)\,u_n > 0, \qquad \tilde{u} = \frac{\gamma + \alpha^2 k}{\gamma + \alpha^2}\,u_n < u_n.$$

Draw the problem in the (u,π) plane, with the indifference curves of L and the Phillips curve through $(u_n, 0)$. Is $(\tilde{\pi}, \tilde{u})$ a rational expectations equilibrium?

2. **The rational expectations equilibrium.** Solve the game by backward induction. Show that the government's best response to any π^e is

$$\pi = \frac{\alpha^2}{\gamma + \alpha^2}\,\pi^e + \frac{\alpha}{\gamma + \alpha^2}(1-k)u_n,$$

and deduce that the unique rational expectations equilibrium is

$$\pi^{RE} = \pi^e = \frac{\alpha}{\gamma}(1-k)\,u_n, \qquad u^{RE} = u_n.$$

Locate it on your graph.

3. **Feasible allocations and commitment.**

   (a) What is the set of allocations attainable under rational expectations? Which one does the government prefer?

   (b) Why does it fail to implement it? Compute the welfare loss from discretion.

   (c) Give two institutional arrangements that would remedy the problem. What does each one change in the model?

Part B – The same argument in the New Keynesian model

Restate the problem in terms of the output gap. The Phillips curve is $\pi_t = \pi_t^e + \kappa\, x_t$ with $\kappa = \lambda(\sigma+\varphi) > 0$, and the policymaker minimizes, period by period,

$$L = \frac{1}{2}\Big[\pi_t^2 + \alpha_x\big(x_t - x^*\big)^2\Big], \qquad \alpha_x = \frac{\kappa}{\varepsilon}, \qquad x^* > 0,$$

where x^* is the output gap the policymaker would like to run.

4. **Discretionary equilibrium.** Taking π_t^e as given, show that the policymaker's best response is $x_t = x^* - \kappa\pi_t/\alpha_x$, and that the rational expectations equilibrium is

$$x_t = 0, \qquad \pi_t = \frac{\alpha_x}{\kappa}\,x^* = \frac{x^*}{\varepsilon}.$$

Compare with Part A. How does the inflation bias depend on κ?

5. **The origin of x^*.** Use Exercise 6 to argue that, without the employment subsidy, the welfare-based loss function contains a linear term in x_t whose coefficient is proportional to $\Phi = 1/\varepsilon$. Why does this imply $x^* > 0$? Which policy instrument removes the inflation bias?

6. **Discounting and the long-run trade-off.** Questions 4 and 5 used a static Phillips curve. In the NK model the Phillips curve is $\pi_t = \beta\,\mathbb{E}_t\pi_{t+1} + \kappa x_t$.

   (a) Show that in a stationary discretionary equilibrium (no shocks, $\mathbb{E}_t\pi_{t+1} = \pi_t$)

$$\pi_t = \frac{\kappa\,x^*}{1-\beta+\kappa^2/\alpha_x} = \frac{\kappa\,x^*}{1-\beta+\kappa\varepsilon}, \qquad x_t = \frac{(1-\beta)\,x^*}{1-\beta+\kappa\varepsilon}.$$

   (b) Check that $\beta \to 1$ gives the result of Question 4. Compute both expressions for $\kappa = 0.3$, $\varepsilon = 6$ and $\beta = 0.99$ (quarterly). Is the inflation bias increasing or decreasing in β? Why?

   (c) For $\beta < 1$ the output gap is positive in equilibrium, because the NKPC implies the long-run relation $x = (1-\beta)\pi/\kappa$. How large is this trade-off for the parameters of (b)? Does it change the conclusion of Question 4?