The Winners and Losers of Climate Policies:
A Sufficient Statistics Approach

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Abstract

To combat global warming, climate policies like carbon taxes, renewable subsidies, and carbon tariffs need to be implemented to phase out fossil-fuel consumption and lower emissions. Who are the winners and losers of such policies? Through a simple Integrated Assessment Model with heterogeneous countries and international trade, we study the costs of climate change through local damages and trade spillovers in international goods and energy markets. We study both the costs of implementing those policies unilaterally, and the local costs and global gains of international policy cooperation. To do so, we express and decompose these welfare changes to first order as a function of *sufficient statistics*, depending on observables and identifiable elasticities, like nations' energy mix, energy rents, trade shares, supply and demand elasticities, and damage parameters. We show that climate change has non-trivial reallocation effects through international trade in goods and energy. Pursuing unilateral policies generates strong leakage effects in goods and energy markets that are an order of magnitude larger than the gains due to reduced emissions. Finally, global climate policy cooperation has a large impact on energy markets, affecting mostly countries reliant on coal and fossil-fuel producers, causing larger welfare losses for those countries than the original costs of climate change.

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1 Introduction

Climate policies must be implemented to phase out fossil fuel consumption and keep the world temperature under $2°C$ to avoid dramatic consequences of global warming (IPCC et al., 2022). To that aim, several policies have been proposed. First, carbon taxation or carbon pricing has been the preferred instrument of economists. It follows from the Pigouvian benchmark, where the externality – and the social cost of carbon emissions – can be internalized by taxation. Second, several countries have been promoting subsidies for renewable energy sources as an alternative to carbon policy, with, for example, the Inflation Reduction Act in the United States. However, carbon taxation may engender "carbon leakage", as economic activity reallocates to other trade partners unaffected by the policy. As a result, countries have also been advocating for international cooperation through climate agreements.

Who are the winners and losers of different climate policies? Can we quantify which countries have the largest losses from climate damages and corrective climate policies such as carbon taxes, renewable energy subsidies, or carbon tariffs? Moreover, how large are the gains from cooperation for the distribution of welfare gains and losses? This paper addresses these questions by quantifying the heterogeneous impacts of those policies and decomposing the welfare impacts for different countries and across different transmission channels.

We develop a framework that allows us to quantify these different margins using sufficient statistics, in the sense of Chetty (2009): a set of observable data moments and elasticities estimated using quasi-experimental variation. In our framework, observables like the energy mix (i.e., the share of oil, gas, coal, and renewables in energy use), the energy rent share of GDP, and trade shares, provide crucial information on whether a country 'wins' or 'loses' as a result of implementing those policies. Using the structure of a quantitative model, we can summarize and decompose the welfare effects of policy: changes in productivity due to climate damages, and welfare changes stemming from general equilibrium effects in energy and goods markets.

To that purpose, we use a climate-economy framework – or Integrated Assessment Model (IAM) – augmented with heterogeneous countries, energy markets, and international trade, closely following Bourany (2025). Individual countries differ in their vulnerability to climate change and temperature, their energy mix in oil, gas, coal, and non-carbon energy, their costs of producing fossil fuels, as well as trade costs in international trade in goods. We approximate the model using a first-order, log-linear decomposition of welfare changes around the current – i.e., status-quo – equilibrium. This allows us to linearly break down the various channels through which climate change and climate policies affect different nations.

These climate policies have unequal impacts across countries, determining their willingness to implement such policies. First, regions are differentially affected by climate change due to differences in local temperature, exposure to global warming, or trade linkages with vulnerable countries. Second, if carbon taxation reduces fossil-fuel consumption, it also has substantial impacts on energy markets: it affects disproportionately the countries that consume a large share of fossil fuels – oil, gas, and coal – or that export those energy sources. Third, countries are heterogeneously exposed to international trade and thus to 'carbon leakage', which reinforces or dampens the gains and losses from the climate policies, especially when implemented unilaterally.

Quantifying the winners and losers from trade policies with our framework – and understanding the underlying mechanisms – requires several key elasticities, in addition to readily available moments in international trade, energy and national accounts data. First, we require estimates of the marginal damages of temperature shocks in different countries on a structural primitive of our model: TFP in traded goods. To identify the parameters of our structural damage function, we implement an estimation strategy inspired by Rudik et al. (2022) that leverages variation in import penetration within bilateral trading partners and changes in local temperature. Second, we require energy supply elasticities in oil-, gas-, and coal-producing sectors. We use time-series variation in local fossil rents and international prices and a simple empirical Bayes shrinkage procedure to recover spatially heterogeneous energy supply elasticities.

Armed with these elasticities and data moments, we use our sufficient statistics formula to study three different experiments for a sample of 193 countries. First, we analyze the effects of increasing greenhouse gas emissions and, hence, global temperature. This has heterogeneous impacts across locations due to differences in temperatures. However, climate change has large spillovers: by changing TFP, it affects production and the endogenous choice of energy inputs. As a result, declines in productivity are the main driver of reduction in CO_2 emissions. Moreover, climate change also reallocates production unequally across international trade partners. Therefore, productivity spillovers through trade represent an important transmission mechanism of global warming across countries, as even cold but open regions can be affected significantly through trade channels.

Second, we study the impacts of four different climate policies. (i) First, we consider the case where each country increases unilaterally the carbon tax, and the revenue of the carbon tax is rebated to the household. As a result, different countries are differently affected through their energy consumption and exports, but the climate impact in terms of emission reduction is limited by the size of each country. Moreover, unilateral carbon taxation leads to carbon leakage effects: it reduces domestic fossil fuel demand through taxation and thus also lowers the global equilibrium price for oil and gas, which then increases the carbon emissions of the countries not affected by the carbon tax. Additionally, by increasing the marginal cost domestically, it also reallocates activity through international trade from regions not affected by the carbon policy. Combining and decomposing all these effects, we can measure the extent of the 'free-riding incentives' that deter individual governments' climate action.

Then, (ii) we consider renewable energy subsidies as an alternative to carbon taxation. Such subsidies lower the relative price of renewable – compared to fossil fuels – and are financed by lump-sum taxes. As a result, it has different general equilibrium implications on energy markets, as well as different welfare effects across countries. Moreover, (iii) to prevent the carbon leakage consequences of unilateral carbon pricing, trade instruments have been at the center of policy discussions, e.g. with the Carbon Border Adjustment Mechanism in the European Union. In a third policy, we study the implementation of carbon tariffs, where the tariff scales with the carbon intensity of the imports and carbon taxes for countries that form a climate club. Carbon tariffs divert trade flows away from high emissions countries and reduce carbon emissions, which we can quantify for a small increase in the carbon price for imports. We analyze this policy for two sets of countries: the European Union (EU), which is already implementing carbon pricing and carbon tariffs, and ASEAN, which gathers southeast Asian countries affected by climate change.

Finally, (iv), we study the distribution of welfare gains and losses from implementing internationally coordinated policies. When all the countries implement carbon taxation together, greenhouse gas emissions are lowered significantly, improving climate and global temperature. In addition, the demand and, hence, the price of fossil fuels change depending on the strength of the substitution between oil, gas, and coal. Such change in oil and gas prices also has strong redistributive effects, as it depletes the energy rents for fossil-fuel exporters. Finally, global climate policy reallocates economic activity and trade patterns, and all these effects attenuate the direct costs of carbon taxation.

This work relates to a lengthy literature on the macroeconomics economics of climate change, specifically the literature that uses large-scale IAMs to evaluate the cost of climate change and the effects of different policies (Nordhaus and Yang, 1996; Barrage and Nordhaus, 2024; Cruz and Rossi-Hansberg, 2024). Our main contribution is to show that, in macroeconomic IAM of Bourany (2025), the effects of many climate policy regimes can be decomposed to the first order to direct effects, effects that operate through the energy market (including changes to energy rents), and leakage effects in international goods markets, and that these effects can be estimated by a set of sufficient statistics readily computable with off-the-shelf macroeconomic data and estimable elasticities. Thus, our work is similar to Lashkaripour (2021), who uses a sufficient statistics approach to estimate the cost of a global trade war, Baqaee and Farhi (2024), who examine how changes to trade barriers reallocate economic activity in general equilibrium, and Kleinman et al. (2024) who derive sufficient statistics for how productivity shocks differentially affect trading partners in constant-elasticity trade models. In essence, we extend this approach to a broad set of climate policy instruments in an environment that also features detailed energy markets, like in Abuin (2024). Additionally, our framework also allows us to derive a local cost of carbon, which accounts for heterogeneity in both damages and the marginal utility of income, like in Cruz and Rossi-Hansberg (2022).

What we do not do is study optimal climate policy at a global level, as in Golosov et al. (2014), or unilaterally optimal policy in an open economy setting, like in Kortum and Weisbach (2021). Unlike Bourany (2025), who studies the optimal design of international climate agreements with carbon taxation and tariffs, or Farrokhi and Lashkaripour (2024), who study the optimal trade policy, either unilaterally or in the context of climate clubs, we exploit the tractability of our sufficient statistics formula to evaluate a large set of climate policies and decompose their effects that operate through different markets. However, our framework is static, which precludes us from studying dynamic policy environments, like in Bourany (2024) who analyze climate policy and redistribution concerns, Hsiao (2022), who studies climate coordination with commitment, or Kotlikoff et al. (2021a) who study the benefit of carbon taxation across generations.

The rest of this paper is organized as follows. In Section 2, we lay out our full macroeconomic IAM. Section 3 derives our first-order decomposition of climate policies in our model and details the policy experiments we have in mind. We describe our data, estimation, and quantification in Section 4. Section 5 details our results for multiple policy counterfactuals, and Section 6 concludes.

2 An integrated assessment model with heterogeneous regions and trade

This model follows the structure of Bourany (2025). We build a simple integrated assessment model (IAM) incorporating multiple dimensions of heterogeneity, climate externality, energy markets, and a realistic trade structure that reproduces the leakage effects of climate policies.

We study a static economy with I countries indexed by $i \in \mathbb{I}$, each with population $\mathcal{P}_i$. All the economic variables are expressed per capita.[1] Each country is composed of five representative agents: (i) a household that consumes the final goods, (ii) a final-good firm producing goods using labor and energy, (iii) a fossil energy firm extracting oil and gas, (iv) a producer of coal energy, and (v) a producer of renewable/non-carbon energy. Moreover, each country has a government that sets taxes, subsidy, and tariffs.

2.1 Household problem

The representative household in country i imports from all countries $j \in \mathbb{I}$ and consumes the aggregate quantity c_i. Each country produces its own good variety. Household preferences have constant elasticity of substitution θ over different varieties, following an Armington structure (Anderson, 1979; Arkolakis, Costinot and Rodriguez-Clare, 2012),

$$\mathcal{U}_i = \max_{\{c_{ij}\}} u\left(\{c_{ij}\}_j\right) = u\left(c_i\right) \ , \qquad c_i = \left(\sum_{j\in\mathbb{I}} a_{ij}^{\frac{1}{\theta}} c_{ij}^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}} \ , \tag{1}$$

where a_{ij} are the preference shifters for country i on the good purchased from country j.[2] We consider standard constant relative risk aversion (CRRA) utility $u(c) = c^{1-\eta}/(1-\eta)$.[3]

Households earn labor income, energy rents, and transfers, and their budget constraints is given by:

$$\sum_{j\in\mathbb{I}} c_{ij} \, (1+\mathrm{t}_{ij}^{b})\tau_{ij}\mathrm{p}_j = w_i\ell_i + \pi_i^e + \ \mathrm{t}_i^{ls} \ , \tag{2}$$

[1] For example, y_i or e_i^f are final output and fossil energy use, respectively, and $\mathcal{P}_i y_i$ and $\mathcal{P}_i e_i^f$ represent the total quantities produced/consumed in the country.

[2] We assume that preferences $\{a_{ij}\}$ and iceberg trade costs $\{\tau_{ij}\}$ are policy-invariant, in particular, they are not sensitive to price changes and tariffs.

[3] We do not include direct effects of climate change on utility, which could proxy for changes to local amenities, or the mortality effects of climate change. It is easy to augment the framework to include direct utility damages by assuming the climate externality affects consumption through a factor $\mathcal{D}_i^u(\mathcal{E})$ which summarizes climate damages, given world emissions $\mathcal{E}$. As labor is internationally immobile, utility damages have no general equilibrium effect, and so we omit them from this analysis. Including utility damages would simply amplify gains and losses stemming from changes in local temperature.

where w_i is the wage rate, ℓ_i the exogenous labor supply is normalized to 1, π_i^e the profit earned from the ownership of the energy firms, and t_i^{ls} the lump-sum transfer received from the government. Households in i imports quantities c_{ij} from country j, purchased at price p_j, and subject to iceberg cost τ_{ij} and to trade tariffs $1+\mathrm{t}_{ij}^b$.

The optimal consumption choice of the household yields the following quantities and trade shares given by:

$$
\begin{aligned}
c_{ij} &= a_{ij} c_i \left(\frac{(1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j}{\mathbb{P}_i} \right)^{-\theta} , \\
s_{ij} \equiv \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i} &= a_{ij} \frac{((1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k)^{1-\theta}} ,
\end{aligned}
\tag{3}
$$

where $p_{ij} = (1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j$ is the effective price for a variety from country j sold in country i, and $\mathbb{P}_i$ is the price index of country i:

$$
\mathbb{P}_i = \left(\sum_{k \in \mathbb{I}} a_{ik} \left((1+\mathrm{t}_{ik}^b)\tau_{ik}\mathrm{p}_k \right)^{1-\theta} \right)^{\frac{1}{1-\theta}} .
$$

As a result, we summarize the budget constraint as $c_i \mathbb{P}_i = \sum_{j \in \mathbb{I}} c_{ij} \, (1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j$, and the per-capita welfare of country i is then summarized by the indirect utility as the utility of income discounted by the price level and climate damages, namely:

$$
\mathcal{U}_i = u\left(c_i\right) = \frac{1}{1-\eta} \left(\frac{w_i \ell_i + \pi_i^e + \mathrm{t}_i^{ls}}{\mathbb{P}_i} \right)^{1-\eta} . \tag{4}
$$

2.2 Final good firm problem

The representative final good producer in country i is producing the domestic variety at price p_i. The firm's profit maximization is:

$$
\max_{\ell_i, e_i^f, e_i^c, e_i^r} \quad \mathrm{p}_i \, \mathcal{D}_i^y(\mathcal{E}) \, z_i \, F(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - q^f(1+\xi^f \mathrm{t}_i^\varepsilon) e_i^f - q_i^c(1+\xi^c \mathrm{t}_i^\varepsilon) e_i^c - q_i^c - q_i^r(1-\mathrm{s}_i^\varepsilon) e_i^r \tag{5}
$$

where the production function $\bar{y}_i = F(\ell_i, e_i^f, e_i^c, e_i^r)$ has constant returns to scale and is concave in all inputs. It uses labor, ℓ_i, at wage w_i, fossil energy, e_i^f, purchased at price, q^f, coal, e^c, at price, q_i^c, and renewable energy, e_i^r, at price, q_i^r. Energy from oil-gas, e_i^f, and coal, e_i^c, differ from renewable in the sense that they emit greenhouse gases, with respective carbon concentration ξ^f and ξ^c, as we will see in 2.4. As a result, there is a motive for taxing oil, gas, and coal energy with the carbon tax t_i^ε. Similarly, as an alternative, we consider renewable energy subsidy, which reduces the price of renewable subsidies by a factor s^ε.

The productivity of the domestic good firm, $y_i = \mathcal{D}_i^y(\mathcal{E}) \, z_i \, \bar{y}_i$, can be decomposed in two terms. First, the TFP, z_i, represents productivity as well as institutional/efficiency differences between countries. This technology wedge accounts for income inequality across countries. These differences in TFP translate into differences in consumption that create redistribution motives for tax policy. The second difference in productivity comes from the climate externality. This is summarized by the net-of-damage function $\mathcal{D}_i^y(\mathcal{E})$, given world emissions $\mathcal{E}$, which is also a reduced-form representation of the climate system from temperatures. It decreases in $\mathcal{E}$ and is country-specific due to differences in costs of climate change, as detailed in Section 4.

The firm input decisions solve the optimality conditions, where we define the marginal product of an input x as $MPx_i \equiv \mathcal{D}_i^y(\mathcal{E})\, z_i F_x(\ell_i, e_i^f, e_i^c, e_i^r)$ for $x \in \{\ell_i, e_i^f, e_i^c, e_i^r\}$. For example, in the case of oil and gas e_i^f, the first-order condition can be written as:

$$\mathrm{p}_i \mathcal{D}_i^y(\mathcal{E})\, z_i\, F_{e^f}(\ell_i, e_i^f, e_i^c, e_i^r) =: \mathrm{p}_i MPe_i^f = q^f (1+\xi^f \mathrm{t}_i^\varepsilon) \ , \tag{6}$$

and similarly for other inputs ℓ_i, e_i^c, e_i^r. Crucially, the private decisions of firms do not internalize climate externalities of their own fossil-fuel energy use and only respond to price, carbon tax t_i^ε, and subsidy s_i^ε.

We consider a nested CES production function. The firm combines labor ℓ_i with a composite of energy e_i, with elasticity σ^y.[4] Second, energy e_i aggregates the different energy sources: oil and gas e^f, coal e_i^c, and renewable/non-carbon e_i^r, with elasticity σ^e.

$$\text{Output:} \qquad y_i = \mathcal{D}^y(\mathcal{E})\, z_i \left(\varepsilon^{\frac{1}{\sigma^y}} (e_i)^{\frac{\sigma^y-1}{\sigma^y}} + (1-\varepsilon)^{\frac{1}{\sigma^y}} (\ell_i)^{\frac{\sigma^y-1}{\sigma^y}} \right)^{\frac{\sigma^y}{\sigma^y-1}} ,$$

$$\text{Energy:} \qquad e_i = \left((\omega^f)^{\frac{1}{\sigma^e}} (e_i^f)^{\frac{\sigma^e-1}{\sigma^e}} + (\omega^c)^{\frac{1}{\sigma^e}} (e_i^c)^{\frac{\sigma^e-1}{\sigma^e}} + (\omega^r)^{\frac{1}{\sigma^e}} (e_i^r)^{\frac{\sigma^e-1}{\sigma^e}} \right)^{\frac{\sigma^e}{\sigma^e-1}} .$$

This allows us to distinguish between the substitution across energy sources and between energy and other inputs like labor, due to climate policies.

2.3 Energy markets

The final-good firm consumes three kinds of energy sources – oil-gas, coal, or renewable (non-carbon) energy – which are supplied by three representative energy firms in each country. Oil-gas sources are traded internationally, and countries can be exporters or importers. Coal and renewable sources are both traded locally, an empirically relevant assumption given the substantial trade costs in coal shipping or electricity transfers. The profits all the energy firms $\pi_i^e = \pi_i^f + \pi_i^c + \pi_i^r$ are redistributed lump-sum to the household.

2.3.1 Fossil production

In each country $i \in \mathbb{I}$, a competitive energy producer extracts fossil fuels – oil and gas – e_i^x and sells it to the international market at price q^f. The energy is extracted at convex cost $\mathcal{C}_i^f(e_i^x)$, where the convex costs are paid in the price of the good of country i.[5] The energy firm's profit

[4] Labor is inelastically supplied $\ell_i = \bar{\ell}_i$ in each country and normalized to 1 – since the country size $\mathcal{P}_i$ is already taken into account. As a result, all the variables can be seen as input per capita.

[5] Alternatively, one could assume that the energy firms use labor inputs, which is equivalent given that there is a direct mapping between prices and wages in Armington models.

maximization problem is given by:

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)\mathrm{p}_i \ , \tag{7}$$

where π_i^f is the fossil energy rent per capita in country i. Since the extraction costs are convex, the production function has decreasing return to scale.[6] Thus, a positive energy rent exists even with competitive firms taking the fossil price as given. For the sake of simplicity, we do not consider that energy firms have market power in the setting of global energy prices as in Bornstein et al. (2023), even though this framework could easily allow for such an extension. We account for misallocation (in the sense of Hsieh and Klenow, 2009) arising due to existing policy distortions that take the form of fossil taxes or subsidies as embedded in extraction productivity in $\mathcal{C}_i^f(\cdot)$, while we capture endogenous misallocation from market power (which can attenuate output elasticities) in our estimates of energy supply elasticities.

Naturally, the optimal extraction decision follows from the optimality condition,

$$q^f = \mathcal{C}_i^{f\,\prime}(e_i^x)\mathrm{p}_i \ , \tag{8}$$

which yields the implicit function $e_i^{x\star} = e^x(q^f/\mathrm{p}_i) = \mathcal{C}_i^{f\prime\,-1}(q^f/\mathrm{p}_i)$. Finally, the energy rent comes from fossil firms' profits $\pi^f(q^f,\mathrm{p}_i) = q^f e^x(q^f/\mathrm{p}_i) - \mathcal{C}_i^f\left(e^x(q^f/\mathrm{p}_i)\right)\mathrm{p}_i > 0$ and depends on the marginal costs as well as the inverse supply elasticity $\nu_i^f = \frac{\mathcal{C}_i^{f\,\prime\prime}(e^x)}{\mathcal{C}_i^{f\,\prime}(e^x)e^x}$. We use the isoelastic extraction function $\mathcal{C}_i^f$,

$$\mathcal{C}_i^f(e_i^x)\mathrm{p}_i = \frac{\bar{\nu}_i^f}{1+\nu_i^f}\left(\frac{e_i^x}{\mathcal{R}_i}\right)^{1+\nu_i^f}\mathcal{R}_i\mathrm{p}_i \ .$$

with $\mathcal{R}_i$ the oil-gas reserves, a fixed factor. This is homogeneous of degree one in $(e_i^x, \mathcal{R}_i)$ and implies a constant elasticity supply function. In this formulation, we think of market power as affecting the supply elasticity by affecting the intensity of reserves in the production of fossil energy. We can write the profit function as,

$$\pi_i^f = \frac{\nu_i^f\bar{\nu}_i^f}{1+\nu_i^f}\left(\frac{e_i^x}{\mathcal{R}_i}\right)^{1+\nu_i^f}\mathcal{R}_i\mathrm{p}_i = \frac{\nu_i^f(\bar{\nu}_i^f)^{-1/\nu_i^f}}{1+\nu_i^f}(q^f)^{1+\frac{1}{\nu_i^f}} \ .$$

As we will see below, the profit π_i^f and its share in income $\eta_i^{\pi f} = \frac{\pi_i{}^f}{y_i\mathrm{p}_i+\pi_i{}^e}$ are key to determine the exposure of a country to carbon taxation. Indeed, reducing carbon emissions by phasing out of fossil fuels reduces energy demand and its price q^f and hence affects energy profit π_i^f and the welfare of large oil and gas exporters.

[6] We can also define a fossil production function with inputs x_i^f such that $e^x = g(x_i^f)$ and profit $\pi = q^f g(x) - x\mathbb{P}_i$ instead of $\pi = q^f e^x - \mathcal{C}(e^x)\mathrm{p}_i$, in which case $g(x) = \mathcal{C}^{-1}(x)$.

2.3.2 International fossil energy markets

We assume that oil and gas are traded frictionlessly in international markets.[7] The market clears such that

$$E^f = \sum_{i\in\mathbb{I}} \mathcal{P}_i e_i^f = \sum_{i\in\mathbb{I}} \mathcal{P}_i e_i^x \ . \tag{9}$$

Countries have different exposure to this fossil energy market. As country i consumes total quantity of fossil fuels $\mathcal{P}_i e_i^f$, produces $\mathcal{P}_i e_i^x$, its net exports of oil-gas are $\mathcal{P}_i(e_i^x - e_i^f) \lesseqgtr 0$.

2.3.3 Coal production

A representative firm in each country produces coal, which is consumed by the final good firm. We differentiate coal from other fossil fuels like oil and gas because coal production typically generates smaller energy rents for producing countries as a share of GDP. Moreover, large coal producers also consume a large fraction of that coal locally, as trade costs for coal transportation are larger. Hence, we make this empirically grounded assumption that coal is not traded.

We again assume production of $\bar{e}_i^c$ is decreasing returns to scale (owing to convex extraction costs) and uses country i final good input. Coal producers' the profit maximization problem is,

$$\pi_i^c = \max_{e_i^c} q_i^c \bar{e}_i^c - \mathcal{C}_i^c(\bar{e}_i^c)\mathrm{p}_i \ ,$$

with the cost function $\mathcal{C}_i^c$, with inverse supply elasticity $\nu_i^c = \frac{\mathcal{C}_i^{c\prime\prime}(e^c)}{\mathcal{C}_i^{c\prime}(e^c)e^c}$. As before, the price for coal and the market clearing condition are given by:

$$q_i^c = \mathcal{C}_i^{c\prime}(\bar{e}_i^c)\mathrm{p}_i \quad , \tag{10}$$

where in equilibrium, $\bar{e}_i^c = e_i^c$. We consider the same isoelastic cost function as for the oil-gas production, with constant inverse elasticity ν_i^c.

2.3.4 Renewable and non-carbon energy production

The final good firm also uses renewable and other low-carbon energy sources, such as solar, wind, or nuclear electricity. A representative firm produces renewable or non-carbon energy, and this supply, $\bar{e}_i^r$, is not traded internationally. This assumption is verified by the fact that electricity is rarely traded across countries – and when it is, it is explained by intermittency rather than structural imbalances. The cost function $\mathcal{C}_i^r(\bar{e}_i^r) = x_i^r$ is paid in country i good at price p_i. Hence,

[7] We refrain from considering a general Armington structure in which each country produces unique, imperfectly substitutable energy varieties. We make the simplifying assumption that fossil fuels produced in different countries are not distinguishable and traded without cost in international markets. That is, crude oil or natural gas from Nigeria, Saudi Arabia, or Russia are perfect substitutes and their movement across borders is costless. In reality, fossil energy has less than an infinite elasticity of substitution due to quality grade differences like sulfur content, and there are trade costs in shipping oil, despite the considerable scale economies in transport on large crude carriers.

the renewable firm maximization problem is:

$$\pi_i^r = \max_{\bar{e}_i^r} q_i^r \bar{e}_i^r - \mathcal{C}_i^r(\bar{e}_i^r)\mathrm{p}_i \ ,$$

with inverse supply elasticity $\nu_i^r = \frac{\mathcal{C}_i^r{}''(e^r)}{\mathcal{C}_i^r{}'(e^r)e^r}$. As a result, the price of renewable and the market clearing are given by:

$$q_i^r = \mathcal{C}_i^r{}'(\bar{e}_i^r)\mathrm{p}_i \ , \qquad \bar{e}_i^r = e^r \ . \tag{11}$$

Again, we consider the same isoelastic cost function as for the oil-gas and coal production, with constant inverse elasticity ν_i^r. When $\nu_i^r = 0$, we have constant return to scale, $\mathcal{C}_i^r{}'$ is a constant and zero profits $\pi_i^r = 0$. This, once again, would return a perfectly elastic supply curve, which is a slightly stronger assumption in the context of renewable energy.

2.4 The climate system

Carbon emissions released from the burning of fossil fuels create an externality as they feed back into the atmosphere, increase temperatures and affect damages. Despite the model being static, we incorporate a simple climate system as in standard Integrated Assessment Models.

We model the climate damage affecting country i's productivity with the structural damage function $\mathcal{D}_i^y(\mathcal{E})$, a reduced-form representation of how rising temperatures (and other correlated weather changes) affect a nation's productivity z_i. The structural damage function maps emissions $\mathcal{E}$ to atmospheric carbon concentration $\mathcal{S}$, which fosters a rise in global temperature, and, in turn, local temperatures T_i, which affect output.

The energy choices yield yearly emissions from fossil fuels, which sum up to,

$$\mathcal{E} = \sum_{i \in \mathbb{I}} \mathcal{P}_i(\xi^f e_i^f + \xi^c e_i^c) \ ,$$

where ξ^f and ξ^c represent the carbon concentration of oil-gas and coal, respectively. These emissions increase the carbon concentration in the atmosphere. We use the scalar $\mathbb{T}$ to convert a static – one year – model to a long-term / dynamic representation of climate.

$$\mathcal{S} = \mathcal{S}_0 + \mathbb{T}\mathcal{E}$$

with $\mathcal{S}_0$ the initial carbon concentration before all the policy decisions are made at the beginning of the 21st century. We assume a linear relationship between the cumulative CO_2 emissions $\mathcal{S}$ and the global temperature anomaly $\mathcal{T}$ compared to preindustrial levels.

$$\mathcal{T} = \chi\mathcal{S} = \chi\left(\mathcal{S}_0 + \mathbb{T}\mathcal{E}\right) \ ,$$

where χ is the climate sensitivity parameter, i.e. how much warming a ton of CO_2 causes, and where $\mathcal{E}$ and $\mathcal{S}$ are measured in carbon units. This specification is rationalized by the climate-sciences literature. For example, Dietz et al. (2021) shows an approximately linear relationship

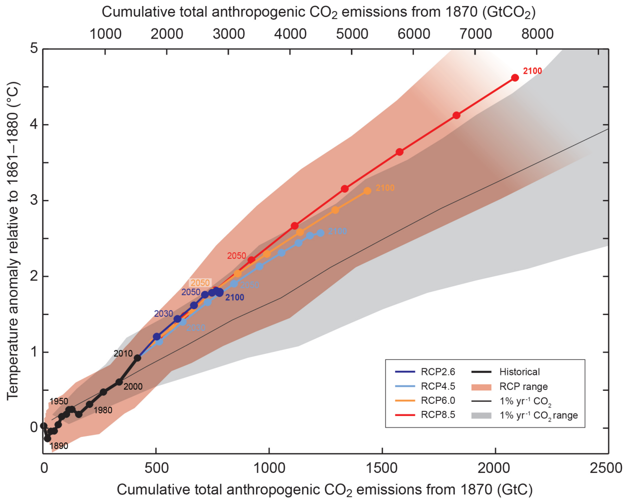

Figure 1: Cumulative emissions and temperature, IPCC et al. (2022)

between $\mathcal{S}$ and $\mathcal{T}$, as shown in Figure 1. It displays the relationship between temperature anomaly and cumulative CO_2 emissions over time, both for historical data in black and a large class of climate models in different Representative Concentration Pathways (RCP).

Moreover, we consider a linear relationship between global and local temperatures, namely,

$$T_i = \Delta_i \mathcal{T} = \Delta_i \chi \mathcal{S} \ ,$$

where Δ_i is a linear pattern scaling parameter that depends on geographical factors such as albedo or latitude.

Finally we consider a period damage function $\hat{\mathcal{D}}(T_{it} - T_i^\star)$ where $T_i^\star$ is the optimal' temperature for country i. The function $\hat{\mathcal{D}}(\hat{T})$ is a reduced-form representation of the economic damage to productivity, with curvature δ

$$\mathcal{D}_i^y(T_i - T_i^\star) = e^{-\gamma^y (T_i - T_i{}^\star)^{1+\delta}} \tag{12}$$

In our baseline quantification, we assume damages are quadratic, i.e. $\delta = 1$, as in the Integrated Assessment Models of Nordhaus' DICE-RICE, Krusell and Smith (2022), Kotlikoff et al. (2021a) and Burke et al. (2015). Such damage creates winners and losers: the countries warmer than the target temperature $T^\star$ are more affected by global warming. In contrast, regions with negative $T_i - T_i^\star$ benefit from a warmer climate. We discuss this quantification in Section 4.

To conclude, the reduced-form static damage functions $\mathcal{D}_i^y(\mathcal{E})$ and $\mathcal{D}_i^u(\mathcal{E})$, for productivity and utility, respectively, we summarize the future costs of climate change in present-discounted value,

$$\mathcal{D}_i^y(\mathcal{E}) = \mathcal{D}_i^y(T_i - T_i^\star) = \mathcal{D}_i^y\left(\Delta_i \chi (\mathcal{S}_0 + \mathbb{T}\mathcal{E}) - T_i^\star\right).$$

The damage function $\mathcal{D}_i^y(\mathcal{E})$ changes endogenously with climate policy choices, as they affect $\mathcal{E}$.

2.5 Equilibrium

To close the model, we need to determine the final good prices for each country p_i. To do so, we consider market clearing for each good i, which happens for the total quantity of goods, and not on a per capita basis, so we must adjust by total population, $\mathcal{P}_i$,

$$\mathcal{P}_i \underbrace{y_i}_{=\mathcal{D}_i^y(\mathcal{E}) z_i F(\cdot)} = \sum_{k \in \mathbb{I}} \mathcal{P}_k \tau_{ki} c_{ki} + \mathcal{P}_i (x_i^f + x_i^c + x_i^r) \tag{13}$$

where x_i^f, x_i^c and x_i^r are the good inputs used in country i to produce fossil and renewable energy, respectively. To summarize, the competitive equilibrium of this economy is defined as follows:

Definition. Competitive equilibrium (C.E.):
For a set of policies $\{\mathrm{t}_i^\varepsilon, \mathrm{t}_{ij}^b, \mathrm{t}_i^{ls}\}_i$ across countries, a C.E. is a set of decisions $\{c_{ij}, e_i^f, e_i^c, e_i^r, e_i^x, \bar{e}_i^c, \bar{e}_i^r\}_{ij}$, and prices $q^f, \{\mathrm{p}_i, w_i, q_i^c, q_i^r\}_i$ such that:

(i) Households choose consumption $\{c_{ij}\}_{ij}$ maximizing utility equation (1) s.t. the budget constraint equation (2), which yield trade shares equation (3)
(ii) Final good firms choose inputs $\{\ell_i, e_i^f, e_i^r\}_i$ to maximize profits, resulting in equation (6)
(iii) Fossil energy firms maximize profits equation (7) and extract/produce $\{e_i^x\}_i$ given by equation (8)
(iv) Renewable and coal energy firms maximize profits and supplies $\{\bar{e}_i^c, \bar{e}_i^r\}$ are given respectively by equation (10) and equation (11)
(v) Energy markets clears for fossils as in equation (9) and for coal and renewable in equation (10) and equation (11)
(vi) Good markets clear for each country as in equation (13), and trade is balanced by Walras Law.

3 Welfare decomposition and climate policies experiments

Different countries have unequal exposure to both climate change and different climate policies. This international heterogeneity depends on how local temperature shocks impact countries' production, countries' exposure to international energy markets, and countries' position in the international goods trade network. In this section, we log-linearize the model to the first order to describe analytically the different transmission channels of several climate policy experiments.

3.1 Summary of the different experiments

In these experiments, we compute the marginal change in welfare $d\mathcal{U}_i$ for different policies, e.g., changes in energy taxes $d\mathrm{t}$. Our preferred welfare measure is the consumption-equivalent change in welfare measured as,

$$\Delta_{\mathrm{t}} \mathcal{U}_i = \frac{d\mathcal{U}_i}{d\mathrm{t}} \frac{1}{u'(c_i)\, c_i}$$

for policy change $d\mathrm{t}$. Note that this welfare metric – in percentage terms – measures the *total derivative* $d\mathcal{U}_i/d\mathrm{t}$ and summarizes all the general equilibrium effects of the policy. While country i's welfare is simply the indirect utility $\mathcal{U}_i$, we compute the world welfare for several experiments,

$$\mathcal{W} = \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \mathcal{U}_i \qquad , \tag{14}$$

with Pareto weights ω_i and population $\mathcal{P}_i$. Similarly, the welfare change – measured in consumption-equivalent units is:

$$\Delta_{\mathrm{t}} \mathcal{W} = \frac{d\mathcal{W}/d\mathrm{t}}{\sum_i \mathcal{P}_i \omega_i \, u'(c_i)\, c_i} = \sum_i \mathcal{P}_i \widehat{\omega}_i \, \Delta_{\mathrm{t}} \mathcal{U}_i \tag{15}$$

with the consumption equivalent weights $\widehat{\omega}_i$,

$$\mathcal{P}_i \widehat{\omega}_i = \frac{\mathcal{P}_i \omega_i u'(c_i)\, c_i}{\sum_j \mathcal{P}_j \omega_j \, u'(\widetilde{c}_j)\, \widetilde{c}_j} \ .$$

Change in carbon emissions and global warming

In a first experiment, we want to understand the impact of climate change on different countries. For that, we write the response of agents welfare $d\mathcal{U}_i$ and decisions dx_i a first-order log-linear change in global emissions, $d \log \mathcal{E} = \frac{d\mathcal{E}}{\mathcal{E}}$, which will impact global temperature $\mathcal{T}$, by an amount $d \log \mathcal{T}$ and hence local temperature T_i. This changes productivity through damage as well as utility. However, the goal of this exercise is to quantify and decompose the welfare gains between (i) the direct effect in terms of TFP and the indirect effects due to (ii) the endogenous choice of inputs, in particular energy sources, and (iii) the reallocation of production through international trade. The total impact of climate is measured in consumption-equivalent percentage change,

$$\Delta_{\mathcal{E}} \mathcal{U}_i = \frac{1}{u'(c_i)\, c_i} \frac{d\mathcal{U}_i}{d\mathcal{E}} \ .$$

As we know from the climate economics literature, summarizing all these effects takes the form of the Social Cost of Carbon (SCC) – accounting for global welfare – and the Local Social Cost of Carbon (LCC) for the local welfare of country i. These represent the monetary cost of extra emissions. The Local Cost of Carbon is,

$$LCC_i = -\frac{\partial \mathcal{U}_i}{\partial \mathcal{E}} \Big/ \frac{\partial \mathcal{U}_i}{\partial c_i} = -\frac{d\mathcal{U}_i}{d\mathcal{E}} \frac{1}{u'(c_i)} \tag{16}$$

where $d\mathcal{U}_i$ is the change in i's welfare due to an 1% increase in emissions, and $u'(c_i)$ is the marginal utility of consumption.

Interestingly, at the first-order and small change in $d\mathcal{E}$, the Local Cost of Carbon is related to the consumption equivalent welfare change:

$$LCC_i = -\frac{d\mathcal{U}_i}{d\mathcal{E}} \frac{1}{u'(c_i)} = -c_i \, \Delta_{\mathcal{E}} \mathcal{U}_i$$

where LCC_i is measured in monetary units and $\Delta_c \mathcal{U}_i$ in percentage change.

Similarly, the global welfare changes for additional carbon emission relate to the Social Cost of Carbon for the welfare objective $\mathcal{W}$, which gives:

$$\begin{aligned} SCC = -\frac{\frac{\partial \mathcal{W}}{\partial \mathcal{E}}}{\frac{\partial \mathcal{W}}{\partial c}} &= \sum_i \mathcal{P}_i \omega_i u'(c_i)\, LCC_i \\ &= -\sum_i \mathcal{P}_i \omega_i u'(c_i) c_i \ \Delta_{\mathcal{E}} \mathcal{U}_i \\ &= -\left(\sum_i \mathcal{P}_i \omega_i u'(c_i) c_i\right) \ \Delta_{\mathcal{E}} \mathcal{W} \end{aligned}$$

As a result, the Social Cost of Carbon is proportional to the consumption equivalent welfare change, where the conversion factor is the household consumption weights by the social welfare weights $\omega_i u'(c_i)$. Details on this calculation in more general Integrated Assessment Models can be found in Bourany (2024).

Unilateral carbon taxation

We now analyze how welfare changes when each country implements a unilateral carbon tax t_i^ε on its own emissions. Such a tax increases the cost of oil-gas by $\xi^f d \log \mathrm{t}_i^\varepsilon \approx \xi^f d\mathrm{t}_i^\varepsilon$ and the cost of coal by $\xi^c d\mathrm{t}_i^\varepsilon$. The consumption-equivalent welfare measure for country i of implementing unilaterally a marginal increase in the carbon tax t_i^ε when other countries are passive $\mathrm{t}_j^\varepsilon = 0, \forall j \neq i$,

$$\Delta_{\mathrm{t}_i^\varepsilon} \mathcal{U}_i = \frac{1}{u'(c_i)\, c_i} \frac{d\mathcal{U}_i}{d\mathrm{t}_i^\varepsilon} \ .$$

Part of these welfare effects can be decomposed between (i) the direct effect on climate – which is limited given the unilateral implementation of the policy – and (ii) the general equilibrium effect through energy prices, oil-gas rents, and good prices. In particular, the leakage effects play a significant role that we can quantify. Naturally, such leakage effects generate welfare gains for country i when country j implements these carbon policies, $\Delta_{\mathrm{t}_j^\varepsilon} \mathcal{U}_i$. As a result, the leakage spillovers for all countries can be summarized by the Jacobian matrix:

$$\boldsymbol{\partial}_{\mathbf{t}} \boldsymbol{\mathcal{U}} = \left\{ \Delta_{\mathrm{t}_j^\varepsilon} \mathcal{U}_i \right\}_{ij}$$

withwelfare change $\Delta_{\mathrm{t}_j^\varepsilon} \mathcal{U}_i$ for country i (row) from country j (column) unilateral carbon policy t^ε.

Unilateral renewable subsidies

Carbon taxation creates an additional cost for fossil fuels and is particularly detrimental to coal energy. Moreover, due to the leakage effect in terms of energy prices and international trade in goods, unilateral carbon taxation induces negative welfare losses, which give rise to policy inaction. We consider an alternative policy where each government unilaterally implements a subsidy on renewable energy $d \log \mathrm{s}_i^\varepsilon \approx d\mathrm{s}_i^\varepsilon$, which reduces the cost of renewable inputs e_i^r for final good firms. This type of industrial policy is financed through lump-sum taxation – maintaining symmetry with our carbon taxation experiments. The welfare changes can be written as,

$$\Delta_{\mathrm{s}_i^\varepsilon} \mathcal{U}_i$$

and similarly for the cross-country spillover Jacobian $\boldsymbol{\partial}_{\mathbf{s}}\mathcal{U} = \left\{\Delta_{\mathrm{s}_j^\varepsilon} \mathcal{U}_i\right\}_{ij}$.

Unilateral carbon tariffs

To dampen the leakage effects through international trade in goods, carbon tariffs have been suggested. We consider the unilateral policy where each country i imposes tariffs that scale with the carbon intensity of the country j it imports from:

$$d\log \mathrm{t}_{ij}^b \approx d\mathrm{t}_{ij}^b = \xi_j^\varepsilon d\mathrm{t}_i^{b,\varepsilon} \qquad\qquad \xi_j^\varepsilon = \frac{\epsilon_i}{y_i \mathrm{p}_i}$$

where ξ_j^ε is the carbon intensity of country i with total emissions ϵ_i, and $d\mathrm{t}_i^{b,\varepsilon}$ is the the marginal increase in carbon price. Unilateral carbon tariffs change the relative prices of goods and has stronger terms-of-trade effects for carbon-intensive countries. The welfare change is denoted $\Delta_{\mathrm{t}_i^b} \mathcal{U}_i$.

Unilateral carbon taxation with carbon tariffs

Our fourth policy experiment is the combination of two unilateral policies : (i) a carbon tax $d\mathrm{t}_i^\varepsilon$ on fossil fuel consumption combined with (ii) a carbon tariff $d\mathrm{t}_i^{b,\varepsilon}$ on the carbon content of the imports, both of them the same size. Such a policy gives rise to a welfare change for country i,

$$\Delta_{\mathrm{t}_i^{b,\varepsilon}} \mathcal{U}_i$$

where the spillovers from j's policy on country i write in matrix form $\boldsymbol{\partial}_{\mathbf{t}^{b,\varepsilon}}\mathcal{U} = \left\{\Delta_{\mathrm{t}_j^{b,\varepsilon}} \mathcal{U}_i\right\}_{ij}$.

Coordinated carbon policy

We now turn to coordinated climate policy. Unilateral policies are limited in their impact on climate due to the small size of countries relative to the world economy, which creates free-riding incentives. We consider the implementation of a marginal increase of the carbon tax in a large set J of countries – roughly, a climate agreement. We consider both an agreement with all the countries $\mathrm{J} = \mathbb{I}$, and agreements within the EU and ASEAN member states, in which nonmembers face a carbon tariff. The optimal design of climate agreements with carbon taxation and tariffs is studied and solved for in Bourany (2025).

The welfare of country i depends on whether it belongs to the agreements $i \in \mathrm{J}$ or not,

$$\Delta_{\mathrm{t}^\varepsilon|\mathrm{J}} \mathcal{U}_i,$$

for each agreement coalition J. In our experiments, we measure changes in welfare to the first-order given increase in the carbon tax. This implies that the cumulative impact of coordination scales linearly with the number of additional countries such that:

$$\begin{aligned}\Delta_{\mathrm{t}^\varepsilon|\mathrm{J}}\,\mathcal{U}_i &= \frac{1}{u'\left(c_i\right)c_i}\,\frac{d\mathcal{U}_i}{d\mathrm{t}^\varepsilon}\Big|_{\mathrm{J}} \\ &= \frac{1}{u'\left(c_i\right)c_i}\sum_{j\in\mathrm{J}}\alpha_{ij}\,\frac{d\mathcal{U}_i}{d\mathrm{t}_j^\varepsilon}\,\underbrace{\frac{d\mathrm{t}_j^\varepsilon}{d\mathrm{t}^\varepsilon}}_{=1} = \sum_{j\in\mathrm{J}}\alpha_{ij}\;\Delta_{\mathrm{t}_j^\varepsilon}\,\mathcal{U}_i,\end{aligned}$$

where α_{ij} are parameters to be determined as a function of the observables, like trade and energy market shares, energy elasticities, and damage parameters, as we will see below. Coordination gains may be nonlinear in terms of welfare – both in the number of countries J and the size of the carbon tax t^ε – as discussed in Bourany (2025).

We now derive the model's welfare decomposition for general experiments. In the next section, we turn to the result for specific policy experiments.

3.2 Observables and sufficient statistics

We now derive the impact of changes in prices and quantities on welfare through the budget constraint. First, we define several objects that are relevant for the decomposition:

- Trade share: $s_{ij} = \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i}$
- Energy share in production: $s_i^e = \frac{e_i q_i^e}{y_i \mathrm{p}_i}$
- Fossil share in energy mix $s_i^f = \frac{e_i^f q^f}{e_i q_i^e}$ and similarly $s_i^c = \frac{e_i^c q_i^c}{e_i q_i^e}$ and $s_i^r = \frac{e_i^r q_i^r}{e_i q_i^e}$
- Fossil energy share: $\eta_i^{\pi f} = \frac{\pi_i^f}{y_i\mathrm{p}_i + \pi_i^e}$ and similarly for $\eta_i^{\pi c}$, and $\eta_i^{\pi r}$.
- Production share/rent share in GDP: $\eta_i^y = \frac{y_i\mathrm{p}_i}{y_i\mathrm{p}_i+\pi_i^e} = 1-\eta_i^{\pi f}-\eta_i^{\pi c}-\eta_i^{\pi r}$
- Consumption expenditure: $x_i = c_i\mathbb{P}_i$
- Consumption share in GDP: $\eta_i^c = \frac{x_i}{y_i\mathrm{p}_i+\pi_i^e}$
- Consumption as a ratio of output: $s_i^{c/y} = \frac{c_i\mathbb{P}_i}{y_i\mathrm{p}_i} = \frac{x_i}{y_i\mathrm{p}_i+\pi_i^f}\frac{y_i\mathrm{p}_i+\pi_i^f}{y_i\mathrm{p}_i} = \frac{\eta_i^c}{1-\eta_i^\pi} = \frac{\eta_i^c}{\eta_i^y}$,
- Energy share as a ratio of consumption: $\frac{e_i q_i^e}{x_i} = \frac{e_i q_i^e}{y_i\mathrm{p}_i}\frac{y_i\mathrm{p}_i}{y_i\mathrm{p}_i+\pi_i^e}\frac{y_i\mathrm{p}_i+\pi_i^f}{x_i} = s_i^e\frac{\eta_i^y}{\eta_i^c}$
- Profit share as a ratio of consumption: $\frac{\pi_i^f}{x_i} = \frac{\pi_i^f}{y_i\mathrm{p}_i+\pi_i^e}\frac{y_i\mathrm{p}_i+\pi_i^f}{x_i} = \frac{\eta_i^{\pi f}}{\eta_i^c}$ and similarly for π_i^c and π_i^r.
- The share of GDP of energy imports and exports, with $v_i = \mathrm{p}_i y_i + q^f(e_i^x - e_i^f)$ and $v^y = \frac{\mathrm{p}_i y_i}{v_i}$, $v^{e^x} = \frac{q^f e_i^x}{v_i}$, $v^{e^f} = \frac{q^f e_i^f}{v_i}$ and $v^{ne} = \frac{q^f(e_i^x - e_i^f)}{v_k}$.

All these variables are measured in the current equilibrium, before implementing climate policies. Moreover, all these variables are observable in international trade and national accounts data.

3.3 Welfare, budget constraint and expenditure

We now compute the welfare of individual country i, $\mathcal{U}_i$, in consumption equivalent terms, accounting for the changes in consumption and climate damages,

$$\frac{d\mathcal{U}_i}{u'(c_i)c_i} = \frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i}\ ,$$

with $x_i = c_i \mathbb{P}_i$ the consumption expenditure c_i. To save on notation, we denote $d\log x_i = \frac{dx_i}{x_i}$ – with a slight abuse of notation.[8] As a result, we expand the budget constraint from equation (2). The carbon tax and subsidies are rebated/taxed lump-sum to the households, and hence, climate policies do not have direct income effects and only act through reallocation and equilibrium effects through price changes.

$$\Delta_t \mathcal{U}_i dt = \frac{\eta_i^y}{\eta_i^c}\Big(\underbrace{d\log \mathcal{D}_i^y}_{\text{climate damage}} + \underbrace{d\log \mathrm{p}_i}_{\text{exports terms of trade}} \Big) - \underbrace{d\log \mathbb{P}_i}_{\text{import terms of trade}} - \frac{\eta_i^y}{\eta_i^c} s_i^e \Big(\underbrace{s_i^f d\log q^f + s_i^c d\log q_i^c + s_i^r d\log q_i^r}_{\text{energy price effects (demand)}} \Big)$$
$$+ \underbrace{\frac{\eta_i^{\pi f}}{\eta_i^c} d\log \pi_i^f + \frac{\eta_i^{\pi c}}{\eta_i^c} d\log \pi_i^c + \frac{\eta_i^{\pi r}}{\eta_i^c} d\log \pi_i^r}_{\text{energy rent effect (supply)}} \quad (17)$$

We see that the welfare gains and losses can be decomposed in five terms: (i) the direct impacts of climate damages production $\mathcal{D}_i^y$, (ii-iii) two terms of trade effects for p_i the sales of good i, and $\mathbb{P}_i$ the price index of the goods purchased by the household from countries j, and (iv-v) the effects of the energy prices changes – for oil-gas, q^f, coal, q_i^c, and renewable q_i^r – through the purchasing cost for production and through the revenue through energy rents. Note that we replace incomes from output y_i at the first-order using the standard Solow decomposition:

$$d\log y_i = d\log \mathcal{D}_i^y + \frac{MPe_i e_i}{y_i} d\log e_i = d\log \mathcal{D}_i^y + \; s_i^e \big[s_i^f d\log e_i^f + s_i^c d\log e_i^c + s_i^r d\log e_i^r \big]$$

where we use the labor supply being inelastic in this Armington trade structure $\ell_i = \bar{\ell}_i$. While each of these terms depends on how equilibrium prices change in a counterfactual, we do not need to simulate the model to recover how log prices change to the first order. Instead, each one of these terms can be computed using observable moments in the data and estimated or calibrated elasticities.

3.4 Climate system and damages

We log-linearize our simple climate system for small changes in emissions $\mathcal{E}$ and energy consumption in oil-gas E^f and in coal E^c. Given the linearity of the climate system $T_i = \Delta_i \mathcal{T} = \Delta_i \chi \mathcal{S}$, we naturally obtain that the percentage increase in temperature reflects the percentage change in carbon concentration,

$$d\log T_i = d\log \mathcal{S} \ .$$

The change in carbon concentration depends on the time horizon $\mathbb{T}$ where $\mathcal{S} = \mathcal{S}_0 + \mathbb{T}\mathcal{E}$, and $s^{\mathcal{E}/\mathcal{S}} = \mathbb{T}\mathcal{E}/\mathcal{S}$ which converges to 1, the longer the horizon we consider,[9]

$$d\log \mathcal{S} = s^{\mathcal{E}/\mathcal{S}} \ d\log \mathcal{E} \xrightarrow[\mathbb{T}\to\infty]{} d\log \mathcal{E} \ .$$

[8] This is not the case, for example, when $x_i < 0$ or changes sign.

[9] If $\mathcal{E}$ represents annual emissions $\sim 35 GTCO_2$/year, given the calibration on 2015 temperature, a horizon $\mathbb{T}$ corresponding to ~ 85 years until 2100 would imply $\mathbb{T}\mathcal{E}/\mathcal{S} \sim 0.7$.

Now, using the damage function with slope γ and curvature δ as in equation (12), we obtain the linearization:

$$
\begin{aligned}
d\log \mathcal{D}_i^y &= -\gamma^y(1+\delta)\big(T_i - T^\star\big)^{\delta} T_i \; d\log T_i \\
d\log \mathcal{D}_i^y &= -\bar{\gamma}_i^y \; d\log \mathcal{E} \; .
\end{aligned}
\tag{18}
$$

with $\bar{\gamma}_i^y = (1+\delta)(T_i - T_i^\star)^\delta T_i$. Therefore, the larger the curvature δ, the more significant the heterogeneity in damages $(T_i - T_i^\star)^\delta$, despite log-linearizing the model around the current equilibrium. Finally, total emissions reacts to changes in aggregate fossil fuels (oil-gas) and coal consumption,

$$
d\log \mathcal{E} = s^{f/E} d\log E^f + s^{c/E} d\log E^c \qquad\qquad s^{f/E} = \frac{\xi^f E^f}{\mathcal{E}} \qquad s^{c/E} = \frac{\xi^c E^c}{\mathcal{E}}
$$

To understand the determination of the equilibrium quantities of oil-gas E^f and E^c we now turn to energy markets.

3.5 Energy markets – Profits, prices, and supply

Using the energy firm problems, we log-linearize the profit change as a function of energy and input prices. For fossil energy (oil and gas), this yields,

$$
d\log \pi_i^f = \left(1 + \frac{1}{\nu_i^f}\right) d\log q^f \; - \; \frac{1}{\nu_i^f} d\log \mathrm{p}_i
\tag{19}
$$

with ν_i^f the fossil energy inverse supply elasticity. We obtain similar formulas for π_i^c and π_i^r as functions of ν_i^c, ν_i^r and q_i^c and q_i^r respectively.

Given that coal and renewables are assumed to be only traded locally (i.e., $\bar{e}_i^c = e_i^c$), we can write the supply curves for coal and renewable energy,

$$
d\log q^c = \nu_i^c \, d\log e_i^c + d\log \mathrm{p}_i
\tag{20}
$$

and similarly for q^r with inverse elasticity ν_i^c. As a result, the price of coal and renewable energy rises both with the input cost p_i and with the quantity demanded e_i^c.

In contrast, oil-gas are traded internationally, where the price q^f clears the world market. Denoting E^f the total oil-gas quantity, changes to the aggregate supply curve are,

$$
d\log E^f = \sum_i \lambda_i^x d\log e_i^x = \sum_i \lambda_i^f d\log e_i^f \; ,
\tag{21}
$$

with $\lambda_i^f = \mathcal{P}_i e_i^f / E^f$ the demand share from country i and $\lambda_i^x = \mathcal{P}_i e_i^x / E^f$ the supply share (or market share) of country i as an exporter,

$$
d\log q^f = \bar{\nu}^f d\log E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\log \mathrm{p}_i \; .
\tag{22}
$$

with the aggregate inverse supply elasticity $\bar{\nu}^f = \left(\sum_i \lambda_i^x \nu_i^{-1}\right)^{-1}$.

3.6 Energy markets – Taxation and demand

The production function equation (7) allows for different, finite elasticities of substitution between both labor and energy (with elasticity σ^y) and between the different energy sources (with elasticity σ^e). In our formulation, we can write the individual country demand curve – for example, for oil and gas – as,

$$d\log e_i^f = \underbrace{-\left(\frac{\sigma^y}{1-s_i^e}s_i^f + (1-s_i^f)\sigma^e\right)\left[d\log q^f + \xi^f dt_i^\varepsilon\right]}_{\text{substitution away from oil-gas}}$$
$$+ \underbrace{\left(\sigma^e - \frac{\sigma^y}{1-s_i^e}\right)\left(s_i^c[d\log q_i^c + \xi^c dt_i^\varepsilon] + s_i^r[d\log q_i^r - ds_i^\varepsilon]\right)}_{\text{substitution away from coal and renewable toward oil-gas}}$$
$$+ \underbrace{\frac{\sigma^y}{1-s^e}(d\log \mathcal{D}_i + d\log \mathrm{p}_i)}_{\substack{\text{energy demand changes due to climate}\\ \text{damage or price changes}}} .$$

We see that a price surge $d\log q^f$ not only decreases fossil demand e^f through substitution across energies σ^e but also away from energy σ^y scaling with the fossil share s_i^f. In comparison, a cost increase for coal also raises the demand for oil-gas e^f through substitution σ^e, but similarly pushes firms away from the energy bundle with elasticity σ^y – the difference of the two elasticities yielding the net effect. Moreover, an increase in productivity $\mathcal{D}_i^y$ or price p_i both increase input demand, as shown in the last two terms.

Finally, replacing input demand, we also summarize the impact on output of changes in prices and policies,

$$d\log y_i = (1+\alpha^y)d\log \mathcal{D}_i^y + \alpha^y d\log \mathrm{p}_i - \alpha^y s_i^f\left[d\log q^f + \xi^f dt_i^\varepsilon\right]$$
$$- \alpha^y s_i^c[d\log q_i^c + \xi^c dt_i^\varepsilon] - \alpha^y s_i^r[d\log q_i^r - ds_i^\varepsilon],$$

with the factor $\alpha^y = \frac{s_i^e\sigma^y}{1-s_i^e}$. We learn two lessons: first, climate damage $d\log \mathcal{D}_i^y$ reduces output more than one for one due to input reallocation away from energy. This multiplier effect $(1+\alpha^y)$ is even stronger when damage reduces when the energy supply curves are flat, i.e. $\nu^c = \nu^r = 0$. However, this effect is dampened when those curves are less elastic $\nu^c \to \infty$: declines in TFP lower the energy price, which facilitates the purchase of additional inputs and improves production, providing some form of adaptation to climate change.

Second, we learn about the sensitivity of output to carbon taxation and other climate policies. The direct exposure of production y_i to the carbon tax t_i^ε is summarized by the factor $\alpha^y\left(\xi^f s_i^f + \xi^c s_i^c\right) = \frac{s_i^e\sigma^y}{1-s^e}(\xi^f s_i^f + \xi^c s_i^c)$, which represent the substitution effect of oil-gas and coal.

To decompose the effect further, we replace the input demand equation (23), with the supply curve equation (20) to find the equilibrium demand for coal and renewables – where the prices q_i^c and q_i^r are solved for. Due to the complex reallocations between the three inputs e_i^f, e_i^c, e_i^r, we keep the general formula for the appendix. The general intuition is that climate policies like carbon taxes $\mathrm{t}_i^{\varepsilon}$ and renewable subsidies reduce demand for oil and coal. However, this effect is dampened by the decline along the supply curve, which reduces the effectiveness of the policy.[10] As a result, the more elastic the supply for coal and renewable, the stronger the quantity – and emissions – response to carbon taxation and climate policies.

Oil-gas energy price

We now combine the individual countries' fossil energy demand, as in equation (23), with the country i share in global demand $\lambda_i^f = \frac{\mathcal{P}_i e_i^f}{E^f}$ as in equation (21), and the supply curve equation (22). We obtain, in general equilibrium, the total demand for oil,

$$\begin{aligned} d\log E^f &= \sum_i \lambda_i^f \, d\log e_i^f \\ &= \frac{1}{1+\bar{\nu}\overline{\lambda}_{\mathbb{I}}^{\sigma,f}} \sum_i \underbrace{\lambda_i^f \left[-\xi^f \Big(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e \Big) + \xi^c s_i^c \Big(\sigma^e - \frac{\sigma^y}{1-s_i^e} \Big) \right]}_{\text{carbon taxation (oil-gas + coal) effect}} \mathrm{J}_i \, d\mathrm{t}^{\varepsilon} \\ &\quad - \frac{1}{1+\bar{\nu}\overline{\lambda}_{\mathbb{I}}^{\sigma,f}} \sum_i \underbrace{\lambda_i^f \, s_i^r \Big(\sigma^e - \frac{\sigma^y}{1-s_i^e} \Big)}_{\text{renewable subsidy effect}} \mathrm{J}_i \, d\mathrm{s}_i^{\varepsilon} + \frac{1}{1+\bar{\nu}\overline{\lambda}_{\mathbb{I}}^{\sigma,f}} \sum_i \underbrace{\lambda_i^f \frac{\sigma^y}{1-s_i^e} \, d\log \mathcal{D}_i^y}_{\text{climate damage effect}} \qquad (23) \\ &\quad + \frac{1}{1+\bar{\nu}\overline{\lambda}_{\mathbb{I}}^{\sigma,f}} \sum_i \lambda_i^f \Bigg(\underbrace{\frac{\sigma^y}{1-s_i^e} d\log \mathrm{p}_i}_{\text{terms-of-trade effect}} + \underbrace{\Big(\sigma^e - \frac{\sigma^y}{1-s_i^e} \Big) (s_i^c \, d\log q_i^c + s_i^r \, d\log q_i^r)}_{\text{substitution from coal and renewable}} \Bigg) \end{aligned}$$

with $\overline{\lambda}_{\mathbb{I}}^{\sigma,f} = \sum_{i\in\mathbb{I}} \lambda_i^f \left(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f) \right) \xi^f$ the aggregate demand elasticity for oil, and the aggregate inverse supply elasticity $\bar{\nu}^f = \left(\sum_i \lambda_i^x \nu_i^{-1} \right)^{-1}$.

This decomposition first reveals that the carbon tax affects oil-gas demand through two channels: the direct substitution away from oil-gas and the indirect substitution away from coal into oil-gas. Comparing the relative strength, the first effect dominates if:

$$\overline{\lambda}_{\mathcal{J}}^{\sigma,f} := \sum_{i\in\mathcal{J}} \lambda_i^f \Big(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e \Big) \, \xi^f > \sum_{i\in\mathcal{J}} \lambda_i^f \Big(\sigma^e - \frac{\sigma^y}{1-s_i^e} \Big) s_i^c \xi^c =: \overline{\lambda}_{\mathcal{J}}^{\sigma,c}$$

where $\mathcal{J}$ is the set of countries imposing the carbon tax.[11] If the largest oil-gas consumers are reducing their demand for oil faster than they substitute away from coal, the net effect on oil-

[10] The formula in the appendix for coal and renewable demand have the following general form. Take an arbitrary energy demand curve $d\log e = -\sigma[d\log q + dt] + \tilde{\sigma} d\log b$, and the supply curve $d\log q = \nu d\log e + \tilde{\sigma} d\log c$, where b and c are demand and supply shifters respectively, we obtain the general demand:

$$d\log e = -\frac{\sigma}{1+\nu\sigma} d\mathrm{t} + \frac{\tilde{\sigma}}{1+\nu\sigma} d\log b - \frac{\sigma}{1+\nu\sigma} d\log b$$

[11] Reorganizing the terms, we can write it as follows – verified in most empirically relevant cases where large oil consumers λ_i^f also have high oil energy share s_i^f:

$$\sum_{i\in\mathcal{J}} \lambda_i^f \left[\frac{\sigma^y}{1-s_i^e} (s_i^f \xi^f + s_i^c \xi^c) + \sigma^e \left((1-s_i^f)\xi^f - s_i^c \xi^c \right) \right] > 0.$$

gas demand declines. Second, we also observe that climate policies, like carbon taxation, t^{ε}, and renewable subsidy, s^{ε}, are stronger when coordinated: oil-gas demand decline more for larger sets of countries implementing the policy J. However, it also generates a *energy price leakage effect*. When country i does not impose carbon policies, the price of oil declines for larger sets of climate agreement participants J – which is beneficial for country i's production and welfare. We quantify this leakage effect below.

To study the direct effect of carbon taxation and the gain for coordination with climate agreement J, we simplify the model further to obtain an analytical formula for the fossil price. In the following, we assume that the energy mix is concentrated on oil and gas $s_i^f = 1, s_i^c = s_i^r = 0$. This implies:

$$d \log E^f = -\frac{1}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i) + \bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \widehat{\lambda}_i^f \mathrm{J}_i d\mathrm{t}_i^{\varepsilon} + \sum_i \widehat{\lambda}_i^f d\log \mathrm{p}_i$$

with energy market shares $\lambda_i^f = \frac{\mathcal{P}_i e_i^f}{E^f}$, and weighted by elasticity $\widetilde{\lambda}_i^f = \lambda_i^f \frac{\sigma^y}{1-s_i^e}$. The average elasticity becomes $\overline{\lambda}^{\sigma,f} = \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}$, the price impact $\widehat{\lambda}_i^f = \widetilde{\lambda}_i^f/(1+\sum_i \widetilde{\lambda}_i^f \gamma_i + \bar{\nu}\overline{\lambda}^{\sigma,f})$, and the covariance is the empirical analog, $\mathbb{C}\mathrm{ov}_i(x_i, y_i) = \sum_i x_i y_i - \sum_i x_i \sum_i y_i$.

The higher the carbon tax $d\mathrm{t}_i^{\varepsilon}$ – at the intensive margin – or the size of a climate policy agreement J – at the extensive margin – the lower the fossil-fuel demand. However, the strength of carbon taxation is muted for three reasons: (i) the more inelastic the supply – with higher curvature ν_i^f and $\bar{\nu}^f$ – the larger the price decline along the supply curve, which then in turn reinforce fossil demand and emissions. This contrasts with the analysis of the oil market in Asker et al. (2024): we claim that the more inelastic the oil supply – due to curvature of costs, concentration, or market power – is detrimental to the effectiveness of coordinated carbon taxation. Moreover, (ii) the energy curve q^f is affected by climate change: higher emissions damage the climate $\bar{\gamma} = \sum_i \bar{\gamma}_i$, which in turn reduces productivity, energy demand and hence emissions. Thus, with damages, the price impact of taxation is lower as it improves both the climate and energy demand in consequence. Additionally, (iii) this vicious effect is strengthened with the distribution of demand $\widetilde{\lambda}_i^f$ and climate damage $\bar{\gamma}_i$. The covariance term $\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i)$ is positive if large energy consumers – with larger market shares λ_i^f and high elasticity σ – are also the most affected by climate change – with higher costs $\bar{\gamma}_i$. The demand effect of taxation is thus muted in those circumstances.

3.7 International trade in goods

Recall the Armington trade block of the model where the good demand is constant elasticity of substitution θ, purchased at a price p_i and subject to tariffs t_{ki}^b for goods from country i. Using the market clearing in equation (13) – reformulated such as countries i sales equal countries k expenditures, coming from incomes in good sales as well as net-exports of fossil energy – the price index, and the trade shares as in equation (3), we obtain the log-linearization of trade patterns:

$$\mathcal{P}_i \mathrm{p}_i y_i = \sum_{k \in \mathbb{I}} \mathcal{P}_k s_{ki} \frac{1}{1+\mathrm{t}_{ki}} [\mathrm{p}_k y_k + q^f (e_k^x - e_k^f)]$$

$$\mathbb{P}_i = \left(\sum_j a_{ij} (\tau_{ij} (1+\mathrm{t}_{ij}^b) \mathrm{p}_j)^{1-\theta} \right)^{\frac{1}{1-\theta}} \qquad s_{ij} = \left(\frac{(1+\mathrm{t}_{ij}^b) \tau_{ij} \mathrm{p}_j}{\mathbb{P}_i} \right)^{1-\theta}$$

$$d \log s_{ij} = (\theta - 1) \left(\sum_k s_{ik} \left(d \log \mathrm{p}_k + d\mathrm{t}_{ik}^b \right) - \left(d \log \mathrm{p}_j + d\mathrm{t}_{ij}^b \right) \right)$$

The linearization of the market clearing is more complex as it also integrates the income effects of the changes in imports and exports of energy $e_i^f - e_i^x$ and energy prices q^f.

$$\underbrace{(d \log \mathrm{p}_i + d \log y_i)}_{\text{sales from } i} = \sum_k \boldsymbol{t}_{ik} \Big[\underbrace{\left(\frac{\mathrm{p}_k y_k}{v_k} \right) (d \log \mathrm{p}_k + d \log y_k)}_{\text{production income of } k} + \underbrace{\frac{q^f e_k^x}{v_k} d \log e_k^x - \frac{q^f e_k^f}{v_k} d \log e_k^f + \frac{q^f (e_k^x - e_k^f)}{v_k} d \log q^f}_{\text{oil-gas income of } k}$$

$$+ \underbrace{\theta \sum_h \left(s_{kh} d\mathrm{t}_{kh}^b - (1+s_{ki}) d\mathrm{t}_{ki}^b \right) + (\theta - 1) \sum_h (s_{kh} d \log \mathrm{p}_h - d \log \mathrm{p}_i)}_{\text{change in trade share in goods purchased by } k \text{ from } h \text{ relative to } i} \Big]$$

with $\mathbf{S} = \{s_{ij}\}_{ij} = \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i}$ the trade share matrix, $\mathbf{T} = \{\boldsymbol{t}_{ij}\}_{ij} = \{\frac{\mathcal{P}_j v_j}{\mathcal{P}_i v_i} s_{ji}\}$ income flow matrix – which represents the share of income v_i from i that is coming from country j – which is identical to the trade/income matrix in Kleinman, Liu and Redding (2024), and the Armington CES θ. This implies that rewritten in matrix notation, we get the price changes as a function of climate damage, carbon tax t^ε and renewable subsidies s^ε, and finally, the trade tariffs policies t^b where the matrix $\mathbf{J}$ represent which countries k impose tariffs on country i.

As for the reaction of oil-gas prices to climate policy, one general lesson from this decomposition relates to the gains from coordination: the larger the coalition of countries implementing carbon taxation and green subsidies, the stronger the reallocation effects. Climate damages lower the productivity $\mathcal{D}_k^y$ and income, which lowers the demand from the most affected countries and redirects the trade patterns. More policy coordination improves global climate, which mitigates those trade disruptions, even for countries that free-ride without implementing carbon policies. However, this free-riding is a double-edged sword: countries i outside climate agreements benefit more from the leakage effect caused by the climate policy in countries k, but when those policies are costly for k's income, that might lower the total demand for country i. All these channels are represented here, and the complete formula can be found in the appendix in Appendix B.

$$d\log \mathrm{p} = \underbrace{\mathbf{A}\Big[(\mathbf{T}v^{y}-\mathbf{I})\delta^{y,z} - \mathbf{T}v^{e^f}\beta^{e,d,f}\Big]}_{\substack{\text{climate impact on output } \delta^{y,z} \\ \text{and energy demand } \beta^{e,d,f}}} d\log \mathcal{D}^y \; + \; \underbrace{\mathbf{A}\Big[-(\mathbf{T}v^{y}-\mathbf{I})\delta^{y,qf} + \mathbf{T}\left(v^{e^x}\tfrac{1}{\nu^f} - v^{e^f}\beta^{e,q}_{f,f} + v^{ne}\right)\Big]}_{\substack{\text{oil-gas price impact on energy output } \delta^{y,qf} \\ \text{and net energy bill } e^x - e^f}} d\log q^f$$
$$+ \underbrace{\mathbf{A}\Big[-\mathbf{T}v^{e^f}\left(-\beta^{e,q}_{f,f}\xi^f + \beta^{e,t}_{f,c}\,\xi^c\right) - (\mathbf{T}v^{y}-\mathbf{I})\delta^{y,t\varepsilon}\Big]}_{\substack{\text{carbon tax impact on output } \delta^{y,t} \text{ and energy} \\ \text{demand away from oil-gas } \beta^{e,q}_{f,f} \text{ and from coal } \beta^{e,q}_{f,c}}} \mathrm{J}\, d\mathrm{t}^{\varepsilon} + \underbrace{\mathbf{A}\Big[\mathbf{T}v^{e^f}\beta^{e,t}_{f,r} + (\mathbf{T}v^{y}-\mathbf{I})\delta^{y,s\varepsilon}\Big]}_{\substack{\text{renewable subsidy impact on oil-gas} \\ \text{demand } \beta^{e,t}_{f,r} \text{ and output } \delta^{y,s\varepsilon}}} \mathrm{J}\, ds^{\varepsilon}$$
$$+ \underbrace{\theta\mathbf{A}\left(\mathbf{TSJ}dt^b - \mathbf{T}(\mathbb{1}+\mathbf{S}')(\mathbf{J}dt^b)'\right)}_{\text{trade reallocation due to tariffs}} \tag{24}$$

with $\mathbf{J}$ a tariff direction matrix – whether country i imposes tariffs on country j, with tariff increasing in the carbon intensity of j in the case of carbon tariffs. Moreover, the general equilibrium (and leakage) effects are summarized in a complicated matrix $\mathbf{A}$ that summarizes the fact that the price p_i also affects energy demand, oil-gas extraction, energy trade balance, and output. Further description can be found in the Appendix B.

3.8 Back to welfare and decomposition

In summary, we use the following welfare decomposition – following equation (17). We decompose the effects of the climate policies into the following four channels: (i) the climate damages or *direct productivity* impacts, (ii) the terms-of-trade effects, or *trade* effects, (iii) the effect on profit from the energy sector or *energy rents* and (iv) the impact on energy prices or *energy cost*.

$$\Delta_{\mathrm{t}}\mathcal{U}_i d\mathrm{t} = \underbrace{\frac{\eta_i^y}{\eta_i^c}\, d\log \mathcal{D}_i^y}_{\substack{\text{direct} \\ \text{productivity}}} + \underbrace{\frac{\eta_i^y}{\eta_i^c} d\log \mathrm{p}_i - d\log \mathbb{P}_i}_{\text{trade effects}} - \underbrace{\frac{\eta_i^y}{\eta_i^c} s_i^e\left(s_i^f\, d\log q^f + s_i^c d\log q_i^c + s_i^r d\log q_i^r\right)}_{\text{energy cost effects}}$$
$$+ \underbrace{\frac{\eta_i^{\pi f}}{\eta_i^c} d\log \pi_i^f + \frac{\eta_i^{\pi c}}{\eta_i^c} d\log \pi_i^c + \frac{\eta_i^{\pi r}}{\eta_i^c} d\log \pi_i^r}_{\text{energy rents effects}} \tag{25}$$

We decompose the welfare impacts through these four different channels, given the other general equilibrium effects of the model: the climate damage as in equation (18), the individual countries' demand for energy as in equation (23), the global price for oil-gas in equation (22) which depends on aggregate quantity of energy consumed in equation (23), the profit of energy firms as in equation (19), and finally the general equilibrium of the good markets driving the terms of trade effects and prices as in equation (24). Before turning to the result of our model, we explain how we estimate the key parameters, such as climate damages and energy supply elasticities.

4 Estimation and quantification

Our main data sources are the World Development Indicators (WDIs), energy data compiled by Our World in Data from the Energy Institute (OWID, Energy Institute, 2024), and the International Trade and Production Database estimation sample (ITPD-E, Borchert et al., 2021) for international trade flows at the sector level, which allows us to remove energy trade from the flow of all other goods internationally. Additional data sources are described in the text. Our estimation covers years 2000-2016 for the trade data, and 1985-2019 for the energy data.

4.1 Estimating the structural damage function

We estimate the structural damage function $\mathcal{D}_i^y$ using temperature shocks and data on trade flows. Typical damage function estimation recovers damage parameters from regressions of GDP/capita on temperature shocks (see, e.g., Burke et al., 2015). In this context, a regression of log GDP per capita on temperature would fail to identify the parameters of the damage function, as GDP contains effects of temperature elsewhere (via their propagation through trade network), as well as general equilibrium effects on energy prices, rents and wages. Subsequently, in our context, an off-the-shelf damage function estimated on GDP recovers parameters that are subject to the Lucas critique. Our estimates of the structural adaptation function are robust to this critique because they net out endogenous adaptation and climate change effects that operate through energy markets.

To estimate T^* and γ, we use the model's gravity regression to estimate the parameters of the damage function, similar to Rudik et al. (2022). To do so, we leverage trade flow data from ITPD-E sample combined with local temperature data. For temperature data, we use Berkeley Earth near-surface temperature data (Rohde et al. (2013), available in $1^\circ \times 1^\circ$ cells), which we aggregate to the country level, population-weighing using population from the Global Human Settlement Layer in 2015 (Pesaresi et al., 2024).

The ITPD-E trade data measures trade flows X_{ij} at the industry level between most country pairs in 2000 through 2016. We use the ITPD-E data as it allows us to drop bilateral trade flows in the energy sector. Through the lens of the model, trade flows from exporter j to importer i are,

$$X_{ij} = p_{ij}c_{ij} = \left(\frac{(1+\mathrm{t}_{ij}^b)\tau_{ij}\mathrm{p}_j}{\mathbb{P}_i}\right)^{1-\theta} \frac{c_i}{\mathbb{P}_i} a_{ij}, \tag{26}$$

where,

$$\mathrm{p}_i = (\mathcal{D}_i^y(T_i)z_i)^{-1}\left(\epsilon(q_i^e)^{1-\sigma^y} + (1-\epsilon)(w_i)^{1-\sigma^y}\right)^{\frac{1}{1-\sigma^y}}$$

where q_i^e is a CES price index for energy prices. We use equation 26 as a basis for estimating $\mathcal{D}_i^y$. Dividing domestic trade, and treating each year t in the data as an equilibrium of the model, equation 26 becomes,

$$\frac{X_{ijt}}{X_{iit}} = (\mathrm{p}_{jt} \,/\, \mathrm{p}_{it})^{1-\theta} \varsigma_{ij}\varsigma_t \tilde{a}_{ijt}\tilde{\tau}_{ijt}$$

where ς_{ij} are all time-invariant bilateral preference or cost shifters, and $\tilde{a}_{ijt}\tilde{\tau}_{ijt}$ represents any time-varying components of bilateral preferences or trade costs. Taking logs and setting $\delta = 1$ generates the estimating equation,

$$\log \mathbb{E}[X_{ij}/X_{ii}] = \beta_0(T_{jt} - T_{it}) + \beta_1(T_{jt}^2 - T_{it}^2) + \varsigma_{ij} + \varsigma_t + \Gamma' W_{it} + \Omega' W_{jt}. \tag{27}$$

In this specification, ς_{ij} is a country-pair fixed effect, while ς_t is a time fixed effect. W_{it} and W_{jt} are importer- and exporter-year controls. These controls are a second order polynomial in log GDP/capita, the share of GDP that is attributed from oil rents, and renewable energy consumption as a percent of total final energy consumption, and are all taken from the WDIs. These controls proxy for the component of factory gate prices p_j driven by energy prices q_i^e or wages w_i. With these controls, the coefficients on temperature β_0 and β_1 identify parameters of the damage function (when $\delta = 1$), namely,

$$-\frac{1}{2}\frac{\beta_0}{\beta_1} = T^*, \quad \frac{-2\beta_1}{\theta - 1} = \gamma.$$

We estimate equation 27 with a Poisson pseudo-maximum likelihood estimator with high dimensional fixed effects to maintain zeros in the trade matrix (Silva and Tenreyro, 2006; Correia et al., 2020). We limit the estimation sample to countries for which domestic trade X_{ii} is not missing in the ITPD-E data and for which W_{it} and W_{jt} is not missing in the WDIs, as well as entities that are present in the ITPD but not other common trade datasets, like CEPII (Conte et al., 2022). This retains 169 countries, primarily dropping territories, dependencies, small island states, historical entities, or special jurisdictions, as well as North Korea.

Identification of β_0 and β_1 comes from within-trading-partner pair variation, tracing out how temperature shocks to bilateral terms-of-trade affect the import penetration ratio, net of year effects common to all pairs and importer- and exporter-year controls (which control for the time-varying component of factor prices). In short, the identification assumption is that time-varying shocks to preferences $\tilde{a}_{ijt}$ and shocks to bilateral cost shifters $\tilde{\tau}_{ijt}$ are such that $\mathbb{E}[\tilde{a}_{ijt}\tilde{\tau}_{ijt} \mid T_{it}, T_{it}^2, T_{jt}, T_{jt}^2, \varsigma_{ij}, \varsigma_t, W_{it}, W_{jt}] = 1$; i.e., conditional on the fixed effects, temperature (and its square), and controls, temperature shocks are uncorrelated with the error term.

Appendix Table 1 reports the results of our estimation. All specifications two-way cluster standard errors at the importer and exporter year to account for serial correlation of temperature within countries. Column (4) reports the results of our preferred specification, which uses differences in temperature $(T_{jt} - T_{it})$ and its square as a regressor, providing more efficient estimators of β_0 and β_1. We estimate $\hat{T}^* = 14.02$ and $\hat{\gamma} = 0.012$. Column (3) reports the results by separately estimating coefficients on T_{it}, T_{jt}, and their squares. Reassuringly, separate estimates have the correct sign and magnitude and are statistically indistinguishable from each other, though estimates on importer temperature are noisier. To estimate T^* and γ with separate coefficients on importer and exporter temperatures, we form precision-weighted averages of the estimands, and recover estimates of T^* and γ that are statistically indistinguishable to those estimated in Column (4).[12]

[12] Columns (1) and (2) use an OLS estimator. With the OLS estimator, the effects of importer temperature on

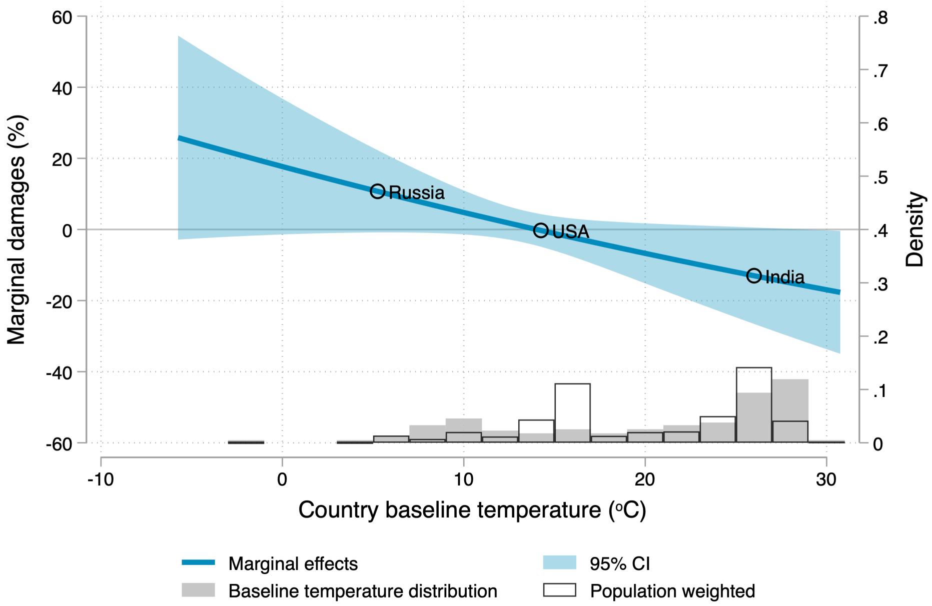

Figure 2: Marginal damages on productivity from a 1 degree change in local temperature versus country baseline temperature, using estimates of equation 27 available in Appendix Table 1, Column (4). 95% CIs computed using standard errors two-way clustered by importer and exporter. Also pictured: the histogram of countries across the baseline temperature distribution, weighted and unweighted by population. The coldest countries in the data are Greenland and Mongolia.

Figure 2 represents the damage function by using the estimates of β_0 and β_1 from Appendix Table 1, Column (4) to compute the marginal damages at each point along the baseline temperature distribution, i.e., the derivative of $\mathcal{D}_i^y$. Russia, which starts from a baseline cold temperature, experiences gains from local warming, while India, a baseline hot country, experiences large losses. This representation of the damage function is common in the literature, and our damage function resembles those estimated using GDP (Burke et al., 2015; Cruz and Rossi-Hansberg, 2024), despite our identification strategy that leverages panel variation in non-energy import penetration and temperature differences.

Finally, to calibrate $T_i^\star$, we also use an intermediary assumption that nests us between the following cases: (i) that T^* represents a global peak temperature for goods production TFP (as in Burke et al., 2015; Kotlikoff et al., 2021a; Krusell and Smith, 2022), (ii) or that global warming affects all the locations symmetrically, where deviation from the local baseline temperature $T_i^\star = T_{it_0}$ damage productivity, as in the representative agent economy of Barrage and Nordhaus (2024). A variant of (ii) is the assumption of full local adaptation to a changing climate, in which local weather shocks (i.e., temperature deviations from a moving average of local temperature) are the only source of damages, an assumption maintained in Kahn et al. (2019). Bilal and Känzig (2024) suggest, in contrast, that climate damages on GDP come in large part from shocks to global

import penetration are substantially noisier. With separate coefficients, we find a peak of around 16 and a flatter damage function $\gamma \approx 0.001$. The effects of temperature differences are indistinguishable from zero in, Column (2).

(rather than local) temperature, as they are associated with extreme events. Our intermediate step and assumes partial local adaptation by assuming,

$$T_i^\star = \alpha^T T^\star + (1-\alpha^T) T_{it_0}$$

where $\alpha^T = 0.5$ and $T^\star = 14.02$, as estimated. We hold γ fixed across countries, implying that all local adaptation affects the peak but not the shape of the damage function.

4.2 Estimating energy supply elasticities

Our goal is to recover the supply elasticities of fossil (oil and gas) and coal production for each country. Our estimation strategy begins with the model-implied relationship that,

$$d\log \pi_i^f = \left(1 + 1/\nu_i^f\right) d\log q^f - \left(1/\nu_i^f\right) d\log \mathrm{p}_i$$

As the world price of fossil is taken as exogenous to producers, a regression of oil rents on international oil-gas prices would recover the oil-gas supply elasticity, provided changes to international oil-gas prices are uncorrelated with changes in traded goods prices. For oil rents, we use data on the GDP share of oil and gas rents from the WDIs, η_i^f. For each country, we construct the effective price of fossil by taking an average of international oil and natural gas prices (from OWID), weighted by the share of oil and gas in the total fossil rents share of GDP. Treating each year as an equilibrium of the model, we leverage the time series to estimate the fossil supply elasticity by estimating,

$$\Delta \log \eta_{it}^f = \rho_i \Delta \log q_{it}^f + \Omega \Delta \log GDP_{it} + \varsigma_i + u_{it}$$

by OLS country-by-county, where here Δ indicates first-differencing and ς_i indicates a country fixed effect, which controls for country-specific time trends intended to capture potentially confounding secular trends in oil rents and international prices within each country. First-differencing implicitly nets out country fixed effects, which absorb all time-invariant shifters of p_i, while controlling for changes to GDP controls for year-over-year changes to p_i, as well as controlling for the denominator of η_{it}^f, so variation in the lefthandside reflects variation in π_i^f.

Estimating equation 4.2 results in very noisy estimates of ρ_i for some countries. Some estimates of ρ_i, while positive, fall below 1 (implying a negative oil-gas supply elasticity), while other estimates even negative, inconsistent with the model and incompatible with the quantification. To ameliorate this, we estimate a pooled estimate of ρ across countries and use an empirical Bayes shrinkage estimator where we impose a truncated normal prior on ρ_i with the truncation beginning at 1. This is tractable, as a normal likelihood (for the coefficient estimates, ρ_i) is conjugate with a truncated normal prior, which allows for easy recovery of the posterior mean. Imposing coefficient sign restrictions in estimation while jointly shrinking away noise is similar to the approach in estimating millions of retail product demand elasticities in Rosenthal-Kay et al. (2024). Histograms of the OLS and empirical Bayes estimates are available in Appendix Figure 11.

To recover coal supply elasticities, we repeat the analysis using the coal rent share of GDP

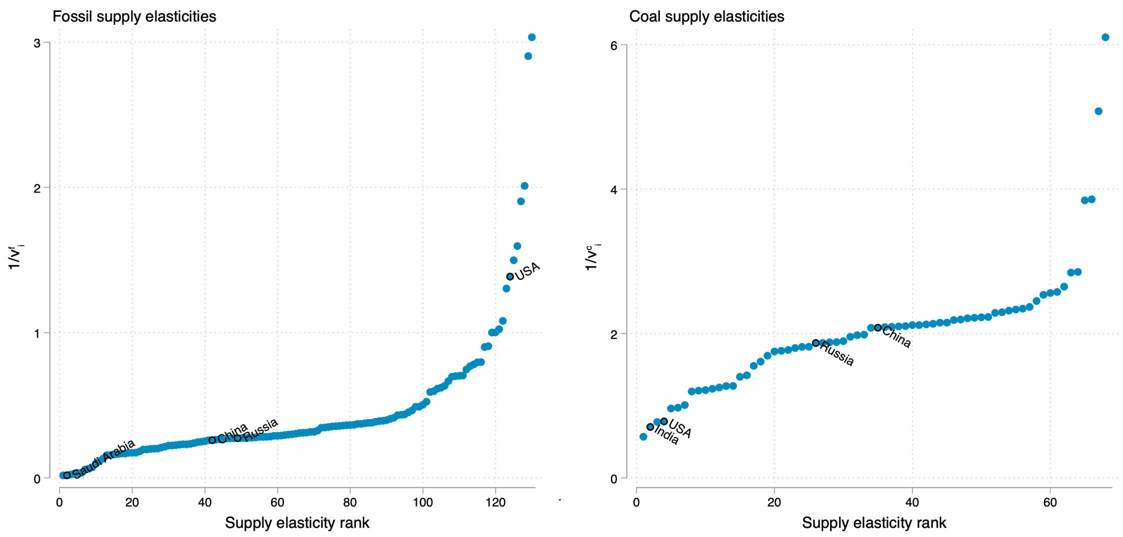

Figure 3: Empircal Bayes estimates of oil-gas and coal energy supply elasticities. Left: Hydrocarbon fossil (oil-gas) elasticities. Saudi Arabia, China, Russia, and the United States are labeled on the plot. Oil-gas energy is inelastically supplied in Saudi Arabia, potentially due to their market power through OPEC. Right: Coal energy supply elasticities.

as a regressand and the international price of coal (from OWID) as a regressor and apply the same empirical Bayes shrinkage routine. While in our model, coal is traded only locally, in reality, there does appear to be an international price of coal: differences in coal prices across countries are small, and movements in coal prices across countries are strongly correlated (Appendix Figure 10), as coal is traded to some extent, and as a commodity, there is significant and sophisticated arbitrage in global markets. With this in mind, we treat the global price of coal as exogenous for estimation.

Figure 3 plots the results of our estimation. Coal supply is substantially more elastic than fossil supply. There is large spatial heterogeneity in supply elasticity estimates: for example, fossil supply is nearly inelastic in OPEC nations like Saudi Arabia as well as Russia and China. Our estimates of energy supply elasticities do not uncover the true resource intensity of extraction technology that, in theory, could be uncovered by a production function estimation routine that accounts for market power. Instead, we view our estimates as 'reduced form' supply elasticities that combine both technology and market power. Market power can attenuate the effective supply elasticity as producers endogenously adjust quantities to move up along their perceived energy demand curve, consistent with low supply elasticities for OPEC nations. In contrast, we find fairly elastically supplied oil-gas in the U.S.

However, coal supply elasticities follow an opposite pattern, in which coal is fairly inelastically supplied in the U.S. and India and more elastically supplied in Russia and China. On average, coal is far more elastically supplied than fossil, generating a flatter supply curve and low coal rents in equilibrium. This is consistent with EPA estimates of the shape of the U.S. coal supply curve (U.S. Environmental Protection Agency, 2023), and the fact that even the largest coal producers do not have coal rents above 1% of GDP.

For countries with no fossil or coal rents in the data, we assign supply elasticities equal to the global pooled estimate (such countries are absent in the figure).

4.3 Externally calibrated parameters

For shares of energy rents in GDP and the other observable shares – as listed in Section 3.2 – and required by our sufficient statistics formula, we use 2000-2016 averages as the baseline, relying on data from the WDIs, OWID, and ITPD. For trade shares, we use data from the ITPD-S dataset for 2019, which fills in missing entries in the trade matrix with temporal smoothing and other extrapolation techniques to compute trade shares. For domestic trade, some entries in small nations with poor data, like Cuba, have implausibly low domestic trade shares. We replace reported domestic trade shares of 5% or less with these predicted domestic trade shares from a logistic regression of trade shares on log GDP/capita, population size, absolute latitude, temperature, and log bilateral flows with the United States (which has good reporting quality). We then renormalize the data so that trade shares sum to one, allowing us to construct $\mathbf{S}$. In practice, the trade data is not balanced, so we renormalize the columns of the $\mathbf{T}$ to mechanically enforce balanced trade in the data.[13]

For the household, we calibrate the CRRA/IES parameter to be $\eta = 1.5$, taken from Barrage and Nordhaus (2024).[14] and set the elasticity of substitution $\theta = 5$, consistent with a trade elasticity of 4, which accords with the estimates of Simonovska and Waugh (2014).

For the production function for goods, we use the average energy cost share $\frac{q_i^e e_i}{\mathrm{p}_i y_i}$ to 10%, as documented by OECD reports and used in Kotlikoff et al. (2021b) and Krusell and Smith (2022). For the elasticity between energy and other inputs, we set $\sigma^y = 0.6$ for all countries, which is in the range of estimates in Papageorgiou et al. (2017), among others.[15] This implies that labor and energy are complementary in production: an increase in the price of energy has a strong impact on output as it is less productive to substitute away toward other inputs – here, labor. This aligns with other empirical and structural evidence on the impact of energy shocks, e.g., Hassler et al. (2019). For energy sources, we calibrate the individual countries' energy mix for oil-gas, coal, and non-carbon (nuclear, hydroelectricity, solar, wind, etc.) to match the energy mix documented in Energy Institute (2024). This allows us to successfully identify countries that are more reliant on specific energy sources: for example, China and India are highly coal-dependent, while Russia, the Middle East, and the United States/Canada are the biggest consumers of oil and gas. Finally, for the elasticity between energy inputs, we use the value $\sigma_e = 2$, following the rest of the literature, i.e. Papageorgiou et al. (2017), Kotlikoff et al. (2021b), and Cruz and Rossi-Hansberg (2024), among others. As we are unable to estimate renewable energy supply elasticities at the country level,

[13]This ensures that the sum of the shares of income spent on traded goods sums to income. Visualizations of these trade matrices for 25 large economies are available in Appendix Figure 12.

[14]This is slightly lower than the standard value $\eta = 2$, for the reason that higher curvature would imply more unequal weights, ω_i, across different countries.

[15]It also aligns with the estimation in Bourany (2022).

we take $1/\nu^r = 2.7$, based on the estimate in Johnson (2014). A summary of these parameters is available in Appendix Table 2.

In addition, we calibrate the climate model described in Section 2.4 to match important features of the relationship between carbon emissions, temperatures, and climate damages. We consider linear models for the relationships between carbon emissions $\mathcal{E}$, carbon concentration $\mathcal{S}$, and global and local temperature $\mathcal{T}$ and T_i, and this implies that we do not require to parametrize the climate sensitivity χ, or the pattern scaling Δ_i.[16] However, the economic model being static, we consider the horizon $\mathbb{T}$ to be 2100 for performing policy experiments. In this context, we analyze the long-term policy impact of climate and trade policies through different channels.

Finally, when we consider welfare, as in equation (14), we consider the weighted sum of individual utilities, with Utilitarian weights $\omega_i = 1$. This implies that the aggregation of the consumption-equivalent welfare change in equation (15) can be aggregated with weights $\mathcal{P}_i\widehat{\omega}_i \propto \mathcal{P}_i\omega_i u'(c_i)c_i = \mathcal{P}_i c_i^{1-\eta}$ for η the inverse of the IES, which control "inequality aversion" in this type of models, as discussed in Anthoff et al. (2009). This, therefore, puts additional weight on the welfare costs of emerging and low-income countries with low consumption and we report his welfare measure in our policy experiments. Alternatively, we also consider Negishi weights $\omega_i = 1/u'(c_i)$. These would undermine these redistributive considerations by putting more weight on advanced economies, yielding $\widehat{\omega}_i = c_i \propto y_i$.

5 Results

In this section, we report the results of our main experiments, using our sufficient statistics formulas, data moments, and estimated damage functions and energy supply elasticities.

5.1 The welfare effects of global warming

We use our sufficient statistics formula to compute the welfare effects of global warming from an emissions impulse that generates 3°C of warming by 2100. The results of this exercise are displayed in Figure 4. In the left panel, we display the spatial distribution of welfare changes around the world. Our results accord with the vast majority of the literature. For example, just as in Cruz and Rossi-Hansberg (2024), the losers of climate change are predominately concentrated in the global south: Africa, Latin America, and South East Asia – all hot countries at baseline – lose, while cooler countries like Canada and Russia win, and the effects of global warming are small in the United States and China.

The right panel of Figure 4 decomposes the welfare changes into those driven by the direct effects of climate change, those driven by change terms-of-trade, changes in energy rents, and exposure to changes in the price of energy. Global warming, by making the world poorer, reduces global demand for energy, which lowers the equilibrium price q^f and provides relief for oil and

[16] Indeed, the log-linearization of the linear climate system yield $d\log T_i = d\log\mathcal{S} \propto d\log\mathcal{E}$ where there is no requirement for climate sensitivity or pattern scaling.

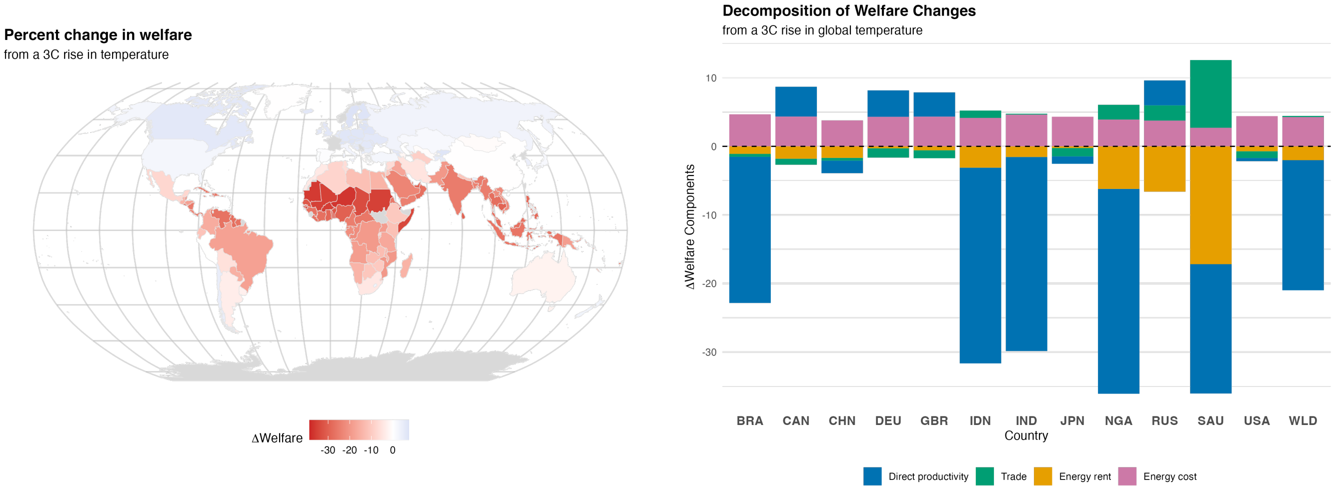

Figure 4: The welfare effects of a 3°C rise in global temperature. Left: map of global welfare changes, in % terms. Red countries lose, while blue countries win. Right: decomposition of welfare changes for several major economies.

gas importing countries. However, it also deteriorates the energy rents of fossil exporters such as Saudi Arabia. However, this baseline hot country loses due to the direct effect on productivity, this loss is partially offset by improving terms-of-trade, as Saudi Arabia is well-connected in the trade matrix to countries that gain, like Japan and Russia.

5.2 Unilateral carbon taxation

We now consider the welfare effects of unilateral carbon taxation. As a case study, we begin with a unilateral carbon tax imposed by China of $50 USD/ton. While this is potentially an unrealistic scenario, this illustrates model mechanisms and the value of including a rich energy sector in a macroeconomic model of climate change. Despite being a large polluter, this moderate carbon tax has virtually no effect on global emissions, which fall by less than 0.1%. Yet, the policy nonetheless creates winners and losers, visible in the left panel of Figure 5. The winners of the policy include Gulf and North African nations, as well as Russia, and interestingly, China.

The reason for the gains in these nations is visible in the right panel. At baseline, China is heavily reliant on coal, reflected in their large share of coal in their energy mix (one of the data moments key to our sufficient statistics exercise). By imposing a carbon tax, China substitutes away from coal toward oil-gas fossil sources, as coal is dirtier than oil-gas ($\xi_c/\xi_f \approx 1.44$), putting upwards pressure on the price of these energy commodities. As a result, the global price of oil-gas rises by approximately 5%, and energy rents rise for fossil exporters as a result of carbon taxation in China. If oil-gas emissions rise by around 5% as well, emissions from coal fall by -7.6%. To sum up, utilitarian welfare falls by 0.14% (0.23% Negishi-weighted).

While the rise in the global price of oil-gas improve the rents of fossil exporters, energy costs inflate in energy importers. For example, European nations 'lose' as their energy costs rise. In China, the overall welfare effect of changing energy prices is positive: the direct effect of taxation drops out in the first-order decomposition as tax revenues are rebated to the household; only the

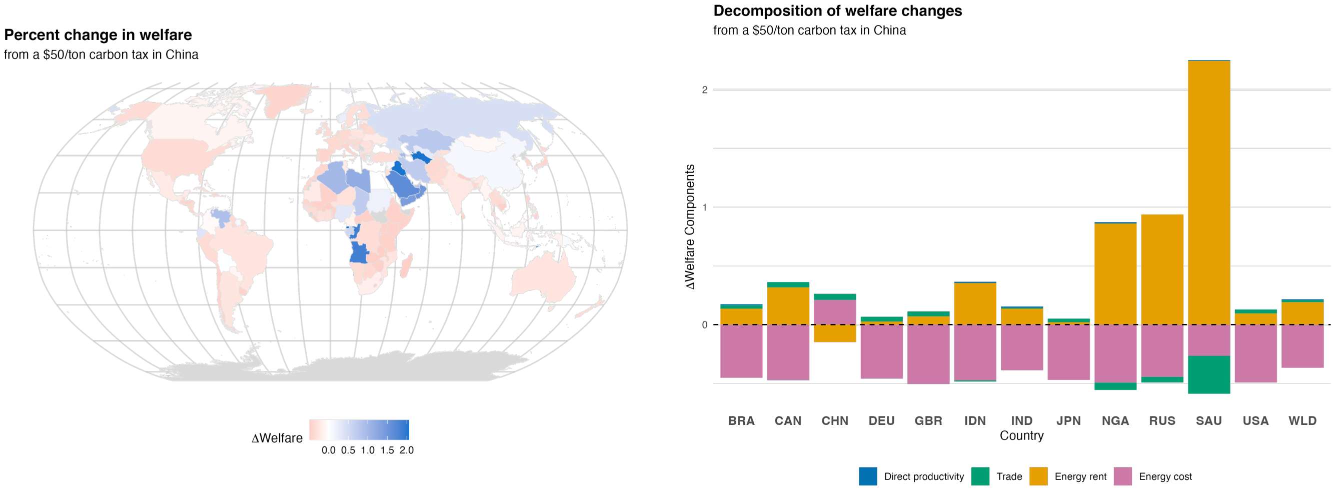

Figure 5: The welfare effects of a \$50/ton carbon tax imposed unilaterally by China. Left: map of global welfare changes, in % terms. Red countries lose, while blue countries win. Right: decomposition of welfare changes for several major economies.

general equilibrium of prices net of carbon taxes affects welfare. Coal prices fall as demand shifts to fossil, and as China has a large share of coal in their energy mix, they benefit.

In Appendix Figure 13, we display the results of a \$50 unilateral carbon tax imposed in the United States. This policy fosters a 0.8% decline in global emissions. This reduction in atmospheric CO_2 has positive welfare effects on nations in the global south and damages the 'winners' of climate change. Interestingly, though Saudi Arabia's energy rents appreciate, as was the case with China, their terms of trade deteriorate as their Middle-Eastern and South Asian trading partners suffer productivity losses. Welfare effects in the U.S. are small but positive: energy prices rise, more than totally offsetting the gains associated with reducing climate damages, but are balanced by improved terms of trade with Canada and European nations.

5.3 Unilateral renewable energy subsidies

In contrast to carbon taxation, we consider renewable energy subsidies. We compare the effect of subsidizing renewables with carbon taxation by plotting each country's welfare change from unilateral policy for a \$50 carbon tax and a 42.6% renewable energy subsidy. We choose this renewable energy subsidy so that the relative price change from policy between the oil-gas and coal bundle and renewables stays the same on average. The main reason why subsidizing renewable energy differs from taxing carbon is that it does not directly cause a reallocation from carbon-intensive coal to oil-gas within the dirty energy bundle, as the tax does not directly alter the oil-gas and coal price ratio. This affects both the aggregate change in emissions, as well as dirty energy exporters' energy rents, depending on their relative dirty energy supply elasticities.

In Figure 6, we plot both each country's consumption equivalent welfare change from pursuing a \$50 dollar carbon tax (left panel) and a 42.6% renewable energy subsidy (right).[17] There are

[17]In Appendix Figure 14, left panel, we plot for each country the global welfare gain – of a Utilitarian planner –

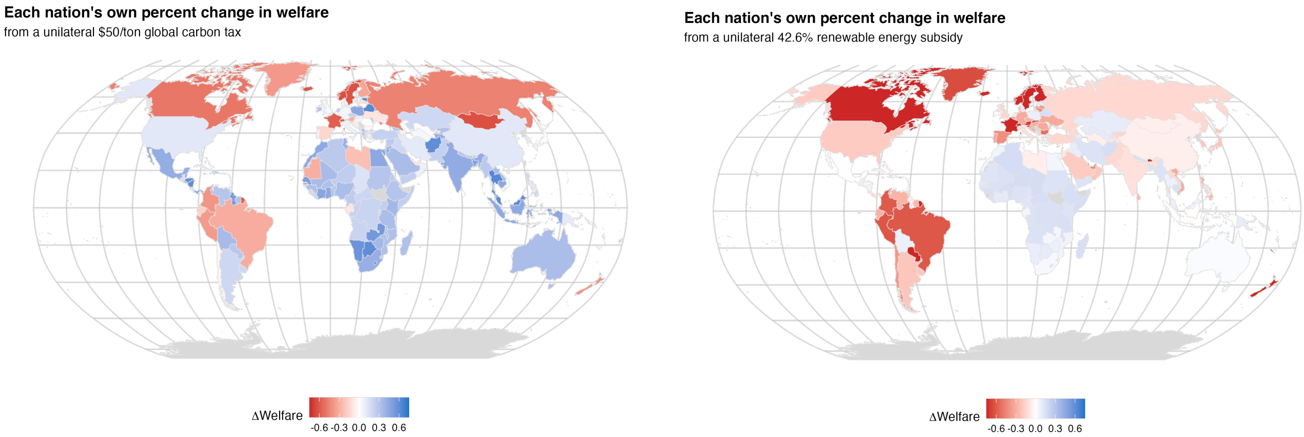

Figure 6: Left: each country's individual welfare change from pursuing unilateral carbon taxation. Right: each country's individual welfare change from pursuing a unilateral renewable subsidy.

large differences across the world in the effects of these two policies. On average, renewable energy subsidies are considerably less effective at raising welfare and cause more harm. For example, a renewable subsidy in France is much worse than a carbon tax, as it disproportionally induces France to move up its renewable energy supply curve. As France has a high share of renewable energy at baseline (over 40%), this effect on energy costs is considerable, as more resources are wasted on renewable 'extraction.' Simply put, the marginal nuclear power plant or solar farm has a high price in France. Likewise, in China, a renewable subsidy generates a large movement up its domestic renewable supply curve, rather than the small movement up the global oil-gas supply curve induced by carbon taxation.

5.4 Coordinated carbon policy

We now consider coordinated climate policy, in which blocs of nations jointly implement carbon taxes and tariffs that take the form of a carbon border adjustment mechanism (CBAMs). CBAMs levy a tariff on the carbon content of imports to each nation in the bloc, but do not place tariffs on bloc members – a 'climate club' as suggested by Nordhaus (2015) and studied in other work (Clausing and Wolfram, 2023; Ernst et al., 2023; Bourany, 2025). The carbon intensity of any nation's exports is observable by knowing the energy mix in production and the carbon intensity of those energy sources, and is readily observable simply by knowing the carbon emissions of any country. Exporting countries only respond to CBAM through the general equilibrium impact on energy prices as well as through terms-of-trade adjustments in the international goods market.

First, we examine the effects of a European Union-wide climate club with a carbon tax and tariff of $50. Figure 7 displays the results of this exercise. The EU is a loser from its own climate club, with only Spain and Portugal benefitting. Aggregate emissions decline by 2.4%, cooling

associated with unilateral carbon policy. While Russia loses from imposing a carbon tax on themselves, the world still benefits, suggesting a considerable gap between governmental and global incentives to internalize the carbon externality. Likewise, in the right panel, we can see similar effects for the renewable energy subsidy.

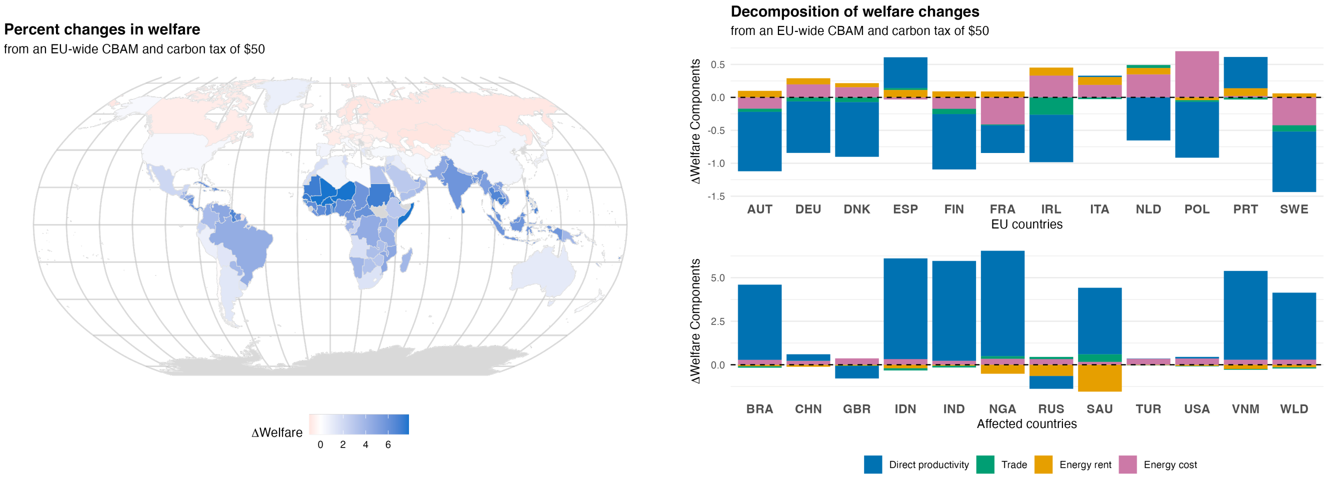

Figure 7: Left: changes in welfare from a EU climate club. Right: Decomposition of welfare changes within the EU (top) and for major trade partners and losers of climate change (bottom).

global temperatures and harming EU nations that benefit from climate change, alongside other cool nations like Canada and Russia. However, the global welfare effect of such a policy is positive, with global utilitarian-weighted welfare rising by 3.9% (a 0.8% with Negishi weights).

In the top right panel, we plot the welfare decomposition for EU member states. Energy cost effects are heterogeneous across the EU, rising in France and falling in Poland. Ireland is particular hurt through international trade, as their major trading partner, Great Britain, faces productivity declines. Countries in the global south benefit from the reduction in world temperature (bottom panel), and most economies benefit from a reduction in energy costs, as demand pressures from Europe in the international fossil hydrocarbon market lessen. This generates positive welfare effects in countries that are mostly unaffected by climate change directly, like the United States and China. Major oil-gas exporters like Saudi Arabia, Nigeria, and Russia lose energy rents as a result, though only Russia is a net loser due to the direct, negative productivity effect brought about by fewer carbon emissions.

In contrast, we consider a climate club composed of ASEAN members with the same policies as the EU climate club. These southeastern Asian countries are losers from climate change at baseline, so internalizing the carbon externality benefits them from the reduction in carbon emissions alone. Figure 8 plots the results of this exercise. The ASEAN climate club reduces global emissions by 0.6% and raises global welfare by 1% (0.2%, Negishi-weighted). By reducing global emissions, the climate club benefits the losers of climate change and harms the winners.

ASEAN members broadly benefit from the policy, owing to the reduction in world temperature. However, gains are heterogeneous not only because of exposure to climate change, but also because of trade in goods and energy, as seen in the top left panel. Energy exports like Brunei have smaller gains as they lose energy rents. Goods trade within the club reallocates, with Indonesia and Myanmar enjoying improved terms of trade at the expense of Malaysia, Singapore, Thailand, and Vietnam. Elsewhere, oil-gas exporters lose energy rents as the fossil fuels price falls by 0.8%.

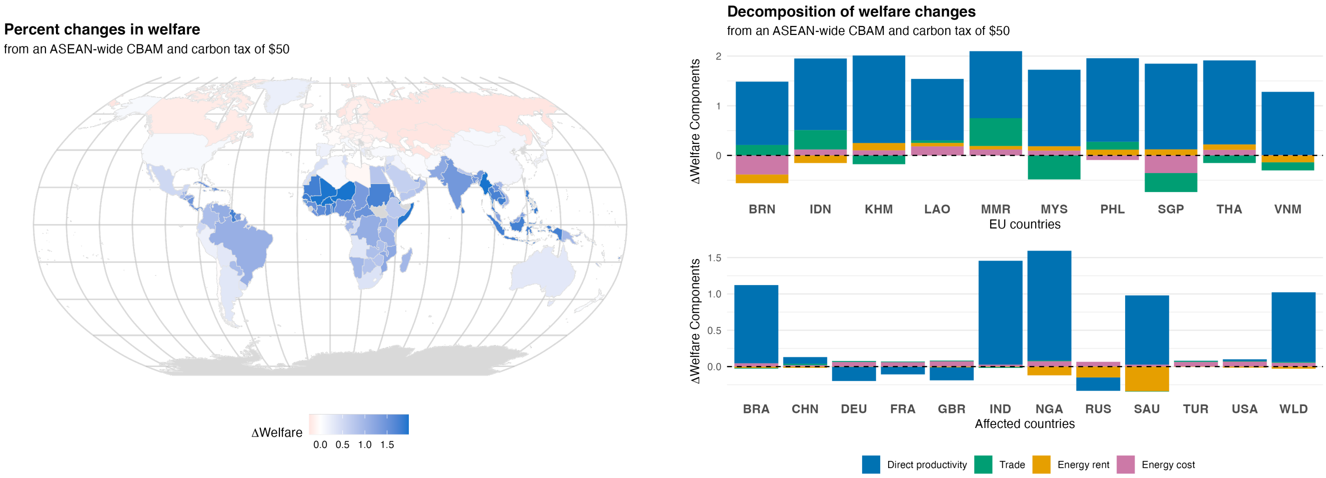

Figure 8: Left: changes in welfare from a ASEAN climate club. Right: Decomposition of welfare changes within ASEAN member states (top) and for major economies and losers of climate change (bottom).

5.5 Global carbon policy

Finally, we examine the case when all nations impose a $50 carbon tax in Figure 9. When all nations participate in carbon taxation, carbon border adjustment mechanisms are not needed due to the targeting principle; the carbon externality is internalized at its source. When we impose this policy, the global price of hydrocarbons rises by 1%. Indeed, this results from the mechanism explained in equation (23), where carbon taxation has a strong impact on coal consumption, and countries substitute toward oil and gas. In net, carbon emissions decline by 4%, which is one order of magnitude larger than for unilateral policies or carbon taxation implemented in small climate clubs. Such global climate policy results in a 6% increase in utilitarian-weighted global welfare. Were we to evaluate welfare using Negishi weights, the global change in welfare is 1%, as rich, cool countries receive a higher weight. Consequently, a large part of the measured welfare gain from implementing global carbon taxation, when evaluating welfare changes from a utilitarian perspective, stems from reducing international inequality.

We plot the results of this exercise in Figure 4. They are the reverse of the effects of climate change: nations in the global south benefit from global carbon taxation while baseline cold nations lose. These losses are offset in Europe, as European nations like Great Britain and Germany have improved terms of trade, while gains are attenuated in Saudi Arabia for the same reason. Saudi energy rents grow as the world substitutes from coal to oil-gas, which puts upward pressure on the international price of hydrocarbon fossil.

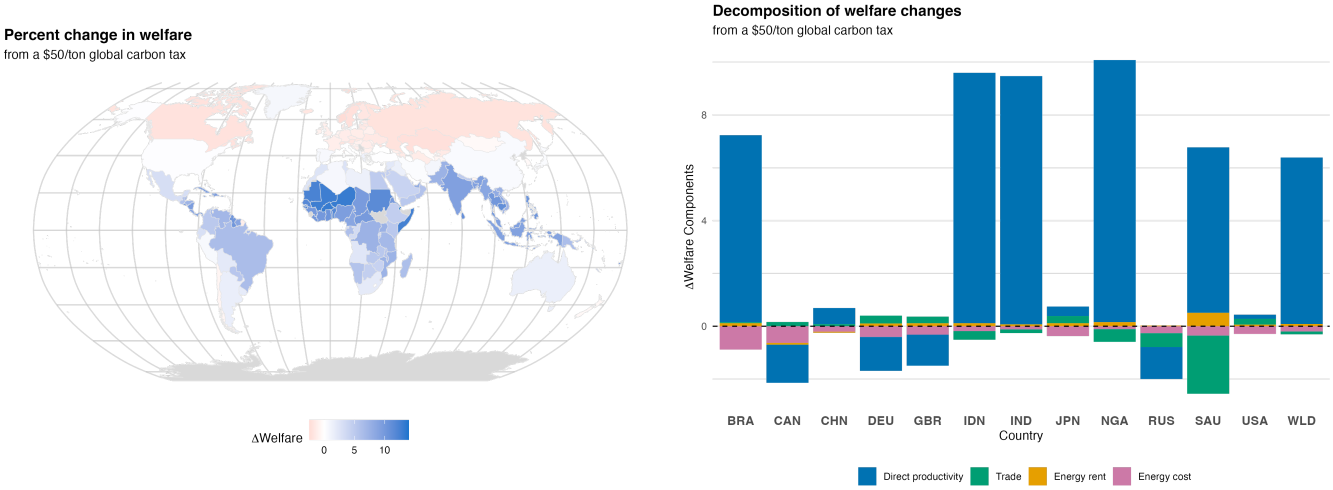

Figure 9: The welfare effects of a $50 carbon tax imposed in every country around the world. Left: map of global welfare changes, in % terms. Red countries lose, while blue countries win. Right: decomposition of welfare changes for several major economies.

6 Conclusion

In this paper, we use a first-order decomposition of the effects of climate policy derived from a macroeconomic IAM to study who wins and loses under various climate policy regimes. Our IAM features trade in both goods and energy markets, and our decomposition is computable with a modest set of sufficient statistics available using freely available trade and national accounts data and estimable elasticities.

We estimate heterogenous damages on productivity from rises in local temperature using bilateral trade data, and estimate a large set of heterogeneous energy supply elasticities for both hydrocarbon fossil and coal producers. We find that this heterogeneity is important not only in capturing heterogeneity in productivity damages across space from climate change but also in the response of nations' energy sectors to climate policy.

We use our estimates to consider a large set of climate policies. First, in agreement with the literature, we find spatially heterogeneous winners and losers from climate change, with baseline hot nations suffering as a result of a hotter planet, and cold nations gaining. These welfare changes are amplified by changes in international goods and energy markets: dirty energy exporters lose energy rents, and the pattern of trade adjusts, improving or deteriorating different nations' terms of trade.

We find that pursuing unilateral policy is often ineffective in combatting climate change, despite often creating positive welfare gains for the nations that pursue such policies. For example, carbon taxation in China does little to affect global emissions, and redirects their energy mix towards oil and gas imports, improving the extraction rents of energy exporters and indirectly generating an improved terms-of-trade for China in international goods markets. As nations' energy mixes differ, carbon taxation and subsidizing renewable energy generate different welfare responses. Broadly, we find that subsidizing renewables is substantially less effective than taxing carbon.

Coordinated climate policy through climate clubs, in which member nations impose a domestic carbon tax and carbon tariffs on imports from non-member nations, better addresses the climate externality. Climate policy in the EU broadly harms EU members but delivers sizable global gains. A climate club of ASEAN members both improves global welfare and the welfare of club members. However, this policy has unequal effects on member nations, as it redirects trade, causing some nations to benefit more than others.

In short, our results suggest that implementing climate policy is difficult unilaterally, as leakage effects can be an order of magnitude larger than the gains from cooling global temperature. International agreements are necessary to combat climate change, but can generate unequal effects among members party to the agreement through these leakage channels. While our sufficient statistics and welfare formulas allow us to quickly compute and decompose the effects of many possible climate clubs, they do not take into account the nonlinear effects induced by imposing large carbon taxes and tariffs. Bourany (2025) moves beyond sufficient statistics and solves for the *optimal* design of these types of climate agreements.

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Appendix

A Additional tables and figures

	OLS		Poisson	
	(1)	(2)	(3)	(4)
Exporter Temperature (C)	0.086***		0.722***	
	(0.030)		(0.203)	
Exporter Temperature^2	-0.002		-0.027***	
	(0.001)		(0.007)	
Importer Temperature (C)	0.058		-0.607	
	(0.117)		(0.826)	
Importer Temperature^2	-0.002		0.020	
	(0.005)		(0.023)	
Exporter-Importer Temperature (C) difference		0.015		0.652**
		(0.061)		(0.331)
Exporter-Importer Temperature^2 difference		0.000		-0.023*
		(0.002)		(0.012)
T^*	15.807	-31.560	13.399	14.016
	(5.0629)	(421.9431)	(0.9911)	(1.9916)
γ	0.001	-0.000	0.010	0.012
	(0.0006)	(0.0012)	(0.0027)	(0.0060)
Importer-Exporter pair FE	✓	✓	✓	✓
Origin GDP/cap control	✓	✓	✓	✓
Origin energy controls	✓	✓	✓	✓
R^2	0.865	0.865		
Pseudo-R^2			0.854	0.854
Observations	366,384	366,384	463,614	463,614

Table 1: Estimates of equation 27. $^{***}p < 0.1, ^{**}p < 0.05, ^{*}p < 0.01$. Standard errors are two-way clustered at the importer and exporter level in parentheses. Dependent variable: importer penetration ration X_{ij}/X_{ii}. Columns (1)-(2) use an OLS estimator with the log of the import penetration ratio on the lefthandside, while (3)-(4) use a Poisson estimator, which retains zeros in the trade matrix. Specifications differ by whether importer and exporter temperature are allowed to have different coefficients, or if the model-implied coefficient restriction is imposed. Computation of T^* and γ assumes $\theta = 5$, as in our quantification.

Parameter	Value	Description	Source
Household preferences			
η	1.5	Coefficient of relative risk aversion	Barrage and Nordhaus (2024)
$\theta - 1$	4	Trade elasticity	Simonovska and Waugh (2014)
Goods production			
T^*	14.02	Global peak temperature	Estimated
γ	0.012	Shape parameter of $\mathcal{D}_i^y$	–
σ^y	0.3	Elasticity of substitution between energy and labor	Papageorgiou et al. (2017)
ω^f	–	Share of oil-gas in production	Calibrated to match energy mix
ω^c	–	Coal share	–
ω^r	–	Renewables share	–
σ_e	2	Elasticity of substitution between energy sources	Papageorgiou et al. (2017); Kotlikoff et al. (2021b)
Energy production			
$1/\nu_i^f$	–	Supply elasticity of oil-gas	Estimated for each country
$1/\nu_i^c$	–	Supply elasticity of coal	–
$1/\nu^r$	2.7	Supply elasticity of renewable energy	Johnson (2014)

Table 2: Summary of estimated and externally calibrated parameters

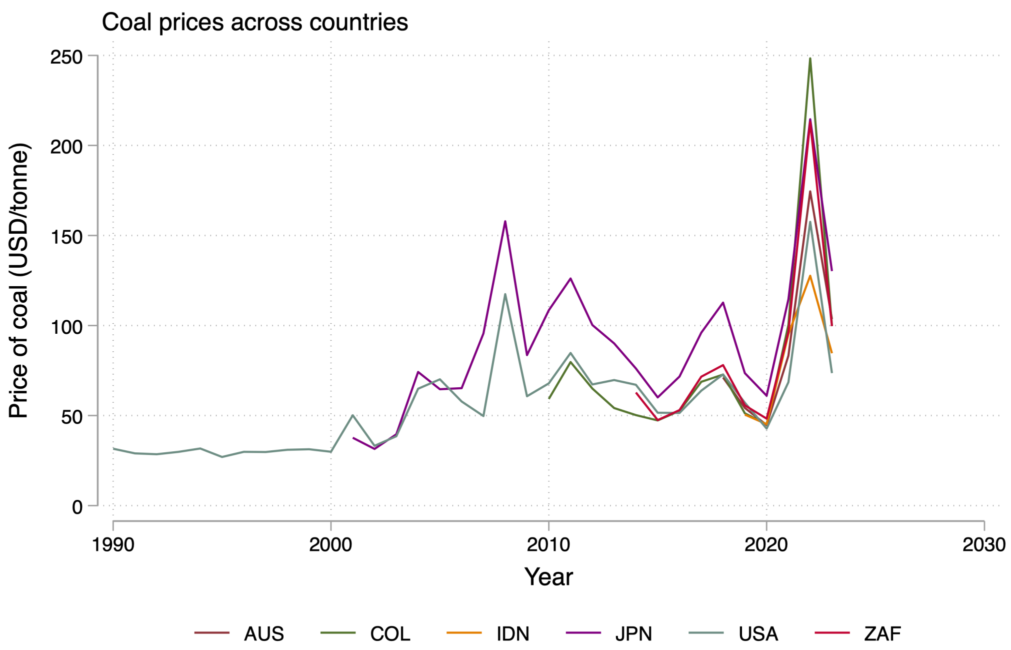

Figure 10: Coal prices in several countries. Source: Our World in Data.

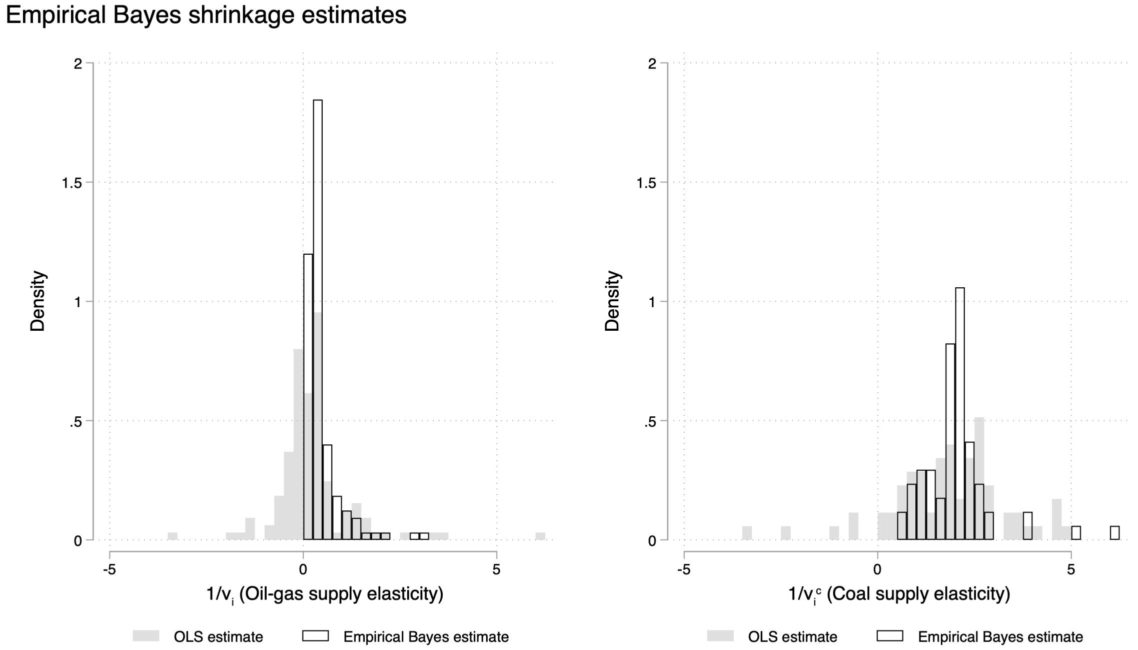

Figure 11: Histograms of the distribution of OLS country-specific energy supply elasticities and the empirical Bayes estimates. Left: oil-gas (hydrocarbon fossil) energy supply elasticities, $1/\nu_i^f$. Right: coal energy supply elasticities $1/\nu_i^c$. See main text for details.

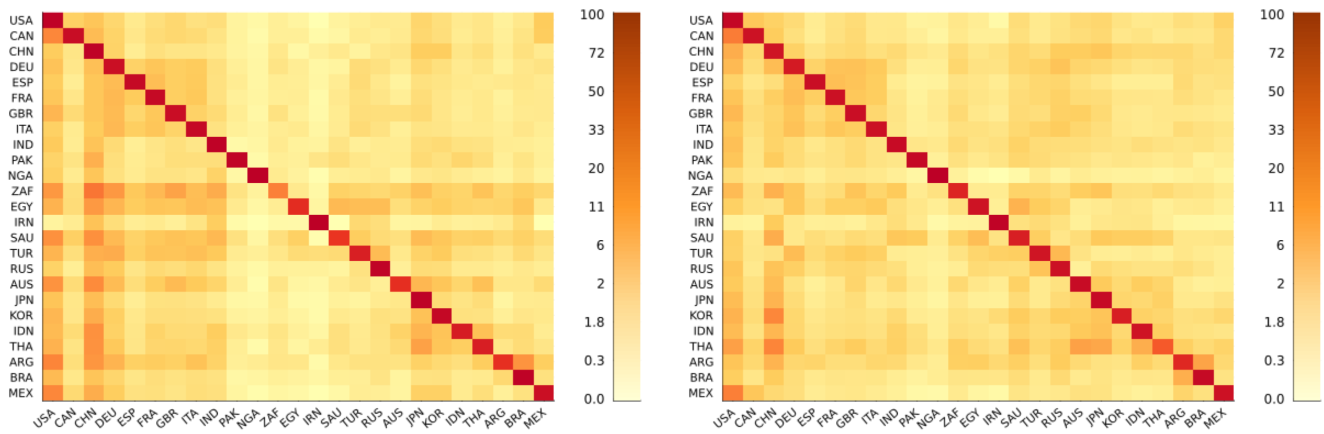

Figure 12: Left: Trade shares matrix, **S**. Right: Income shares matrix **T**. See main text for details.

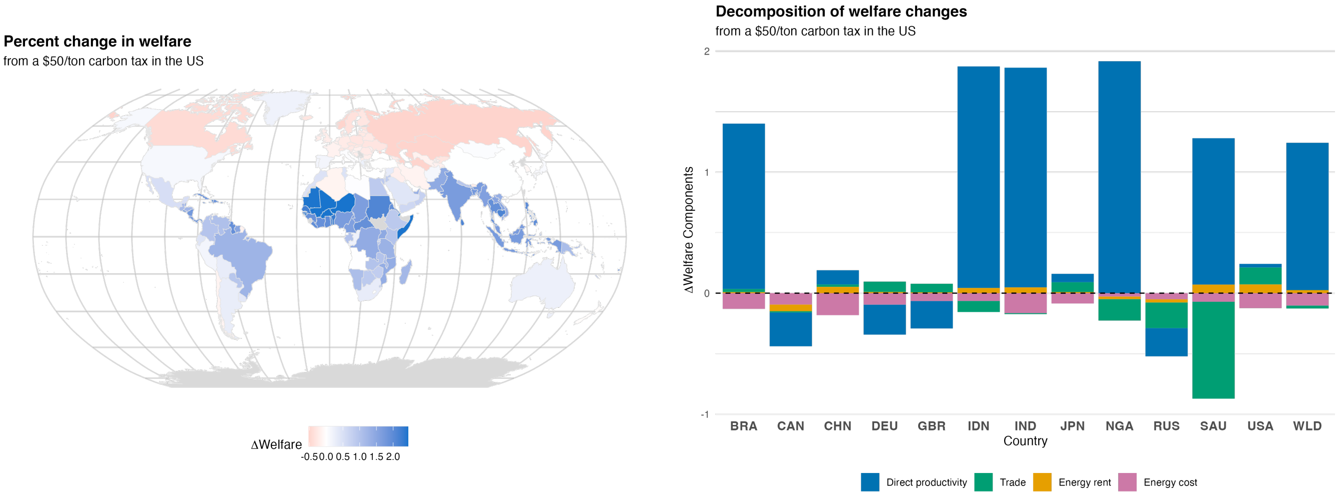

Figure 13: The welfare effects of a $50 carbon tax imposed unilaterally by the United States. Left: map of global welfare changes, in % terms. Red countries lose, while blue countries win. Right: decomposition of welfare changes for several major economies.

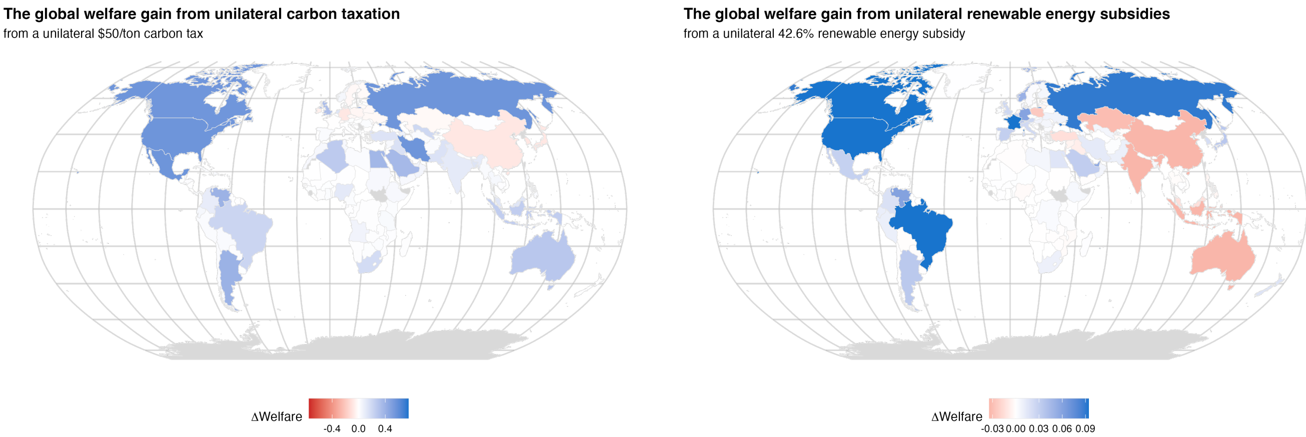

Figure 14: Left: Map of the global utilitarian welfare change associated with each country's unilateral $50 carbon tax. Right: the same, for a 42.6% renewable energy subsidy.

B Welfare decomposition

B.1 Model summary

First let us summarize the model, as presented above

$$
\begin{aligned}
c_i \mathbb{P}_i = x_i &= w_i \ell_i + \pi_i^x + \mathrm{t}_i^{ls} = \mathrm{p}_i z_i \mathcal{D}_i(T_i) F(e_i, \ell_i) - q_t^e e_i + \frac{1}{\mathcal{P}_i}\Big(q^e e_i^x - \mathrm{p}_i \mathcal{C}^f(\mathcal{P}_i e_i^x, \mathcal{R}_i)\Big) + \mathrm{t}_i^{ls} \\
\mathcal{P}_i \mathrm{p}_i y_i &= \sum_{k \in \mathbb{I}} \mathcal{P}_k s_{ki} \frac{v_k}{1 + \mathrm{t}_{ki}} \\
v_i &= \mathrm{p}_i y_i + q^f (e_i^x - e_i^f) + \mathrm{t}_i^{ls} \\
\pi_i^f &= \frac{1}{\mathcal{P}_i} \frac{\nu_i^f \bar{\nu}^{-1/\nu_i^f}}{1+\nu_i^f} \mathcal{R}_i (q^f)^{1+\frac{1}{\nu_i^f}} \mathrm{p}_i^{-1/\nu_i^f} \\
\pi_i^c &= \frac{1}{\mathcal{P}_i} \frac{\nu_i^c \bar{\nu}^{-1/\nu_i^c}}{1+\nu_i^c} (q_i^c)^{1+\frac{1}{\nu_i^c}} \mathrm{p}_i^{-1/\nu_i^c} \\
\pi_i^r &= \frac{1}{\mathcal{P}_i} \frac{\nu_i^r \bar{\nu}^{-1/\nu_i^r}}{1+\nu_i^r} (q_i^r)^{1+\frac{1}{\nu_i^r}} \mathrm{p}_i^{-1/\nu_i^r} \\
\sum_k \mathcal{P}_i e_i^f &= \sum_k e_i^x = (q^f)^{1/\nu} \sum_k \mathcal{R}_i \bar{\nu}_i^{-1/\nu} \mathrm{p}_i^{-1/\nu} \\
F_i(\varepsilon(e^f, e^c, e^r), \ell) &= \Big[(1-\epsilon)^{\frac{1}{\sigma_y}} \big(\bar{k}^\alpha \ell^{1-\alpha}\big)^{\frac{\sigma_y - 1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \big(z_i^e \ \varepsilon_i(e^f, e^c, e^r)\big)^{\frac{\sigma_y - 1}{\sigma_y}}\Big]^{\frac{\sigma_y}{\sigma_y - 1}} \\
\varepsilon(e^f, e^c, e^r) &= \Big[(\omega_i^f)^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e - 1}{\sigma_e}} + (\omega_i^c)^{\frac{1}{\sigma_e}} (e^c)^{\frac{\sigma_e - 1}{\sigma_e}} + (\omega_i^r)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e - 1}{\sigma_e}}\Big]^{\frac{\sigma_e}{\sigma_e - 1}}
\end{aligned}
$$

B.2 Change in welfare – experiments

We compute the change in the welfare of each country for different experiments. The model is linearized around an equilibrium where climate change is not realized yet $\mathcal{T} = 0$, and where the policies are identical to the "status quo": $\mathrm{t}^\varepsilon = \bar{\mathrm{t}}^\varepsilon = 0$ and $\mathrm{t}_{ij}^b = \bar{\mathrm{t}}_{ij}^b$. As a result, this corresponds to the competitive equilibrium.

We consider first-order deviations where we increase either (i) the impact of climate change and (ii) climate policy instruments by a small amount. To save on notation, we denote $d \ln x_i = \frac{dx_i}{x_i}$ – with a slight abuse of notation[18]

Effects of climate change

I consider a first-order change in global warming, which will impact global temperature $\mathcal{T}$, by an amount $d \ln \mathcal{T}$ and hence local temperature T_i and local productivity $d \ln z_i = \frac{dz_i}{z_i}$.

Unilateral climate policies – Carbon tax and Renewable subsidy

In addition, I consider a first-order change in local climate policies: carbon tax t_i^ε and renewable subsidies s_i^ε. As a result, the policy change we consider is $d \ln \mathrm{t}_i^\varepsilon = \frac{d\mathrm{t}_i^\varepsilon}{1+\mathrm{t}_i^\varepsilon} = d\mathrm{t}_i^\varepsilon$ where we consider a multiplicative carbon tax on fossil fuel $q_i^f(1+\mathrm{t}_i^\varepsilon)$. Similalry, for small renewable subsidy: $d \ln \mathrm{s}_i^\varepsilon = d\mathrm{s}_i^\varepsilon$.

[18] This is the case, for example, when $x_i < 0$ or change sign.

I consider the case where those policies are implemented unilaterally, i.e., for country i but not for country $j \neq i$, and compare the cost of such implementation in the presence of trade leakage.

Coordinated climate policies

Then, we consider the case of coordinated climate policies, where a large set of countries implement the policy jointly. I consider a set $\mathcal{J}$ of J countries that are linked by a coordination mechanism, e.g. a climate agreement. In matrix notation, these changes in carbon tax are noted:

$$\mathbf{J} d\mathbf{t}^{\varepsilon} = \left\{ \mathbb{1}_{\{i \in \mathcal{J}\}} \, d \ln \mathrm{t}_i^{\varepsilon} \right\}_i$$

with $\mathbf{J} = \mathrm{J}_i = \mathbb{1}_{\{i \in \mathcal{J}\}}$ the column vector that is one if $i \in \mathcal{J}$ and zero otherwise.

Carbon Border Adjustment Mechanism / Carbon tariffs

I also consider a first-order change in tariffs t_{ij}^{b}, imposed by country i on the goods from country j. As a result, the policy change we consider is $d \ln \mathrm{t}_{ij}^{b} = \frac{d\mathrm{t}_{ij}^{b}}{1+\mathrm{t}_{ij}^{b}} = d\mathrm{t}_{ij}^{b}$ for multiplicative tariffs. The tariff scale with carbon intensity of the country one imports from: $\mathrm{t}_{ij}^{b} = \xi_j^{\varepsilon} \mathrm{t}_i^{\varepsilon}$ with the carbon intensity of country j, $\xi_j^{\varepsilon} = \frac{\varepsilon_j}{y_j \mathrm{p}_j}$ with ε_j the per capita carbon emissions.

I consider three cases: First, this policy is implemented unilaterally for country i but not for country $j \neq i$. Second, the policy is implemented in coordination with the carbon tax at the same carbon price t^{ε}, again unilaterally. Third, this carbon tax + carbon tariff policy is coordinated among countries within a club $\mathcal{J}$, e.g. European Union or OECD countries against non-member countries, $\overline{\mathbf{J}} \equiv \mathrm{J}_{ij} = \mathbb{1}_{\{i \in \mathcal{J}, j \notin \mathcal{J}\}}$.

Welfare change

I now compute the welfare of individual country i, defined as the indirect utility, accounting for change in consumption and climate damages: $\mathcal{U}_i = u(\{c_{ij}\}_j, T_i) = u\big(c_i \mathcal{D}_i^u(T_i)\big)$. This changes writes as:

$$d\mathcal{U}_i = du\Big(c_i \mathcal{D}_i^u\Big) = u'(c_i \mathcal{D}_i^u)(c_i \mathcal{D}_i^u)\Big(\frac{dc_i}{c_i} + \frac{d\mathcal{D}_i^u}{\mathcal{D}_i^u}\Big) = u'(\widetilde{c}_i)\widetilde{c}_i\Big(\frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\mathcal{D}_i^u}{\mathcal{D}_i^u}\Big)$$

with $x_i = c_i \mathbb{P}_i$ the consumption expenditure and $\widetilde{c}_i = c_i \mathcal{D}_i^u$. As a result, in the main text, we display the result in consumption equivalent:

$$\frac{d\mathcal{U}_i}{u'(\widetilde{c}_i)\widetilde{c}_i} = \Big(\frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\mathcal{D}_i^u}{\mathcal{D}_i^u}\Big)$$

B.3 Climate externality

To see the effects of a change in emissions and carbon policies on climate, we unpack the damage $d\mathcal{D}_i$. In this section, we consider the following simplified climate system:

$$T_i = \Delta_i \mathcal{T} = \Delta_i \chi \mathcal{S}$$
$$\mathcal{S} = \mathcal{S}_0 + \mathbb{T}\mathcal{E} = \mathcal{S}_0 + \xi^f E^f + \xi^c E^c$$

where

$$\mathcal{E} = \sum_i \mathcal{P}_i(\xi^f e_i^f + \xi^c e_i^c)$$

is a representation of yearly emissions $\mathcal{E}$ due to oil-gas and coal. We scale those yearly emissions by a factor $\mathbb{T}$ to represent a specific horizon – say 50 or 100 years. Moreover, the parameters ξ^f and ξ^c represent the carbon contents for different fossil fuels per unit of energy supplied.

We use a damage function, conventionally used in Integrated Assessment models, with curvature δ and slope γ:

$$\mathcal{D}_i^y(T_i) = e^{-\frac{\gamma^y}{1+\delta}(T_i - T_i^\star)^{1+\delta}}$$

and similarly for $\mathcal{D}_i^u(T_i)$. The linear approximation of this climate system implies:

$$\frac{d\mathcal{D}_i^y}{\mathcal{D}_i^y} = -\gamma^y(T_i - T_i^\star)^\delta dT_i = -\gamma(T_i - T_i^\star)^\delta T_i \frac{dT_i}{T_i}$$

we notice that despite the approximation being linear – and hence abstracting from the curvature δ of damages – we still have that a higher curvature imply more heterogeneous damages between warm and cold regions based on $(T_i - T_i^\star)^\delta$.

Regarding the change in temperature caused by emissions, we get:

$$dT_i = \Delta_i \chi d\mathcal{S} = \Delta_i \chi(\xi^f dE^f + \xi^c dE^c)$$

$$\Rightarrow \quad \frac{dT_i}{T_i} = \frac{\Delta_i \chi d\mathcal{E}}{\Delta_i \chi(\mathcal{S}_0 + \mathbb{T}\mathcal{E})} = s^{E/S}\Big(s^{f/E}\frac{dE^f}{E^f} + s^{c/E}\frac{dE^c}{E^c}\Big)$$

$$\text{with } s^{E/S} = \frac{\mathbb{T}\mathcal{E}}{\mathcal{S}_0 + \mathbb{T}\mathcal{E}} \qquad s^{f/E} = \frac{\xi^f E^f}{\mathcal{E}} \qquad s^{c/E} = \frac{\xi^c E^c}{\mathcal{E}}$$

As a result, to summarize, the change in damage depends on the total energy used in fossil (oil-gas) and coal.

$$d\ln \mathcal{D}_i^y = -\bar{\gamma}_i{}^y\big(s^{f/E} d\ln E^f + s^{c/E} d\ln E^c\big) \qquad \bar{\gamma}_i^y = \gamma(T_i - T_i^\star)T_i\, s^{E/S}$$

and similarly for $d\ln \mathcal{D}_i^y$ where $\bar{\gamma}_i^y$ and $\bar{\gamma}_i^y$ summarize in simple parameters – as sufficient statistics – the heterogeneous impacts of climate change on output and utility.

B.4 Production

We now derive the impact of changes in prices and quantities on welfare through the budget constraint. First, we define several objects – like shares – that are relevant for the decomposition:

- Energy share in production: $s_i^e = \frac{e_i q_i^e}{y_i \mathrm{p}_i}$
- Fossil share in energy mix $s_i^f = \frac{e_i^f q^f}{e_i q_i^e}$ and similarly $s_i^c = \frac{e_i^c q_i^c}{e_i q_i^e}$ and $s_i^r = \frac{e_i^r q_i^r}{e_i q_i^e}$
- Production share/rent share in GDP: $\eta_i^y = \frac{y_i \mathrm{p}_i}{y_i \mathrm{p}_i + \pi_i^f} = 1 - \eta_i^\pi$
- Consumption share in GDP: $\eta_i^c = \frac{x_i}{y_i \mathrm{p}_i + \pi_i^f}$
- Consumption as a ratio of output: $s_i^{c/y} = \frac{c_i \mathbb{P}_i}{y_i \mathrm{p}_i} = \frac{x_i}{y_i \mathrm{p}_i + \pi_i^f} \frac{y_i \mathrm{p}_i + \pi_i^f}{y_i \mathrm{p}_i} = \frac{\eta_i^c}{1 - \eta_i^\pi} = \frac{\eta_i^c}{\eta_i^y}$,

- Energy share as a ratio of consumption: $\frac{e_i q_i^e}{x_i} = \frac{e_i q_i^e}{y_i \mathrm{p}_i} \frac{y_i \mathrm{p}_i}{y_i \mathrm{p}_i + \pi_i^f} \frac{y_i \mathrm{p}_i + \pi_i^f}{x_i} = s_i^e \frac{\eta_i^y}{\eta_i^c}$
- Profit share as a ratio of consumption: $\frac{\pi_i^f}{x_i} = \frac{\pi_i^f}{y_i \mathrm{p}_i + \pi_i^f} \frac{y_i \mathrm{p}_i + \pi_i^f}{x_i} = \frac{\eta_i^\pi}{\eta_i^c}$
- The share of GDP of energy imports and exports, with $v_i = \mathrm{p}_i y_i + q^f(e_i^x - e_i^f)$ and $v^y = \frac{\mathrm{p}_i y_i}{v_i}$, $v^{e^x} = \frac{q^f e_i^x}{v_i}$, $v^{ef} = \frac{q^f e_i^f}{v_i}$ and $v^{ne} = \frac{q^f(e_i^x - e_i^f)}{v_k}$.

Returning to our decomposition, we start from the budget constraint:

$$
\begin{aligned}
c_i \mathbb{P}_i = x_i &= \mathrm{p}_i z_i \mathcal{D}_i(T_i) F(e_i, \ell_i) - q_i^e e_i + \frac{1}{\mathcal{P}_i} \sum_\ell \left(q^\ell \bar{e}_i^\ell - \mathrm{p}_i \mathcal{C}^\ell(e_i^\ell) \right) + \mathrm{t}_i^{ls} \\
&= \mathrm{p}_i z_i \mathcal{D}_i(T_i) F(e_i, \ell_i) - \left(q^f (1+\mathrm{t}_i^\varepsilon) e_i^f + q_i^c (1+\mathrm{t}_i^\varepsilon) e_i^c + q_i^r (1-\mathrm{s}_i^\varepsilon) e_i^r \right) + \\
&\qquad \frac{1}{\mathcal{P}_i} \sum_\ell \left(q^\ell \bar{e}_i^\ell - \mathrm{p}_i \mathcal{C}^\ell(e_i^\ell) \right) + \widetilde{\mathrm{t}}_i^{ls} + q^f \mathrm{t}_i^\varepsilon e_i^f + q_i^c \mathrm{t}_i^\varepsilon e_i^c - q_i^r \mathrm{s}_i^\varepsilon e_i^r
\end{aligned}
$$

Since the revenues of the carbon tax and the renewable subsidy are redistributed/taxed lump-sum to the Household, we do not see any direct redistributive effect of carbon taxation, e.g. as the terms $q^f \mathrm{t}_i^\varepsilon e_i^f$ cancel out.

Taking the first-order expansion of the budget constraint, we obtain:

$$
\begin{aligned}
\frac{dc_i}{c_i} = \frac{dx_i}{x_i} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} &= \frac{\mathrm{p}_i y_i}{x_i} \left(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i} \right) - \frac{e_i q_i^e}{x_i} \Big(\frac{e_i^f q^f}{e_i q_i^e} \left(\frac{de^f}{e^f} + \frac{dq^f}{q^f} \right) + \frac{e_i^c q^c}{e_i q_i^e} \left(\frac{de^c}{e^c} + \frac{dq^c}{q^c} \right) + \frac{e_i^r q^r}{e_i q_i^e} \left(\frac{de^r}{e^r} + \frac{dq^r}{q^r} \right) \Big) \\
&\quad + \frac{\pi^f}{x_i} \frac{d\pi_i^f}{\pi^f} + \frac{\widetilde{\mathrm{t}}_i^{ls}}{x_i} \left(\frac{d\widetilde{\mathrm{t}}_i^{ls}}{\widetilde{\mathrm{t}}_i^{ls}} \right) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i} \\
\frac{dc_i}{c_i} &= \frac{\eta_i^y}{\eta_i^c} \left(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i} \right) - s_i^e \frac{\eta_i^y}{\eta_i^c} \Big(s_i^f \left(\frac{de^f}{e^f} + \frac{dq^f}{q^f} \right) + s_i^c \left(\frac{de^c}{e^c} + \frac{dq^c}{q^c} \right) + s_i^r \left(\frac{de^r}{e^r} + \frac{dq^r}{q^r} \right) \Big) \\
&\quad + \frac{\eta_i^{\pi f}}{\eta_i^c} \frac{d\pi_i^f}{\pi_i^f} + \frac{\eta_i^{\pi c}}{\eta_i^c} \frac{d\pi_i^c}{\pi_i^c} + \frac{\eta_i^{\pi r}}{\eta_i^c} \frac{d\pi_i^r}{\pi_i^r} + \frac{\widetilde{\mathrm{t}}_i^{ls}}{x_i} \left(\frac{d\widetilde{\mathrm{t}}_i^{ls}}{\widetilde{\mathrm{t}}_i^{ls}} \right) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i}
\end{aligned}
$$

First, using output changes, approximating the production function:

$$
\frac{dy_i}{y_i} = \frac{d\mathcal{D}_i^y}{\mathcal{D}_i} + \frac{MPe_i e_i}{y_i} \frac{de_i}{e_i} = \frac{d\mathcal{D}_i^y}{\mathcal{D}_i^y} + s_i^e \left[s_i^f \frac{de_i^f}{e_i^f} + s_i^c \frac{de_i^c}{e_i^c} + s_i^r \frac{de_i^r}{e_i^r} \right]
$$

we see that Hulten's theorem implies a first-order impact of a change in energy price that scales with the share of energy in production s_i^e, which is typically around $5-10\%$ and the share of fossils in the energy mix s_i^f, s_i^c, which sum to above 85%.

B.5 Energy markets – Profits, and prices for Coal and Renewable

Using the fossil-energy firm problem, we get the profit change as a function of the price:

$$
\frac{d\pi_i^f}{\pi_i^f} = \left((1+\frac{1}{\nu_i^f}) \frac{dq^f}{q^f} - \frac{1}{\nu_i^f} \frac{d\mathrm{p}_i}{\mathrm{p}_i} \right)
$$

The energy rent is affected by changes in the aggregate fossil energy price dq^f. Since the cost also depends on imported inputs, the prices of goods $\mathbb{P}_i$ also matter for profit and welfare.

Similarly, for coal and renewable, we obtain the same formulation for profit:

$$\frac{d\pi_i^c}{\pi_i^c} = (1+\tfrac{1}{\nu_i^c})\frac{dq_i^c}{q_i^c} - \frac{1}{\nu_i^c}\frac{d\mathrm{p}_i}{\mathrm{p}_i}$$

and similarly for $d\ln\pi_i^r$ as a function of $d\ln q_i^r$.

Now, we use the production function for coal and renewable, which implies the simple supply curve $q_i^c = \mathcal{C}_i^{c\prime}(\bar{e}_i^c)\mathrm{p}_i = (e_i^c)^{\nu_i^c}\mathrm{p}_i$ and $q_i^r = \upsilon_i^r\mathbb{P}_i$, we get

$$\frac{dq^r}{q^r} = \nu_i^r\frac{de_i^r}{e_i^r} + \frac{d\mathrm{p}_i}{\mathrm{p}_i} \qquad \text{and} \qquad \frac{dq^c}{q^c} = \nu_i^c\frac{de_i^c}{e_i^c} + \frac{d\mathrm{p}_i}{\mathrm{p}_i}$$

the price of both coal and renewable energy are directly exposed to changes in the price of the domestic good used in production.

As a result, the profits from coal and renewable can also be rewritten in function of quantities:

$$\frac{d\pi_i^c}{\pi_i^c} = (1+\tfrac{1}{\nu_i^c})\Big[\nu_i^r\frac{de_i^r}{e_i^r} + \frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big] - \frac{1}{\nu_i^c}\frac{d\mathrm{p}_i}{\mathrm{p}_i} = (1+\nu_i^c)\frac{de_i^r}{e_i^r} + \frac{d\mathrm{p}_i}{\mathrm{p}_i}$$

B.6 Returning to the budget/expenditure

Accounting for these different effects dramatically simplifies the change in consumption:

$$\begin{aligned}\frac{dc_i}{c_i} &= \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{dy_i}{y_i}\Big) - s_i^e\frac{\eta_i^y}{\eta_i^c}\Big(s_i^f(\frac{de^f}{e^f}+\frac{dq^f}{q^f})+s_i^c(\frac{de^c}{e^c}+\frac{dq^c}{q^c})+s_i^r(\frac{de^r}{e^r}+\frac{dq^r}{q^r})\Big)\\ &\qquad + \sum_\ell \frac{\eta_i^{\pi\ell}}{\eta_i^c}\Big((1+\frac{1}{\nu_i^\ell})\frac{dq^\ell}{q^\ell} - \frac{1}{\nu_i^\ell}\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big) + \frac{d\tilde{\mathrm{t}}_i^{ls}}{x_i}) \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i}\\ &= \frac{\eta_i^y}{\eta_i^c}\Big(\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{d\mathcal{D}_i}{\mathcal{D}_i}\Big) - \frac{\eta_i^y}{\eta_i^c}s_i^e\Big[s_i^f\frac{dq^f}{q^f} + s_i^c\frac{dq^c}{q^c} + s_i^r\frac{dq^r}{q^r}\Big] + \sum_\ell\frac{\eta_i^{\pi\ell}}{\eta_i^c}\Big((1+\frac{1}{\nu_i^\ell})\frac{dq^\ell}{q^\ell} - \frac{1}{\nu_i^\ell}\frac{d\mathrm{p}_i}{\mathrm{p}_i}\Big) + \frac{d\tilde{\mathrm{t}}_i^{ls}}{x_i} \quad - \frac{d\mathbb{P}_i}{\mathbb{P}_i}\end{aligned}$$

$$\frac{dc_i}{c_i} = \Big[\frac{\eta_i^y}{\eta_i^c} - \sum_\ell\frac{1}{\nu_i^\ell}\frac{\eta_i^{\pi\ell}}{\eta_i^c}\Big]\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{\eta_i^y}{\eta_i^c}\frac{d\mathcal{D}_i^y}{\mathcal{D}_i^y} + \sum_\ell\Big[\frac{\eta_i^{\pi\ell}}{\eta_i^c}\Big(1+\frac{1}{\nu_i^\ell}\Big) - \frac{\eta_i^y}{\eta_i^c}s_i^e s_i^\ell\Big]\frac{dq^\ell}{q^\ell} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\tilde{\mathrm{t}}_i^{ls}}{x_i}$$

B.7 Energy markets

We now turn to energy where the demand and equilibrium effect on prices will be of first-oder importance for our welfare decomposition.

Energy demand

To examine the demand side of the market, we compute the elasticities of demand for each energy source, which are determined jointly by the firm First-Order Conditions. Thanks to our nested CES formulation, we can compute the elasticity $\varepsilon^{\ell}_{q^k} = \frac{\partial e^{\ell}_i}{\partial q^k}\frac{q^k}{e^{\ell}_i}$ as :

$$\begin{bmatrix} \varepsilon^f_{q^f} & \varepsilon^f_{q^c} & \varepsilon^f_{q^r} \\ \varepsilon^c_{q^f} & \varepsilon^c_{q^c} & \varepsilon^c_{q^r} \\ \varepsilon^r_{q^f} & \varepsilon^r_{q^c} & \varepsilon^r_{q^r} \end{bmatrix} = (\widetilde{H}^e)^{-1} = -\frac{\sigma^y}{1-s^e}\begin{bmatrix} s^f & s^c & s^r \\ s^f & s^c & s^r \\ s^f & s^c & s^r \end{bmatrix} + \sigma^e \begin{bmatrix} -(1-s^f) & s^c & s^r \\ s^f & -(1-s^c) & s^r \\ s^f & s^c & -(1-s^r) \end{bmatrix}$$

where the first part correspond to the change in aggregate price of energy q^e, since $\frac{\partial q^e_i}{\partial q^k}\frac{q^k}{q^e_i} = s^k_i$, which reduces demands for overall energy, according to elasticity $\frac{\sigma^y}{1-s^e_i}$ where s^e_i is the cost share of energy and σ^y the elasticity between energy and other inputs. Second, the later part summarizes the substitution effect across energy sources, negative along the diagonal and positive out of diagonal, due to positive cross-elasticity in the CES framework.

Moreover, the energy demand also depends on aggregate TFP (and hence climate damage), and the price level at which the final good is sold. As a result, the productivity elasticities and the final good price elasticity write:

$$\begin{bmatrix} \varepsilon^f_z \\ \varepsilon^c_z \\ \varepsilon^r_z \end{bmatrix} = \frac{\sigma^y}{1-s^e}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} \varepsilon^f_p \\ \varepsilon^c_p \\ \varepsilon^r_p \end{bmatrix} = \frac{\sigma^y}{1-s^e}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

which again is standard in the Nested CES framework.

As a result, we can express the energy demand as a function of the other endogenous variables:

$$\begin{aligned} d\ln e^f_i &= -(\frac{\sigma^y}{1-s^e_i}s^f_i + (1-s^f_i)\sigma^e)[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] + (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^c_i[d\ln q^c_i + \xi^c \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] + (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^r_i[d\ln q^r_i - \mathrm{J}_i d\ln \mathrm{s}^{\varepsilon}_i] \\ &\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i \\ d\ln e^c_i &= (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^f_i[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] - (\frac{\sigma^y}{1-s^e_i}s^c_i + (1-s^c_i)\sigma^e)[d\ln q^c_i + \xi^c \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] + (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^r_i[d\ln q^r_i - \mathrm{J}_i d\ln \mathrm{s}^{\varepsilon}_i] \\ &\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i \\ d\ln e^r_i &= (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^f_i[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] + (\sigma^e - \frac{\sigma^y}{1-s^e_i})s^c_i[d\ln q^c_i + \xi^c \mathrm{J}_i d\ln \mathrm{t}^{\varepsilon}] - (\frac{\sigma^y}{1-s^e_i}s^r_i + (1-s^r_i)\sigma^e)[d\ln q^r_i - \mathrm{J}_i d\ln \mathrm{s}^{\varepsilon}_i] \\ &\quad + \frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i \end{aligned}$$

Those endogenous energy demands can be reintegrated into the production function to obtain the change in output as a function of good prices, energy prices, and productivity:

$$
\begin{aligned}
d\ln y_i &= d\ln \mathcal{D}_i + s_i^e\left[s_i^f d\ln e_i^f + s_i^r d\ln e_i^c + s_i^r d\ln e_i^r\right] \\
&= (1+\frac{s_i^e\sigma^y}{1-s_i^e})d\ln\mathcal{D}_i + \frac{s_i^e\sigma^y}{1-s_i^e}d\ln \mathrm{p}_i - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^f[\xi^f d\ln q^f + d\ln \mathrm{t}_i^\varepsilon] \\
&\qquad - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r[d\ln q_i^c + \xi^c d\ln \mathrm{t}_i^\varepsilon] - s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r[d\ln q_i^r - \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon]
\end{aligned}
$$

$$
d\ln y_i = \alpha^{y,z}d\ln z_i + \alpha^{y,p}d\ln \mathrm{p}_i - \alpha^{y,qf}\left[d\ln q^f + d\ln \mathrm{t}_i^\varepsilon\right] - \alpha^{y,qc}\left[d\ln q_i^c + d\ln \mathrm{t}_i^\varepsilon\right] - \alpha^{y,qr}\left[d\ln q_i^r - d\ln \mathrm{s}_i^\varepsilon\right]
$$

$$
\alpha_i^{y,z} = 1+\frac{s_i^e\sigma^y}{1-s_i^e} \qquad \alpha_i^{y,p} = \frac{s_i^e\sigma^y}{1-s_i^e}
$$

$$
\alpha_i^{y,qf} = s_i^e\frac{\sigma^y}{1-s^e}s_i^f \qquad \alpha_i^{y,qc} = s_i^e\frac{\sigma^y}{1-s^e}s_i^c \qquad \alpha_i^{y,qr} = s_i^e\frac{\sigma^y}{1-s_i^e}s_i^r
$$

$$
\begin{aligned}
d\ln y_i &= \alpha^{y,z}d\ln z_i + \alpha^{y,p}d\ln \mathrm{p}_i - \alpha^{y,qf}d\ln q^f \\
&\quad - (\xi^f\alpha^{y,qf}+\xi^c\alpha^{y,qc})d\ln \mathrm{t}_i^\varepsilon + \alpha^{y,qr}d\ln \mathrm{s}_i^\varepsilon - \alpha^{y,qc}d\ln q_i^c - \alpha^{y,qr}d\ln q_i^r
\end{aligned}
$$

where this last equation uses the supply curve of coal and renewable. We can see the exposure of country i's output of carbon tax: $\xi^f\alpha^{y,qf}+\xi^c\alpha^{y,qc} = s_i^e\frac{\sigma^y}{1-s^e}(\xi^f s_i^f + \xi^c s_i^c)$, through the price and substitution effect of oil, gas and coal.

Coal and renewable energy markets

We write the demand curves in matrix forms:

$$
dq_i^{c,r} = \begin{bmatrix} d\ln q_i^c \\ d\ln q_i^r \end{bmatrix} = \begin{bmatrix} \nu_i^c & 0 \\ 0 & \nu_i^r \end{bmatrix}\begin{bmatrix} d\ln e_i^c \\ d\ln e_i^r \end{bmatrix} + d\ln \mathrm{p}_i
$$

$$
de_i^{c,r} = \begin{bmatrix} d\ln e_i^c \\ d\ln e_i^r \end{bmatrix} = A\,dq_i^{c,r} + \mathrm{J}_i A\begin{bmatrix} \xi^c d\ln \mathrm{t}_i \\ -d\ln \mathrm{s}_i \end{bmatrix} + (\sigma^e - \frac{\sigma^y}{1-s_i^e})s_i^f[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e}d\ln\mathcal{D}_i + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i
$$

with

$$
A = \begin{bmatrix} -\left(\frac{\sigma^y}{1-s_i^e}s_i^c + (1-s_i^c)\sigma^e\right) & (\sigma^e - \frac{\sigma^y}{1-s_i^e})s_i^r \\ (\sigma^e - \frac{\sigma^y}{1-s_i^e})s_i^c & -\left(\frac{\sigma^y}{1-s_i^e}s_i^r + (1-s_i^r)\sigma^e\right) \end{bmatrix}
$$

We can summarize and solve the system, where the vector b compiles the other terms (productivity and oil price effects):

$$
\begin{aligned}
dq_i^{c,r} &= \boldsymbol{\nu} de_i^{c,r} + d\ln \mathrm{p}_i \\
de_i^{c,r} &= A\,dq_i^{c,r} + A d\ln \mathrm{t}_i^{c,r} + \frac{\sigma^y}{1-s^e}d\ln \mathrm{p}_i + b_i
\end{aligned}
$$

$$
de_i^{c,r} = [\mathbb{I} - A\boldsymbol{\nu}]^{-1}Ad\ln \mathrm{t}_i^{c,r} + [\mathbb{I} - A\boldsymbol{\nu}]^{-1}[A + \frac{\sigma^y}{1-s^e}\mathbb{1}]d\ln \mathrm{p}_i + [\mathbb{I} - A\boldsymbol{\nu}]^{-1}b_i
$$

Solving in matrix form yields the price and quantity expression for coal and renewables.

$$
dq_i^{c,r} = [\mathbb{I} - A\boldsymbol{\nu}]^{-1}\boldsymbol{\nu}A\ d\ln \mathrm{t}_i^{c,r} + [\mathbb{I} - A\boldsymbol{\nu}]^{-1}\boldsymbol{\nu}b_i + [\mathbb{I} - A\boldsymbol{\nu}]^{-1}[\frac{\sigma^y}{1-s^e}\boldsymbol{\nu} + 1]\mathbb{1}d\ln \mathrm{p}_i
$$

with $b_i = (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i$.

We can write the energy and price as follows:

$$dq_i^c = -\beta^{q,t}_{c,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon - \beta^{q,t}_{c,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{q,b,c}[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i] + \beta^{q,p,c} d\ln \mathrm{p}_i$$

$$dq_i^r = +\beta^{q,t}_{r,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon + \beta^{q,t}_{r,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{q,b,r}\Big[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i\Big] + \beta^{q,p,r} d\ln \mathrm{p}_i$$

Where β^q are complicated parameters function of A and $\boldsymbol{\nu}$. For $\nu_i^r = \nu_i^r = 0$, we can simplify to have $\beta^{q,t}_{\ell,\ell'} = \beta^{q,b,\ell} = 0$ and $\beta^{q,p,\ell} = 1$, where we simply obtain $d\ln q_i^c = d\ln q_i^r = d\ln \mathrm{p}_i$. For quantities, we get the demand curve as a function of the policies:

$$de_i^c = -\beta^{e,t}_{c,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon - \beta^{e,t}_{c,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{e,p,c} d\ln \mathrm{p}_i + \beta^{e,b,c}\Big[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i\Big]$$

$$de_i^r = \beta^{e,t}_{r,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon + \beta^{e,t}_{r,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{e,p,r} d\ln \mathrm{p}_i + \beta^{e,b,r}\Big[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i\Big]$$

where, β^e are again complicated parameters function of A and $\boldsymbol{\nu}$. Again, with $\nu_i^r = \nu_i^r = 0$, we obtain $\beta^{e,t}_{\ell,\ell'} = A_{\ell,\ell'}$ and $\beta^{e,p,\ell} = \sum_{\ell'} A_{\ell,\ell'} + \frac{\sigma^y}{1-s^e}, \forall\, \ell$ and $\beta^{e,b,\ell} = 1, \forall\, \ell$

As a result, the energy demand for fossil can also be rewritten:

$$d\ln e_i^f = -(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e)[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^c [d\ln q_i^c + \xi^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^r [d\ln q_i^r - \mathrm{J}$$
$$+ \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i + \frac{\sigma^y}{1-s^e} d\ln \mathrm{p}_i$$

$$dq_i^c = -\beta^{q,t}_{c,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon - \beta^{q,t}_{c,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{q,b,c}[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i] + \beta^{q,p,c} d\ln \mathrm{p}_i$$

$$dq_i^r = \beta^{q,t}_{r,c}\ \xi^c d\ln \mathrm{t}_i^\varepsilon + \beta^{q,t}_{r,r}\ d\ln \mathrm{s}_i^\varepsilon + \beta^{q,b,r}\Big[(\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^f [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i\Big] + \beta^{q,p,r} d\ln \mathrm{p}_i$$

$$d\ln e_i^f = \Big\{ -(\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e) + \Big[s_i^c \beta^{q,b,c} + s_i^r \beta^{q,b,r}\Big](\sigma^e - \frac{\sigma^y}{1-s_i^e})^2 s_i^f \Big\}[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon]$$
$$+ \Big\{s_i^c - s_i^c \beta^{q,t}_{c,c} + s_i^r \beta^{q,t}_{r,c}\Big\}(\sigma^e - \frac{\sigma^y}{1-s_i^e})\, \xi^c \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon + \Big\{ - s_i^r - s_i^c \beta^{q,t}_{c,r} + s_i^r \beta^{q,t}_{r,r}\Big\}(\sigma^e - \frac{\sigma^y}{1-s_i^e})\, \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon$$
$$\frac{\sigma^y}{1-s^e}\Big\{1 + (\sigma^e - \frac{\sigma^y}{1-s_i^e})\Big[s_i^c \beta^{q,b,c} + s_i^r \beta^{q,b,r}\Big]\Big\} d\ln \mathcal{D}_i \ + \ \Big\{\frac{\sigma^y}{1-s^e} + (\sigma^e - \frac{\sigma^y}{1-s_i^e})\Big[s_i^c \beta^{q,p,c} + s_i^r \beta^{q,p,r}\Big]\Big\} d\ln \mathrm{p}_i$$

$$d\ln e_i^f = -\beta^{e,q}_{f,f,i}\, [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \beta^{e,t}_{f,c,i}\ \xi^c \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon - \beta^{e,t}_{f,r,i}\ \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon + \beta^{e,d,f}_i\ d\ln \mathcal{D}_i \ + \beta^{e,d,f}_i\ d\ln \mathrm{p}_i$$

This equation shows the layers of general equilibrium effects happening in the three energy markets. An increase in the price of fossil (oil-gas) decreases directly the oil-gas quantity consumed. However, it also creates substitution effects, as it now also increases both the demand and hence the price of coal and renewable – with magnitude $\beta^{q,b,c}$ and $\beta^{q,b,r}$ which then triggers a substitution effect away from those sources, and toward fossil, which then mitigates the drop in oil-gas demand. Similar effects arise for the carbon tax on coal $\xi^c \mathrm{t}_i^\varepsilon$ which increases demand for oil-gas, or subsidy for renewables s_i^ε that decreases it, accounting for all reallocation channels.

If we assume $\nu_i^c = \nu_i^r = 0$, the coal and renewable supply curves are perfectly elastic, and then, we obtain a simplified formula as a function of primitives:

$$
\begin{aligned}
d\ln e_i^f = & -\underbrace{(\frac{\sigma^y}{1-s_i^e}s_i^f+(1-s_i^f)\sigma^e)}_{=\beta^{e,q}_{f,f}}[d\ln q^f+\xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \underbrace{s_i^c(\sigma^e-\frac{\sigma^y}{1-s_i^e})}_{=\beta^{e,t}_{f,c}}\xi^c \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon - \underbrace{s_i^r\,(\sigma^e-\frac{\sigma^y}{1-s_i^e})}_{=\beta^{e,t}_{f,r}}\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
& + \underbrace{\frac{\sigma^y}{1-s^e}}_{=\beta^{e,d,f}}d\ln \mathcal{D}_i + \underbrace{\Big[\frac{\sigma^y}{1-s^e}+(\sigma^e-\frac{\sigma^y}{1-s_i^e})(s_i^c+s_i^r)\Big]}_{=\beta^{e,d,f}}d\ln \mathrm{p}_i
\end{aligned}
$$

Going back to the general case, we can rewrite output – substituting the price of coal and renewable – as a function of policies:

$$
\begin{aligned}
d\ln y_i &= \alpha^{y,z}d\ln z_i+\alpha^{y,p}d\ln \mathrm{p}_i-\alpha^{y,qf}d\ln q^f \\
&\quad -(\xi^f\alpha^{y,qf}+\xi^c\alpha^{y,qc})d\ln \mathrm{t}_i^\varepsilon+\alpha^{y,qr}d\ln \mathrm{s}_i^\varepsilon-\alpha^{y,qc}d\ln q_i^c-\alpha^{y,qr}d\ln q_i^r \\
dq_i^c &= -\beta^{q,t}_{c,c}\,\xi^c d\ln \mathrm{t}_i^\varepsilon-\beta^{q,t}_{c,r}\,d\ln \mathrm{s}_i^\varepsilon+\beta^{q,b,c}[-(\sigma^e-\frac{\sigma^y}{1-s_i^e})s_i^f[d\ln q^f+\xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon]+\frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i]+\beta^{q,p,c}d\ln \mathrm{p}_i \\
dq_i^r &= \beta^{q,t}_{r,c}\,\xi^c d\ln \mathrm{t}_i^\varepsilon+\beta^{q,t}_{r,r}\,d\ln \mathrm{s}_i^\varepsilon+\beta^{q,b,r}\Big[(\sigma^e-\frac{\sigma^y}{1-s_i^e})s_i^f[d\ln q^f+\xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon]+\frac{\sigma^y}{1-s^e}d\ln \mathcal{D}_i\Big]+\beta^{q,p,r}d\ln \mathrm{p}_i \\
\Rightarrow\ d\ln y_i &= \Big[\alpha^{y,z}-\frac{\sigma^y}{1-s^e}(\alpha^{y,qc}\beta^{q,b,c}+\alpha^{y,qr}\beta^{q,b,r})\Big]d\ln \mathcal{D}_i+\Big[\alpha^{y,p}-(\alpha^{y,qc}\beta^{q,p,c}+\alpha^{y,qr}\beta^{q,p,r})\Big]d\ln \mathrm{p}_i \\
&\quad +\Big[-\alpha^{y,qf}+(\alpha^{y,qc}\beta^{q,b,c}+\alpha^{y,qr}\beta^{q,b,r})(\sigma^e-\frac{\sigma^y}{1-s_i^e})s_i^f\Big]d\ln q^f \\
&\quad +\Big[-(\xi^f\alpha^{y,qf}+\xi^c\alpha^{y,qc})+(\alpha^{y,qc}\beta^{q,t}_{c,c}-\alpha^{y,qr}\beta^{q,t}_{r,c})\xi^c+(\alpha^{y,qc}\beta^{q,b,c}+\alpha^{y,qr}\beta^{q,b,r})(\sigma^e-\frac{\sigma^y}{1-s_i^e})s_i^f\xi^f\Big]d\ln \mathrm{t}_i^\varepsilon \\
&\quad +\Big[\alpha^{y,qr}+(\alpha^{y,qc}\beta^{q,t}_{c,r}-\alpha^{y,qr}\beta^{q,t}_{r,r})\Big]d\ln \mathrm{s}_i^\varepsilon
\end{aligned}
$$

this combines all the demand and supply substitution patterns arising in the three energy markets. To give an idea for the mechanism, take the parameter for the carbon tax t^ε, it shows different effects: (i) the direct impact of carbon taxation on the consumption of oil-gas and coal $(\xi^f\alpha^{y,qf}+\xi^c\alpha^{y,qc})$, (ii) the indirect impact of the decline in coal demand due to this tax $\xi^c\mathrm{t}^\varepsilon$ on respectively the price of coal q_i^c and the price of renewable q_i^r, hence change the input choices: $(\alpha^{y,qc}\beta^{q,t}_{c,c}-\alpha^{y,qr}\beta^{q,t}_{r,c})\xi^c$, and (iii) the indirect impact of the decline in *oil-gas* demand due to the tax $\xi^d\mathrm{t}^\varepsilon$ on the price of coal q_i^c and the price of renewable q_i^r respectively, i.e. $(\alpha^{y,qc}\beta^{q,b,c}+\alpha^{y,qr}\beta^{q,b,r})(\sigma^e-\frac{\sigma^y}{1-s_i^e})s_i^f\xi^f$, the price of energies c and r affecting output with magniture $\alpha^{y,qc}$ and $\alpha^{y,qr}$ respectively.

To save on notation, I compile these effects under the new parameters δ

$$
d\ln y_i=\delta^{y,z}d\ln \mathcal{D}_i+\delta^{y,p}d\ln \mathrm{p}_i-\delta^{y,qf}d\ln q^f-\delta^{y,t\varepsilon}d\ln \mathrm{t}_i^\varepsilon+\delta^{y,s\varepsilon}d\ln \mathrm{s}_i^\varepsilon
$$

Fossil energy market – demand

The energy demand in fossil is the sum of individual countries demand, where we denote the share of country i in global production $\lambda_i^f = \frac{\mathcal{P}_i e_i^f}{E^f}$

$$
\begin{aligned}
dE^f &= \sum_i \mathcal{P}_i de_i^f \\
d\ln E^f &= \sum_i \lambda_i^f d\ln e_i^f \\
&= -\sum_i \lambda_i^f (\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e)[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \lambda_i^f (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^c [d\ln q_i^c + \xi^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] \\
&\quad + \sum_i \lambda_i^f (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^r [d\ln q_i^r - \mathrm{J}_i d\ln \mathrm{s}^\varepsilon] + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathcal{D}_i + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathrm{p}_i
\end{aligned}
$$

We see that carbon taxation decreases demand for oil and gas by direct substitution but can also increase it if the substitution away for coal is strong enough. The first effect dominates the second – up to the first order – if:

$$
\overline{\lambda}_{\mathcal{J}}^{\sigma,f} := \sum_{i\in\mathcal{J}} \lambda_i^f (\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e)\ \xi^f > \sum_{i\in\mathcal{J}} \lambda_i^f (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^c \xi^c =: \overline{\lambda}_{\mathcal{J}}^{\sigma,c}
$$

which depend, among others, on the covariance $\mathbb{C}\mathrm{ov}_i(\lambda_i^f, 1-s_i^f)$ and $\mathbb{C}\mathrm{ov}_i(\lambda_i^f, s_i^c)$, since the substitution effect is stronger than the income effect $\sigma^e > \sigma^y/(1-s_i^e)$, in most empirically-relevant cases.

However, that simple condition only summarizes the direct effects. When we consider the indirect effects, accounting for the changes in prices for coal and renewable $d\ln q^c$ and $d\ln q^r$.

$$
\begin{aligned}
d\ln E^f &= \sum_i \lambda_i^f d\ln e_i^f \\
&= -\sum_i \lambda_i^f \beta_{f,f,i}^{e,q} [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon] + \sum_i \lambda_i^f \beta_{f,c,i}^{e,t}\ \xi^c \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon \\
&\quad + \sum_i \lambda_i^f \beta_{f,r,i}^{e,t}\ \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon + \sum_i \lambda_i^f \beta_i^{e,d,f}\ d\ln \mathcal{D}_i\ + \sum_i \lambda_i^f \beta_i^{e,d,f}\ d\ln \mathrm{p}_i
\end{aligned}
$$

The condition becomes:

$$
\begin{aligned}
\overline{\lambda}_{\mathcal{J}}^{\sigma,f} := \sum_{i\in\mathcal{J}} \lambda_i^f \Big\{ (\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e) + \Big[s_i^c \beta_i^{q,b,c} + s_i^r \beta_i^{q,b,r}\Big] (\sigma^e - \frac{\sigma^y}{1-s_i^e})^2 s_i^f \Big\}\ \xi^f \\
> \sum_{i\in\mathcal{J}} \lambda_i^f \Big\{ s_i^c - s_i^c \beta_{c,c}^{q,t} + s_i^r \beta_{r,c}^{q,t} \Big\} (\sigma^e - \frac{\sigma^y}{1-s_i^e})\ \xi^c =: \overline{\lambda}_{\mathcal{J}}^{\sigma,c}
\end{aligned}
$$

We obtain that, if $\overline{\lambda}_{\mathcal{J}}^{\sigma,f} > \overline{\lambda}_{\mathcal{J}}^{\sigma,c}$, the direct effect of a carbon tax $\xi^f \mathrm{t}_i^\varepsilon$ on oil-gas outweighs the reallocation from coal to oil-gas due to the larger increase $\xi^c \mathrm{t}_i^\varepsilon$.

Fossil energy market – supply

Now, the energy supply curve can also be recast as the sum of individual extraction $E^f = \sum_i \mathcal{P}_i e_i^f = \sum_i \mathcal{P}_i e_i^x$, and, with the share of fossil production $\lambda_i^x = \mathcal{P}_i e_i^x / E^f$, it hence derives as follow:

$$e_i^x = (q^f)^{1/\nu_i} \mathcal{R}_i \nu_i^{-1/\nu_i} \mathrm{p}_i^{-1/\nu_i}$$
$$d\ln E^f = \sum_i \lambda_i^x d\ln e_i^x = \sum_i \lambda_i^x \frac{1}{\nu_i}[d\ln q^f - d\ln \mathrm{p}_i]$$
$$\Rightarrow \qquad d\ln q^f = \bar{\nu} d\ln E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i$$

with the aggregate supply elasticity $\bar{\nu} = \left(\sum_i \lambda_i^x \nu_i^{-1}\right)^{-1}$, that we already encountered in the second best optimal Ramsey policy.

Now, replacing the energy demand quantity $d\ln E^f$ into the energy supply/price curve, we obtain:

$$\begin{aligned}
d\ln q^f &= \bar{\nu} d\ln E^f + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i \\
&= -\bar{\nu}\overline{\lambda}^{\sigma,f} d\ln q^f + \bar{\nu} \sum_i \lambda_i^{\sigma,f}\, \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon + \bar{\nu} \sum_i \lambda_i^f (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^c [d\ln q_i^c + \xi^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] \\
&\qquad + \bar{\nu} \sum_i \lambda_i^f (\sigma^e - \frac{\sigma^y}{1-s_i^e}) s_i^r [d\ln q_i^r - d\ln \mathrm{s}_i^\varepsilon] + \bar{\nu} \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} [d\ln \mathcal{D}_i + d\ln \mathrm{p}_i] + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i
\end{aligned}$$

$$\begin{aligned}
d\ln q^f &= -\bar{\nu} \underbrace{\sum_i \lambda_i^f \beta_{f,f,i}^{e,q}}_{=\overline{\lambda}^{\sigma,f}} [d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \bar{\nu} \sum_i \lambda_i^f \beta_{f,c,i}^{e,t}\, \xi^c \mathrm{J}_i d\ln \mathrm{t}_i^\varepsilon \\
&\qquad + \bar{\nu} \sum_i \lambda_i^f \beta_{f,r,i}^{e,t}\, \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon + \bar{\nu} \sum_i \lambda_i^f \beta_i^{e,d,f}\, d\ln \mathcal{D}_i\ + \bar{\nu} \sum_i \lambda_i^f \beta_i^{e,d,f}\, d\ln \mathrm{p}_i + \sum_i \lambda_i^x \frac{\bar{\nu}}{\nu_i} d\ln \mathrm{p}_i
\end{aligned}$$

$$\begin{aligned}
d\ln q^f &= \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \mathrm{J}_i [-\xi^f \beta_{f,f,i}^{e,q} + \xi^c \beta_{f,c,i}^{e,t}] d\ln \mathrm{t}^\varepsilon + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f\, \beta_{f,r,i}^{e,t}\, \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
&\qquad + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \beta_i^{e,d,f}\, d\ln \mathcal{D}_i\ + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i [\lambda_i^f \beta_i^{e,p,f}\ + \lambda_i^x \frac{\bar{\nu}}{\nu_i}] d\ln \mathrm{p}_i
\end{aligned}$$

where $\overline{\lambda}^{\sigma,f} = \overline{\lambda}_{\mathbb{I}}^{\sigma,f}$, for $\mathbb{I}$ the whole world. As before, we see that carbon taxation decreases the oil-gas energy price if $\overline{\lambda}_{\mathcal{J}}^{\sigma,f} > \overline{\lambda}_{\mathcal{J}}^{\sigma,c}$. Moreover, we see that a change in the good price $d\ln \mathrm{p}_i$ of all the countries change the aggregate price of oil and gas because it both increases the price of renewable and coal, increases demand for oil-gas by substitutions – the terms $\bar{\nu}\lambda_i^{\sigma,c}$ and $\bar{\nu}\lambda_i^{\sigma,r}$ – and it also increases the price of the input – through the term $\lambda_i^x \frac{\bar{\nu}}{\nu_i}$.

If we assume that again $\nu^c = \nu^r = 0$, this implies:

$$\begin{aligned}
d\ln q^f &= \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \mathrm{J}_i \Big[-\xi^f (\frac{\sigma^y}{1-s_i^e} s_i^f + (1-s_i^f)\sigma^e) + \xi^c s_i^c (\sigma^e - \frac{\sigma^y}{1-s_i^e})\Big] d\ln \mathrm{t}^\varepsilon + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f\, s_i^r\, (\sigma^e - \frac{\sigma^y}{1-s_i^e})\, \mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
&\qquad + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \lambda_i^f \frac{\sigma^y}{1-s^e} d\ln \mathcal{D}_i\ + \frac{\bar{\nu}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}} \sum_i \Big[\lambda_i^f \Big(\frac{\sigma^y}{1-s^e} + (\sigma^e - \frac{\sigma^y}{1-s_i^e})(s_i^c + s_i^r)\Big) + \lambda_i^x \frac{\bar{\nu}}{\nu_i}\Big] d\ln \mathrm{p}_i
\end{aligned}$$

Similalry, we can write total energy demand as:

$$d\ln E^f = \frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f \mathrm{J}_i\Big[-\xi^f(\frac{\sigma^y}{1-s_i^e}s_i^f+(1-s_i^f)\sigma^e)+\xi^c s_i^c(\sigma^e-\frac{\sigma^y}{1-s_i^e})\Big]d\ln \mathrm{t}^\varepsilon + \frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f\, s_i^r\,(\sigma^e-\frac{\sigma^y}{1-s_i^e})\;\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon$$
$$+\frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f\frac{\sigma^y}{1-s^e}\,d\ln\mathcal{D}_i\;+\frac{1}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i\Big[\lambda_i^f\Big(\frac{\sigma^y}{1-s^e}+(\sigma^e-\frac{\sigma^y}{1-s_i^e})(s_i^c+s_i^r)\Big)\Big]d\ln \mathrm{p}_i$$

B.8 Trade *à la* Armington

To investigate how the price indices $\mathbb{P}_i$ and the good price p_i are determined, we should now consider the market for goods.

$$\mathcal{P}_i\mathrm{p}_i y_i = \sum_{k\in\mathbb{I}}\mathcal{P}_k s_{ki}\frac{v_k}{1+\mathrm{t}_{ki}}$$

Using the CES framework, we obtain that:

$$\mathbb{P}_i = \Big(\sum_j a_{ij}(\tau_{ij}(1+\mathrm{t}_{ij}^b)\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$
$$\frac{d\mathbb{P}_i}{\mathbb{P}_i} = \sum_j s_{ij}(\frac{d\mathrm{p}_j}{\mathrm{p}_j}+\frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b})$$
$$s_{ij} = \frac{c_{ij}(1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j}{\sum_k c_{ik}(1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k} = a_{ij}\frac{\big((1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j\big)^{1-\theta}}{\sum_k\big((1+\mathrm{t}_{ik})\tau_{ik}\mathrm{p}_k\big)^{1-\theta}} = \Big(\frac{(1+\mathrm{t}_{ij})\tau_{ij}\mathrm{p}_j}{\mathbb{P}_i}\Big)^{1-\theta}$$
$$\frac{ds_{ij}}{s_{ij}} = (\theta-1)\Big(\frac{d\mathbb{P}_i}{\mathbb{P}_i}-(\frac{d\mathrm{p}_j}{\mathrm{p}_j}+\frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b})\Big)$$
$$\frac{ds_{ij}}{s_{ij}} = (\theta-1)\Big(\sum_k s_{ik}(\frac{d\mathrm{p}_k}{\mathrm{p}_k}+\frac{d\mathrm{t}_{ik}^b}{1+\mathrm{t}_{ik}^b})-(\frac{d\mathrm{p}_j}{\mathrm{p}_j}+\frac{d\mathrm{t}_{ij}^b}{1+\mathrm{t}_{ij}^b})\Big)$$

Using those formulas, the market clearing linearizes as follows:

$$\frac{d[\frac{s_{ij}v_i}{1+t_{ij}}]}{\frac{s_{ij}v_i}{1+\mathrm{t}_{ij}}} = \Big[d\ln v_i+\theta\sum_k\big(s_{ik}d\ln\mathrm{t}_{ik}-(1+s_{ij})d\ln\mathrm{t}_{ij}\big)+(\theta-1)\sum_{k\neq j}\big(s_{ik}d\ln\mathrm{p}_k-d\ln\mathrm{p}_j\big)\Big]$$
$$\text{with}\qquad v_i = \mathrm{p}_i y_i+q^f(e_i^x-e_i^f)$$

This implies:

$$\mathcal{P}_i\widetilde{v}_i(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}) = \sum_k\mathcal{P}_k\frac{s_{ki}v_k}{1+\mathrm{t}_{ki}}d\ln[\frac{s_{ki}v_k}{1+t_{ki}}]$$
$$(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}) = \sum_k\frac{\mathcal{P}_k v_k}{\mathcal{P}_i v_i}s_{ki}\Big[d\ln v_k+\theta\sum_h\big(s_{kh}d\ln\mathrm{t}_{kh}-(1+s_{ki})d\ln\mathrm{t}_{ki}\big)+(\theta-1)\sum_h\big(s_{kh}d\ln\mathrm{p}_h-d\ln\mathrm{p}_i\big)\Big]$$
$$(\frac{d\mathrm{p}_i}{d\mathrm{p}_i}+\frac{dy_i}{y_i}) = \sum_k\boldsymbol{t}_{ik}\Big[(\frac{\mathrm{p}_k y_k}{v_k})(d\ln\mathrm{p}_k+d\ln y_k)+\frac{q^f e_k^x}{v_k}d\ln e_k^x-\frac{q^f e_k^f}{v_k}d\ln e_k^f+\frac{q^f(e_k^x-e_k^f)}{v_k}d\ln q^f$$
$$+\theta\sum_h\big(s_{kh}d\ln\mathrm{t}_{kh}-(1+s_{ki})d\ln\mathrm{t}_{ki}\big)+(\theta-1)\sum_h\big(s_{kh}d\ln\mathrm{p}_h-d\ln\mathrm{p}_i\big)\Big]$$

with $\boldsymbol{t}_{ik} = \frac{\mathcal{P}_k v_k}{\mathcal{P}_i v_i} s_{ki}$, which is analogous to the same matrix in Kleinman et al. (2024). Using the fact that $\sum_k \boldsymbol{t}_{ik} = 1$ we factorize the p_i. This implies, rewritten in matrix notation:

$$
\begin{aligned}
(\theta d\ln \mathrm{p}_i + d\ln y_i) &= \sum_k \boldsymbol{t}_{ik}\Big[(\frac{\mathrm{p}_k y_k}{v_k})(d\ln \mathrm{p}_k + d\ln y_k) + \frac{q^f e^x_k}{v_k} d\ln e^x_k - \frac{q^f e^f_k}{v_k} d\ln e^f_k + \frac{q^f(e^x_k - e^f_k)}{v_k} d\ln q^f\Big] \\
&\quad + \theta \sum_k \boldsymbol{t}_{ik} \sum_h \left(s_{kh} d\ln \mathrm{t}_{kh} - (1{+}s_{ki}) d\ln \mathrm{t}_{ki}\right) + (\theta{-}1) \sum_k \boldsymbol{t}_{ik} \sum_h s_{kh} d\ln \mathrm{p}_h \\
\theta d\ln \mathrm{p} + d\ln y &= \mathbf{T} v^y [d\ln \mathrm{p} + d\ln y] + \mathbf{T} v^{e^x} d\ln e^x - \mathbf{T} v^{e^f} d\ln e^f + \mathbf{T} v^{ne} d\ln q^f + (\theta{-}1)\mathbf{TS} d\ln \mathrm{p} \\
&\quad + \theta\big(\mathbf{T}(\mathbf{S}{\odot}\mathbf{J}{\odot}d\mathrm{t}^b)\mathbb{1} - \mathrm{diag}[\mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}d\mathrm{t}^b)']\big) \\
\Big[\theta\mathbf{I}{-}\mathbf{T}{\odot}v^y - (\theta{-}1)(\mathbf{TS})\Big] d\ln \mathrm{p} &= (\mathbf{T}{\odot}v^y{-}\mathbf{I}) d\ln y + \mathbf{T} v^{e^x} d\ln e^x_k - \mathbf{T} v^{e^f} d\ln e^f_k + \mathbf{T} v^{ne} d\ln q^f \\
&\quad + \theta\big(\mathbf{T}(\mathbf{S}{\odot}\mathbf{J}{\odot}d\mathrm{t}^b)\mathbb{1} - \mathrm{diag}[\mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}d\mathrm{t}^b)']\big)
\end{aligned}
$$

with $v^y = \frac{\mathrm{p}_i y_i}{v_i}$, $v^{e^x} = \frac{q^f e^x_i}{v_i}$, $v^{e^f} = \frac{q^f e^f_i}{v_i}$ and $v^{ne} = \frac{q^f(e^x_i - e^f_i)}{v_k}$.

Recall fossil demand and supply and production of goods y_i,

$$
\begin{aligned}
d\ln e^f_i &= -\gamma^{e,q}_{f,f,i}\,[d\ln q^f + \xi^f \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \gamma^{e,t}_{f,c,i}\,\xi^c \mathrm{J}_i d\ln \mathrm{t}^\varepsilon_i + \gamma^{e,t}_{f,r,i}\,\mathrm{J}_i d\ln \mathrm{s}^\varepsilon_i + \gamma^{e,d,f}_i\, d\ln \mathcal{D}_i\ + \gamma^{e,d,f}_i\, d\ln \mathrm{p}_i \\
d\ln e^x_i &= \frac{1}{\nu_i}[d\ln q^f - d\ln \mathrm{p}_i] \\
d\ln y_i &= \alpha^{y,z} d\ln z_i + \alpha^{y,p} d\ln \mathrm{p}_i - \alpha^{y,qf} d\ln q^f - (\xi^f \alpha^{y,qf} {+} \xi^c \alpha^{y,qc}) d\ln \mathrm{t}^\varepsilon_i - (\alpha^{y,qc} {+} \alpha^{y,qr}) d\ln \mathrm{p}_i
\end{aligned}
$$

We can replace that in the general equilibrium for prices:

$$
\begin{aligned}
\Big[\theta\mathbf{I}{-}\mathbf{T}{\odot}v^y - (\theta{-}1)(\mathbf{TS})\Big] d\ln \mathrm{p} &= (\mathbf{T} v^y - \mathbb{I}) d\ln y + \mathbf{T} v^{e^x} \frac{1}{\nu^f}[d\ln q^f - d\ln \mathrm{p}] \\
&\quad - \mathbf{T} v^{e^f}\big[- \beta^{e,q}_{f,f}\, d\ln q^f + \big(- \beta^{e,q}_{f,f}\xi^f + \beta^{e,t}_{f,c}\ \xi^c\big) \mathrm{J} d\ln \mathrm{t}^\varepsilon - \beta^{e,t}_{f,r}\ \mathrm{J} d\ln \mathrm{s}^\varepsilon + \beta^{e,d,f}\, d\ln \mathcal{D}^y\ + \\
&\quad + \mathbf{T} v^{ne} d\ln q^f
\end{aligned}
$$

Rewriting and rearranging:

$$
\begin{aligned}
\Big[\theta\mathbf{I}{-}\mathbf{T}{\odot}v^y - (\theta{-}1)(\mathbf{TS}){+}\mathbf{T}{\odot}(v^{e^f}{\odot}\beta^{e,d,f}{+}v^{e^x}{\odot}\tfrac{1}{\nu^f})\Big] d\ln \mathrm{p} &= (\mathbf{T}{\odot}v^y{-}\mathbf{I}) d\ln y \\
&\quad + \mathbf{T}{\odot}[v^{e^x}\frac{1}{\nu^f} - v^{e^f}\beta^{e,q}_{f,f}\ + v^{ne}] d\ln q^f - \mathbf{T} v^{e^f} \beta^{e,d,f}\, d\ln \mathcal{D}^y \\
&\quad - \mathbf{T}{\odot}v^{e^x}\,[(- \beta^{e,q}_{f,f}\xi^f + \beta^{e,t}_{f,c}\ \xi^c) \mathrm{J} d\ln \mathrm{t}^\varepsilon + \beta^{e,t}_{f,r}\ \mathrm{J} d\ln \mathrm{s}^\varepsilon] \\
&\quad + \theta\big(\mathbf{T}(\mathbf{S}{\odot}\mathbf{J}{\odot}d\mathrm{t}^b)\mathbb{1} - \mathrm{diag}[\mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}d\mathrm{t}^b)']\big)
\end{aligned}
$$

Replacing output y

$$\Big[\theta\mathbf{I}{-}\mathbf{T}{\odot}v^{y} - (\theta{-}1)(\mathbf{TS}){+}\mathbf{T}{\odot}(v^{e^{f}}{\odot}\beta^{e,d,f}{+}v^{e^{x}}{\odot}\tfrac{1}{\nu^{f}})\Big]d\ln \mathrm{p} =$$
$$(\mathbf{T}{\odot}v^{y}{-}\mathbf{I})\Big[\,\delta^{y,z}d\ln\mathcal{D}^{y}_{i} + \delta^{y,p}d\ln \mathrm{p}_{i} - \delta^{y,qf}d\ln q^{f} - \delta^{y,t\varepsilon}d\ln \mathrm{t}^{\varepsilon}_{i} + \delta^{y,s\varepsilon}d\ln \mathrm{s}^{\varepsilon}_{i}\Big]$$
$$+\,\mathbf{T}{\odot}[v^{e^{x}}\frac{1}{\nu^{f}} - v^{e^{f}}\beta^{e,q}_{f,f} + v^{ne}]d\ln q^{f} - \mathbf{T}v^{e^{f}}\beta^{e,d,f}\,d\ln\mathcal{D}^{y}$$
$$-\,\mathbf{T}{\odot}v^{e^{f}}[(- \beta^{e,q}_{f,f}\xi^{f} + \beta^{e,t}_{f,c}\ \xi^{c})\mathrm{J}d\mathrm{t}^{\varepsilon} + \beta^{e,t}_{f,r}\ \mathrm{J}d\mathrm{s}^{\varepsilon}]$$
$$+\,\theta\big(\mathbf{T}(\mathbf{S}{\odot}\mathbf{J}{\odot}dt^{b})\mathbb{1} - \mathrm{diag}[\mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}dt^{b})']\big)$$

As a result, the general equilibrium effects on the goods markets yield the following change in prices:

$$\begin{aligned}\Big[(\theta + \delta^{y,p})\mathbf{I}{-}\mathbf{T}{\odot}v^{y}(1{+}\delta^{y,p}) - (\theta{-}1)(\mathbf{TS}) + \mathbf{T}{\odot}(v^{e^{f}}{\odot}\beta^{e,d,f}{+}v^{e^{x}}{\odot}\tfrac{1}{\nu^{f}})\Big]d\ln \mathrm{p} &= \Big[\,(\mathbf{T}{\odot}v^{y}{-}\mathbf{I})\ \delta^{y,z} - \mathbf{T}v^{e^{f}}\beta^{e,d,f}\Big]d\ln\mathcal{D}^{y}_{i}\\ &+ \Big[-(\mathbf{T}{\odot}v^{y}{-}\mathbf{I})\delta^{y,qf} + \mathbf{T}{\odot}(v^{e^{x}}\frac{1}{\nu^{f}} - v^{e^{f}}\beta^{e,q}_{f,f} + v^{ne})\Big]d\ln q^{f}\\ &+ \Big[-\mathbf{T}{\odot}v^{e^{f}}\big((- \beta^{e,q}_{f,f}\xi^{f} + \beta^{e,t}_{f,c}\ \xi^{c})\big) - (\mathbf{T}{\odot}v^{y}{-}\mathbf{I})\delta^{y,t\varepsilon}\Big]\mathrm{J}d\mathrm{t}^{\varepsilon}\\ &+ \Big[\mathbf{T}{\odot}v^{e^{f}}\beta^{e,t}_{f,r} + (\mathbf{I}{-}\mathbf{T}{\odot}v^{y})\delta^{y,s\varepsilon}\Big]\mathrm{J}ds^{\varepsilon}\\ &+ \theta\big(\mathbf{T}(\mathbf{S}{\odot}\mathbf{J}{\odot}dt^{b})\mathbb{1} - \mathrm{diag}[\mathbf{T}(\mathbb{1}{+}\mathbf{S}'){\odot}(\mathbf{J}{\odot}dt^{b})']\big)\end{aligned} \tag{28}$$

Again, this compile all the different channels of transmissions that arise in our model. For example, taking the parameter for the carbon tax t^{ε} we see that it changes the price in multiple ways: (i) first, it lowers the oil-gas expenditure for country k, by a factor $\beta^{e,q}_{f,f}\xi^{f}$, (ii) however, it increases the oil-gas bill as a substitution away from coal $\beta^{e,t}_{f,c}\ \xi^{c}$. In addition, (iii) taxing carbon reduces output by a factor $\delta^{y,t\varepsilon}$, which then reduces the revenues in net by a factor $(\mathbf{T}{\odot}v^{y}{-}\mathbf{I})$.

B.9 Back to welfare and climate damage

From the budget/consumption expenditure, we saw that welfare is written as:

$$\frac{d\mathcal{U}_{i}}{u'(c_{i})c_{i}} = (d\ln c_{i} + d\ln\mathcal{D}^{u}_{i})$$
$$\frac{d\mathcal{U}_{i}}{u'(c_{i})c_{i}} = \Big[\frac{\eta^{y}_{i}}{\eta^{c}_{i}} - \sum_{\ell}\frac{1}{\nu^{\ell}_{i}}\frac{\eta^{\pi\ell}_{i}}{\eta^{c}_{i}}\Big]\frac{d\mathrm{p}_{i}}{\mathrm{p}_{i}} + \frac{\eta^{y}_{i}}{\eta^{c}_{i}}d\ln\mathcal{D}^{y}_{i} + \sum_{\ell}\Big[\frac{\eta^{\pi\ell}_{i}}{\eta^{c}_{i}}\Big(1{+}\frac{1}{\nu^{\ell}_{i}}\Big) - \frac{\eta^{y}_{i}}{\eta^{c}_{i}}s^{e}_{i}s^{\ell}_{i}\Big]\frac{dq^{\ell}}{q^{\ell}} - \frac{d\mathbb{P}_{i}}{\mathbb{P}_{i}} + \frac{d\widetilde{\mathrm{t}}^{ls}_{i}}{x_{i}} + d\ln\mathcal{D}^{u}_{i}$$

with the damage

$$d\ln\mathcal{D}^{y}_{i} = -\bar{\gamma}^{y}_{i}\big(s^{f/E}d\ln E^{f} + s^{c/E}d\ln E^{c}\big)$$

and similarly with $\bar{\gamma}^{u}_{i}$ for $\mathcal{D}^{u}_{i}$

The oil-gas energy price is central for summarizing all the general equilibrium forces for fossil-fuel demand. As a result, since the aggregate supply curve for oil and gas is upward sloping, a higher price implies a higher demand, and hence higher quantity consumed and greenhouse gas emitted.

$$
\begin{aligned}
d\ln q^f &= \frac{\bar{\nu}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f \mathrm{J}_i\,[-\xi^f\gamma^{e,q}_{f,f,i}+\xi^c\gamma^{e,t}_{f,c,i}]d\ln \mathrm{t}^\varepsilon + \frac{\bar{\nu}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f\;\gamma^{e,t}_{f,r,i}\;\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
&\quad + \frac{\bar{\nu}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f\gamma_i^{e,d,f}\,d\ln \mathcal{D}_i \;+ \frac{\bar{\nu}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i [\lambda_i^f\gamma_i^{e,p,f}+\lambda_i^x\frac{\bar{\nu}}{\nu_i}]d\ln \mathrm{p}_i \\
d\ln E^f &= \frac{1}{\bar{\nu}}\big[d\ln q^f - \sum_i \lambda_i^x\frac{\bar{\nu}}{\nu_i}d\ln \mathrm{p}_i\big]
\end{aligned}
$$

Similarly, for total coal consumption, we can aggregate, with weights $\lambda_i^c = \frac{\mathcal{P}_i e_i^c}{E_i^c}$

$$
\begin{aligned}
d\ln E^c = \sum_i \lambda_i^c d\ln e_i^c &= \sum_i \lambda_i^c\big[-\beta^{e,t}_{c,c,i}\;\xi^c+\beta_i^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1{-}s_i^e})s_i^f\xi^f\big]\mathrm{J}_i d\ln \mathrm{t}^\varepsilon - \sum_i \lambda_i^c\beta^{e,t}_{c,r,i}\;\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon + \sum_i \beta_i^{e,p,c}d\ln \mathrm{p}_i \\
&\quad + \sum_i \lambda_i^c\beta_i^{e,b,c}\Big[(\sigma^e{-}\frac{\sigma^y}{1{-}s_i^e})s_i^f d\ln q^f \;+\frac{\sigma^y}{1{-}s^e}d\ln \mathcal{D}_i\Big]
\end{aligned}
$$

As a result, we can write damage depending on a complicated combination of supply and demand effects:

$$
\begin{aligned}
d\ln \mathcal{D}_i^y &= -\bar{\gamma}_i^y\big(s^{f/E}d\ln E^f + s^{c/E}d\ln E^c\big) \\
&= -\bar{\gamma}_i^y\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \lambda_i^f\,[-\xi^f\gamma^{e,q}_{f,f,i}+\xi^c\gamma^{e,t}_{f,c,i}]\mathrm{J}_i + s^{c/E}\sum_i \lambda_i^c\big[-\beta^{e,t}_{c,c,i}\;\xi^c+\beta_i^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1{-}s_i^e})s_i^f\xi^f\big]\mathrm{J}_i\Big\}d\ln \mathrm{t}^\varepsilon \\
&\quad -\bar{\gamma}_i^y\sum_i\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\lambda_i^f\;\gamma^{e,t}_{f,r,i}\;+s^{c/E}\lambda_i^c\beta^{e,t}_{c,r,i}\Big\}\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
&\quad -\bar{\gamma}_i^y\sum_i\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\lambda_i^f\gamma_i^{e,d,f}\;+\;s^{c/E}\lambda_i^c\beta_i^{e,b,c}\frac{\sigma^y}{1{-}s^e}\Big\}d\ln \mathcal{D}_i^y\;+s^{c/E}\sum_i \lambda_i^c\beta_i^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1{-}s_i^e})s_i^f d\ln q^f \\
&\quad -\bar{\gamma}_i^y\sum_i\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}[\lambda_i^f\gamma_i^{e,p,f}\;-\bar{\nu}\overline{\lambda}^{\sigma,f}\lambda_i^x\frac{\bar{\nu}}{\nu_i}]+s^{c/E}\lambda_i^c\beta_i^{e,p,c}\Big\}d\ln \mathrm{p}_i
\end{aligned}
$$

with $\widetilde{\lambda}_i^f = \lambda_i^f + \lambda_i^c\beta_i^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1-s_i^e})s_i^f\,\bar{\nu}\,\frac{s^{c/E}}{s^{f/E}}$, we can rewrite without the price of oil-gas q^f

$$
\begin{aligned}
d\ln \mathcal{D}_i^y &= -\bar{\gamma}_i^y\big(s^{f/E}d\ln E^f + s^{c/E}d\ln E^c\big) \\
\Big[1+\bar{\gamma}_i^y\sum_i\Big\{\tfrac{s^{f/E}}{1+\bar{\nu}\overline{\lambda}^{\sigma,f}}\widetilde{\lambda}_i^f\gamma_i^{e,d,f}&+s^{c/E}\lambda_i^c\beta_i^{e,b,c}\tfrac{\sigma^y}{1-s^e}\Big\}\Big]d\ln \mathcal{D}_i^y = \\
&-\bar{\gamma}_i^y\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\sum_i \widetilde{\lambda}_i^f\,[-\xi^f\gamma^{e,q}_{f,f,i}+\xi^c\gamma^{e,t}_{f,c,i}]\mathrm{J}_i + s^{c/E}\sum_i \lambda_i^c\big[-\beta^{e,t}_{c,c,i}\;\xi^c+\beta_i^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1{-}s_i^e})s_i^f\xi^f\big]\mathrm{J}_i\Big\}d\ln \mathrm{t}^\varepsilon \\
&-\bar{\gamma}_i^y\sum_i\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}\widetilde{\lambda}_i^f\;\gamma^{e,t}_{f,r,i}\;+s^{c/E}\lambda_i^c\beta^{e,t}_{c,r,i}\Big\}\mathrm{J}_i d\ln \mathrm{s}_i^\varepsilon \\
&-\bar{\gamma}_i^y\sum_i\Big\{\frac{s^{f/E}}{1{+}\bar{\nu}\overline{\lambda}^{\sigma,f}}[\lambda_i^f\gamma_i^{e,p,f}\;-\bar{\nu}\overline{\lambda}^{\sigma,f}\lambda_i^x\frac{\bar{\nu}}{\nu_i}]+s^{c/E}\Big(\lambda_i^c\beta_i^{e,p,c}+\big(\sum_k \beta_k^{e,b,c}(\sigma^e{-}\frac{\sigma^y}{1{-}s_k^e})s_k^f\big)\lambda_i^x\frac{\bar{\nu}}{\nu_i}\Big)\Big\}d\ln \mathrm{p}_i
\end{aligned}
$$

Damages change with the aggregate consumption of oil, gas, and coal, which each depends on various general equilibrium effects. The carbon tax and renewable subsidies create substitution effects away from fossil and toward renewable. Moreover, the price level p_i increases the terms of trade for countries that consume oil-gas, and reduces production for exporters, which then affects the equilibrium prices of oil. All these effects are accounted for in this formula, which then enter in the welfare calibration above.

B.10 Further simplification

To simplify the welfare formula even further, in the following we consider that energy is only composed of oil-gas. In practice, oil and gas compose the largest share of energy, with oil representing close to 35% of energy use and natural gas close to 20% at the world level.

We consider that $s_i^f = 1$ and $s_i^r = s_i^c = 0$ in all the formulas above.

This assumption simplify our setting dramatically. The previous welfare decomposition reduces to :

$$\frac{d\mathcal{U}_i}{u'(c_i)c_i} = \frac{dc_i}{c_i} = \Big[\frac{\eta_i^y}{\eta_i^c} - \frac{\eta_i^\pi}{\eta_i^c}\frac{1}{\nu_i}\Big]\frac{d\mathrm{p}_i}{\mathrm{p}_i} + \frac{\eta_i^y}{\eta_i^c}\frac{d\mathcal{D}_i}{\mathcal{D}_i} - \frac{\eta_i^y}{\eta_i^c}s_i^e\frac{dq^f}{q^f} + \frac{\eta_i^\pi}{\eta_i^c}\Big(1+\frac{1}{\nu}\Big)\frac{dq^f}{q^f} - \frac{d\mathbb{P}_i}{\mathbb{P}_i} + \frac{d\widetilde{\mathrm{t}}_i^{ls}}{x_i}$$

where the damage rewrite:

$$d\ln\mathcal{D}_i = -\bar{\gamma}_i d\ln E^f$$

with the average damage is defined as $\bar{\gamma} = \sum_i \bar{\gamma}_i$. And the oil-gas demand curve write:

$$\begin{aligned}
d\ln E^f &= \sum_i \lambda_i^f d\ln e_i^f \\
&= -\sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e}[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln\mathcal{D}_i + \sum_i \lambda_i^f \frac{\sigma^y}{1-s_i^e} d\ln \mathrm{p}_i \\
&= -\sum_i \widetilde{\lambda}_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \widetilde{\lambda}_i^f d\ln\mathcal{D}_i + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i
\end{aligned}$$

where, to simplify notations, we denote $\widetilde{\lambda}_i^f = \lambda_i^f \frac{\sigma^y}{1-s_i^e}$, and it's average $\overline{\lambda}^{\sigma,f} = \sum_i \widetilde{\lambda}_i^f \frac{\sigma^y}{1-s_i^e}$. As a result, the demand now rewrites:

$$d\ln E^f = \frac{1}{1+\bar{\gamma}+\mathbb{C}\mathrm{ov}_i(\widetilde{\lambda}_i^f, \bar{\gamma}_i)}\Big[-\sum_i \widetilde{\lambda}_i^f[d\ln q^f + \mathrm{J}_i d\ln \mathrm{t}^\varepsilon] + \sum_i \widetilde{\lambda}_i^f d\ln \mathrm{p}_i\Big]$$

We can see that the energy demand curve is affected by climate change: more emission imply larger damage, which in turn reduce energy demand and hence emissions. Moreover, the covariance term indicates that if the large energy producers (with a larger share of the market, and high elasticity σ) are also the most affected by climate change, this effect is stronger and the demand curve is even steeper / more inelastic.