Climate Change, Inequality, and the Optimal Climate Policy

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November 2026

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- However, relatively little is known on how (and whether!) climate policy should depend on inequality across countries.
- Indeed, energy taxation and climate policy have strong distributional consequences:
  - (i) between higher income countries and developing economies
  - (ii) between warm and cold countries, and differential exposure to climate change
  - (iii) between energy importer and exporters, and dependence on energy consumption and rents

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$\Rightarrow$ What is the optimal carbon policy in the presence of climate externality and inequality?

  - The optimal design of Pigouvian taxation crucially depends on redistribution instruments i.e. lump-sum transfers across countries

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- What is the optimal carbon policy with inequality?
- Study an Integrated Assessment Model (IAM) with heterogeneous countries to:
  - Evaluate the welfare costs of global warming and the Social Cost of Carbon
  - Solve for general *optimal Ramsey policies* for carbon taxation
  - Provide a novel numerical methodology for IAMs with *heterogeneity*

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- Preview of the results:
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  $\Rightarrow$ ***Theoretically:*** effect *ambiguous*. ***Quantitative:*** carbon tax should be *lower* with inequality
  - Country-specific taxes: poorer countries should pay relatively lower taxes

Roadmap

- Simplified set-up: Toy example
- Quantitative model:
  - Set-up: Dynamic IAMs with heterogeneity
  - Quantification
- First-best carbon taxation: all instruments available
- Second-best carbon taxation: Constrained efficient without lump-sum transfers
  - Uniform carbon taxation
  - Heterogeneous carbon prices across countries
- Extensions
- Optimal Climate Agreements

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Quantitative Model – Household & Firms

Static deterministic Neoclassical economy

- countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature T_i, energy extraction cost C_i
- In each country, five agents:

(i) Rep. household, (ii) final good firm, (iii-v) oil-gas (fossil), coal and renewable energy producers

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1. Representative household problem in i:

$$\mathcal{V}_{it_0} = \max_{\{c_{it}\}_t} \int_{t_0} e^{-\rho t} u(c_{it}) dt \qquad \dot{a}_{it} = \underbrace{w_{it}\ell_i}_{\substack{\text{labor} \\ \text{income}}} + \underbrace{r^k_{it} a_i}_{\substack{\text{capital} \\ \text{income}}} + \underbrace{\pi^f_i}_{\substack{\text{fossil firm} \\ \text{profit}}} + \underbrace{\mathrm{t}^{ls}_i}_{\substack{\text{lump-sum} \\ \text{transfers}}}$$

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2. Competitive final good producer in country i

$$\max_{\{e^f_{it}, e^c_{it}, e^r_{it}, k_{it}\}} \underbrace{\mathcal{D}(T_i)}_{\substack{\text{climate}\\ \text{damage}}} \underbrace{z_i}_{TFP} f(k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) - w_{it}\ell_i - r^k_{it}k_{it} - \underbrace{(q^f_t + \mathrm{t}^{\varepsilon}_{it}) e^f_{it}}_{\substack{\text{fossil}\\ \text{(oil-gas)}}} - \underbrace{(q^c_{it} + \mathrm{t}^{\varepsilon}_{it}) e^c_{it}}_{\text{coal}} - \underbrace{q^r_{it} e^r_{it}}_{\substack{\text{non-carbon}\\ \text{energy}}}$$

- Climate policy: Carbon tax $\mathrm{t}^{\varepsilon}_i$

Model – Energy markets & Emissions

3. Competitive fossil fuels energy producer, selling on international fossil market:
   - Supply of fossil energy e^x_{it} by extraction at cost $\mathcal{C}^f_{it}$

$$\pi^f_{it} = \max_{e^x_{it}} q^f_t e^x_{it} - \mathcal{C}^f_i(e^x_{it}, \mathcal{R}_{it}) \qquad \dot{\mathcal{R}}_{it} = -e^x_{it}$$

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4. Coal energy firm: elastic supply e^c_i at price $q^c_{it} = z^c_i$
5. Renewable energy firm: elastic supply e^r_i at price $q^r_{it} = z^r_i e^{-g_r t}$

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- Climate system: GHGs from oil-gas and coal affect temperatures

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta \mathcal{S}_t = \sum_{\mathbb{I}} \underbrace{(e^f_i + e^c_i)}_{\text{GHG from energy}} - \delta_s \mathcal{S}_t$$

$$T_i = \bar{T}_{i0} + \Delta_i = \bar{T}_{i0} + \underbrace{\Delta_i}_{\substack{\text{pattern} \\ \text{scaling}}} \mathcal{S}_t$$

Model – Equilibrium

- Given policies $\{\mathrm{t}^f_{it}, \mathrm{t}^{ls}_{it}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_{it}, e^f_{it}, e^r_{it}, e^x_{it}\}_i$, states $\{\mathcal{S}_t, T_{it}\}_i$ and prices $\{q^f_t, q^c_{it}, q^r_{it}, w_{it}\}_i$ such that:

- Households choose $\{c_{it}\}_i$ to max. utility s.t. budget constraint
- Firm choose inputs $\{e^f_{it}, e^c_{it}, e^r_{it}\}_i$ to max. profit
- Oil-gas firms extract/produce $\{e^x_{it}\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e^c_{it}, e^r_{it}\}$
- Emissions $\mathcal{E}_t$ affects climate $\{\mathcal{S}_t, T_{it}\}_i$.
- Government budget clear $\sum_i \mathrm{t}^{ls}_{it} = \sum_i \mathrm{t}^{\varepsilon}_{it}(e^f_{it} + e^c_{it})$
- Prices $\{q^f_t, q^c_{it}, q^r_{it}\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e^x_{it} = \sum_{\mathbb{I}} e^f_{it}$, and e^c_{it}, e^r_{it}
  The good market clearing holds by Walras law.

Competitive equilibrium

- Key objects:
  - Marginal value of wealth $\lambda_{it}^k := \frac{d\mathcal{V}_{it}}{dk_{it}} = u'(c_{it})$
  - Marginal value of carbon $\lambda_{it}^{\mathcal{S}} = \frac{d\mathcal{V}_{it}}{d\mathcal{S}_t}$ for country i
  - "Local social cost of carbon" (LCC) for region i:

$$LCC_{it} := -\frac{\partial \mathcal{V}_{it}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}/\partial a_{it}} = -\frac{\lambda_{it}^{\mathcal{S}}}{\lambda_{it}} = \int_t^\infty e^{-(\rho+\delta_s)\tau} \frac{\lambda_{i\tau}^k}{\lambda_{it}^k} \Delta_i \gamma \left(T_{i\tau} - T_i^\star\right) y_{i\tau} d\tau \ > 0$$

    - w/ Nordhaus-style damage $\mathcal{D}_i(T) = e^{-\gamma(T-T_i^\star)^2}$
    - Stationary eqbm closed-form formula, as in GHKT (2014) Closed Form Solution

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From Status-Quo to Optimal Policy

- Optimal policy: Maximizing "world" welfare:

$$\mathcal{W}_{t_0} = \max_{\mathbf{t}} \sum_{\mathbb{I}} \omega_i \int_{t_0} e^{-\rho t}\, u(c_{it}) dt = \sum_{\mathbb{I}} \omega_i \mathcal{V}_{it_0}$$

- Social Cost of Carbon

$$SCC_t := -\frac{d\mathcal{W}_t / d\mathcal{S}_t}{d\mathcal{W}_t / dk_t}$$

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$$SCC_t := -\frac{d\mathcal{W}_t/d\mathcal{S}_t}{d\mathcal{W}_t/dk_t} \stackrel{?}{=} -\sum_{\mathbb{I}} \omega_i \frac{d\mathcal{V}_{it}/d\mathcal{S}_t}{d\mathcal{V}_{it}/dk_{it}} =: \sum_{\mathbb{I}} \omega_i LCC_{it}$$

- Policies:
  - Competitive Equilibrium: no policies $\mathrm{t}_i^{\varepsilon} = 0$
  - First-Best Allocation: allocation + decentralization *with all instruments*
  - Second-Best Policy: *only* carbon tax instruments

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Dynamic models using the Pontryagin Max. Principle

- Every dynamic model can be expressed as a Forward Backward system of ODEs: states $\{k_{it}, \mathcal{S}_t\}$ and costates $\{\lambda^k_{it}, \psi^{\mathcal{S}}_t\}$. Ex NCG:

$$\begin{cases} \dot{k}_{it} &= f(k_{it}, e_{it}) - \delta k_{it} - q^e_{it} e_{it} - c_{it} \\ \dot{\lambda}^k_{it} &= -\big(f_k(k_{it}, e_{it}) - \delta - \rho\big)\lambda_{it} \end{cases}$$

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- *Novelty*:
  - *Heterogeneity*: exante, *arbitrary*, expost: *number of state/costate*
  - Large state space: as a *sequential* approach, no need for grid, just follow the agents
  - *Optimal policy* (Ramsey):
    policies usually expressed as fct of multipliers $\Leftrightarrow$ costates: $\mathrm{t}^{\varepsilon} = SCC = -\lambda^{\mathcal{S}}_t / \lambda^k_t$

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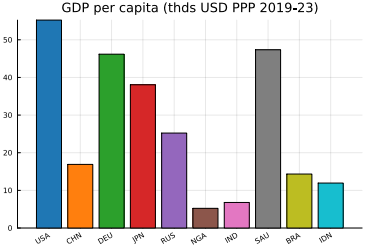
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  - *Global* method $\rightarrow$ eqbm potentially far from steady state: non-linearity/tipping points?
  - *Caveat:* Rely (for now) on non-linear solvers.

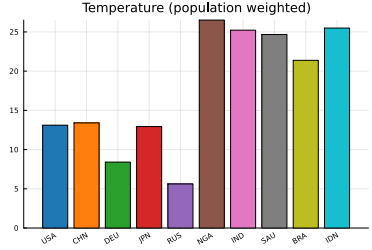
Quantification - 1

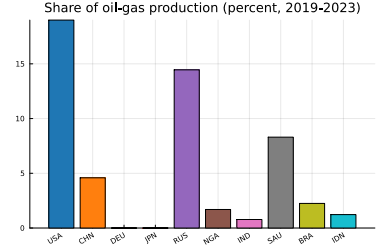
- Standard functional forms: $\triangleright$ $u(\cdot)$ CRRA η, $\triangleright$ $f(\cdot)$ Cobb-Douglas (k, e),
  $\triangleright$ energy e CES (e^f, e^c, e^r) with parameter σ,
  $\triangleright$ extraction cost $\mathcal{C}^f(e^x)$ isoelastic with inverse elasticity ν_i
- Damage function à la Nordhaus: $\mathcal{D}_i(T) = e^{-\gamma(T-T_i^\star)^2}$
  - Calibration $\gamma = 3 \times \gamma_{DICE}$ as in Bourany, Rosenthal-Kay (2026)
- Growth: population $\dot{\mathcal{P}}_i = n\mathcal{P}_i$ and TFP $\dot{z}_{it} = \bar{g} z_{it}$
- Sample of 25 countries + 7 regions: incl. US, China, India, Nigeria, Saudi Arabia, Russia, Japan, Brazil, etc..

Quantification - 2

- Parameters matching country-level data, avg. 2018-2023, Details Quantification
- TFP $z_i \Rightarrow$ GDP y_i, Population $\mathcal{P}_i$, Temperature T_{it_0}, Pattern scaling Δ_i
- Mix: oil-gas e_i^f, Coal e_i^c, Low-carbon e_i^r, energy share ε_i, oil-gas prod° e_i^x, reserves $\mathcal{R}_i$, rents π_i^f







World welfare and Pareto weights

- Recall welfare objective: $\mathcal{W}_t = \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \mathcal{V}_{it}$
- ***Crucial:*** how to chose ω_i ?

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- ***Two benchmarks:***
  - ▷ Uniform Pareto weights $\omega_i = 1$

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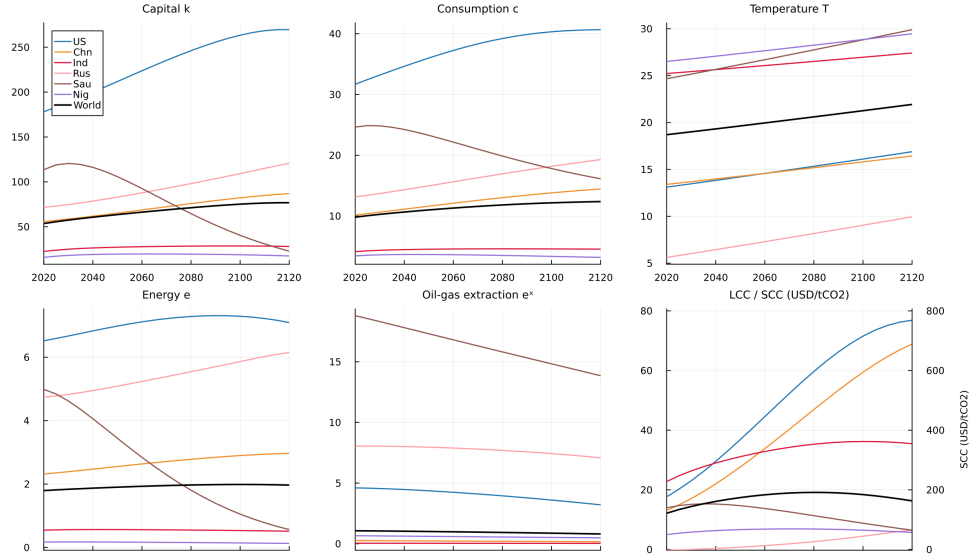
▷ Uniform Pareto weights $\omega_i = 1$

▷ Negishi weights $\omega_i = \frac{1}{u'(\bar{c}_i)}$
  - with $\bar{c}_i$ conso in initial eqbm 2018-23 *without climate change*
  - implies no redistribution motive: $\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \qquad \forall i, j \in \mathbb{I}$
  - Climate change + carbon tax have redistributive effects $\Rightarrow$ change allocation c_i



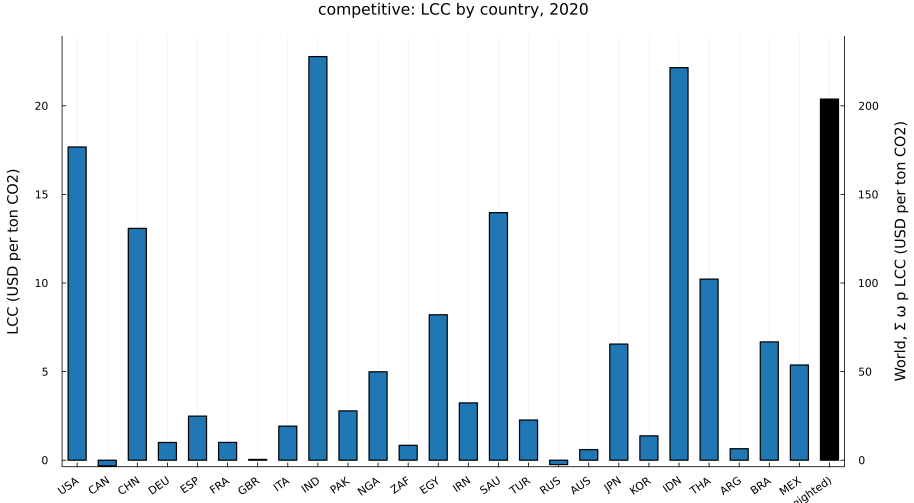
Dynamics in Competitive Equilibrium

$\triangleright\ \mathcal{T}_{2100} = 3.75°C$



Competitive Equilibria - Cost of Climate

▷ $LCC_{it} = -\lambda_{it}^{\mathcal{S}}/\lambda_{it}^{k}$ and ▷ "naive" $SCC_{it} = \sum_{\mathbb{I}} \mathcal{P}_i \omega_i LCC_{it}$



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First-Best, Optimal policy with transfers

- Planner's Welfare maximization:

$$\mathcal{W}_{t_0}^{fb} = \max_{\{\boldsymbol{c},\boldsymbol{e}\}_{i,t}} \sum_{\mathbb{I}} \omega_i \int_{t_0} e^{-\rho t}\, u(c_{it}) dt = \sum_{\mathbb{I}} \omega_i \mathcal{V}_{it_0}^{fb}$$

  - *Allocation:* choice of controls $\{c_{it}, e_{it}\}_{i,t}$
    s.t. resources constraints (i) Good market ϕ_t^k, (ii) Energy markets, (iii) Climate dynamics ϕ_{it}^S

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    carbon tax(es) *and cross-countries* transfers

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- Key objects: Local vs. Global Social Cost of Carbon,

$$SCC_t^{fb} := -\frac{\partial \mathcal{W}_t^{fb}/\partial \mathcal{S}_t}{\partial \mathcal{W}_t^{fb}/\partial k} = -\frac{\phi_t^{\mathcal{S}}}{\overline{\phi}_t} = -\frac{\sum_{\mathbb{I}} \phi_{it}^{\varepsilon}}{\frac{1}{I}\sum_{\mathbb{I}} \phi_{it}^{k}} \qquad LCC_i := -\frac{\partial \mathcal{V}_{it}^{fb}/\partial \mathcal{S}_t}{\partial \mathcal{V}_{it}^{fb}/\partial k_{it}} = -\frac{\phi_{it}^{\mathcal{S}}}{\phi_{it}^{k}}$$

First-Best, Optimal policy with transfers

- *Proposition 1:* Optimal carbon tax:

$$\mathrm{t}_t^{\varepsilon} = SCC_t^{fb}$$

  - Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC_t^{fb} = \frac{\phi_t^{\mathcal{S}}}{\bar{\phi}_t} = \sum_{\mathbb{I}} \frac{\phi_{it}^{\varepsilon}}{\phi_{it}} = \sum_{\mathbb{I}} LCC_{it}$$

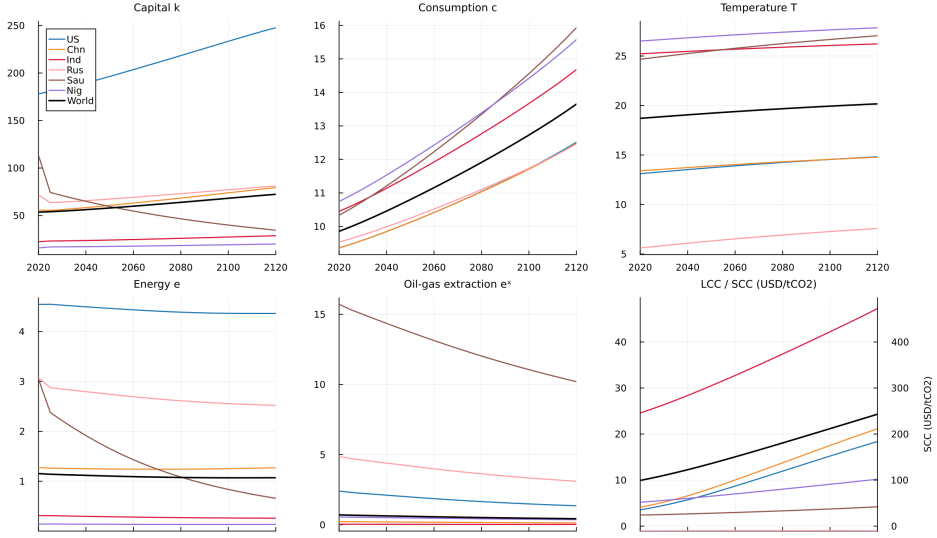
- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_{it}) = \phi_{it} = \bar{\phi}_t = \phi_{jt} = \omega_j u'(c_{jt}) \quad \forall i,j \in \mathbb{I} \qquad \forall\, t > t_0$$

  - Imply cross-countries lump-sum transfers $\exists\, i,j,t > t_0 \;\; s.t.\; \mathrm{t}_{it}^{ls} \geq 0$ or $\exists\, j \;\; s.t.\; \mathrm{t}_{jt}^{ls} \leq 0$

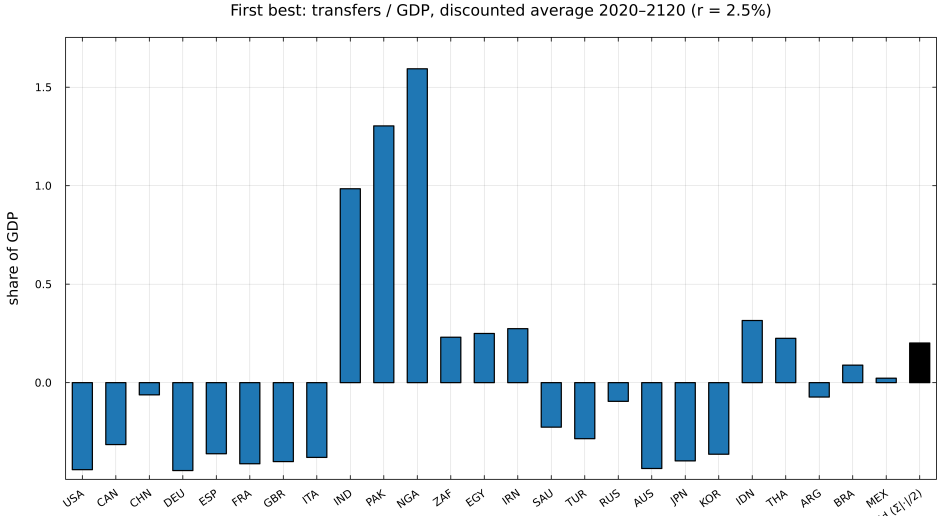
First-Best Allocation

$\triangleright$ c_{it} $\approx$ equalized, $\triangleright$ fossil fuels / emissions: -46%, $\triangleright$ $\mathcal{T}_{2100} = 2.4°C$



First Best and Transfers

▷ First-best w/ transfers: $\Rightarrow \quad t^{\varepsilon} = SCC^{fb}$, and $|t_{it}^{ls}| \gg 0$



Roadmap

- Simplified set-up: Toy example
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- **Second-best carbon taxation: Constrained efficient without lump-sum transfers**
  - **Uniform carbon taxation**
  - Heterogeneous carbon prices across countries
- Extensions
- Optimal Climate Agreements

Ramsey policy with limited transfers

- Second best without lump-sum transfers: choose carbon tax $\{\mathrm{t}^{\varepsilon}_t\}_t$ to maximizing welfare:

$$\mathcal{W}^{sb}_t = \max_{\{\mathrm{t}^{\varepsilon}_s\}_{s\geq t}} \sum_i \omega_i \mathcal{V}_{it}$$

  - with tax receipts redistributed lump-sum: $\mathrm{t}^{ls}_{it} = \mathrm{t}^{\varepsilon} e^f_{it}$ and subject to:
    - agents' budget ψ^k_{it},
    - climate system $\psi^{\mathcal{S}}_{it}$
    - agents optimality (FOC) conditions, e.g. e^f_{it} oil-gas: v^f_{it}
    - energy markets clearings, etc.

- Welfare relevant-measure of inequality across regions:

$$\widehat{\psi}_{it} = \frac{\omega_i \psi^k_{it}}{\bar{\psi}^k_t} \approx \frac{\omega_i u'(c_i)}{\frac{1}{I}\sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1$$

  $\Rightarrow$ ceteris paribus, poorer countries have higher $\widehat{\psi}_i$

Ramsey policy and Social Cost of Carbon

- Social Cost of Carbon integrates these inequalities $\widehat{\psi}_{it} := \omega_i \psi_{it}^k / \bar{\psi}_t^k$:
- *Proposition 2:*

$$SCC_t^{sb} = \sum_{\mathbb{I}} \widehat{\psi}_{it} \, LCC_{it}$$

$$SCC_t^{sb} = \textstyle\sum_{\mathbb{I}} \omega_i LCC_{it} + I \, \mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}, LCC_{it}\right)$$

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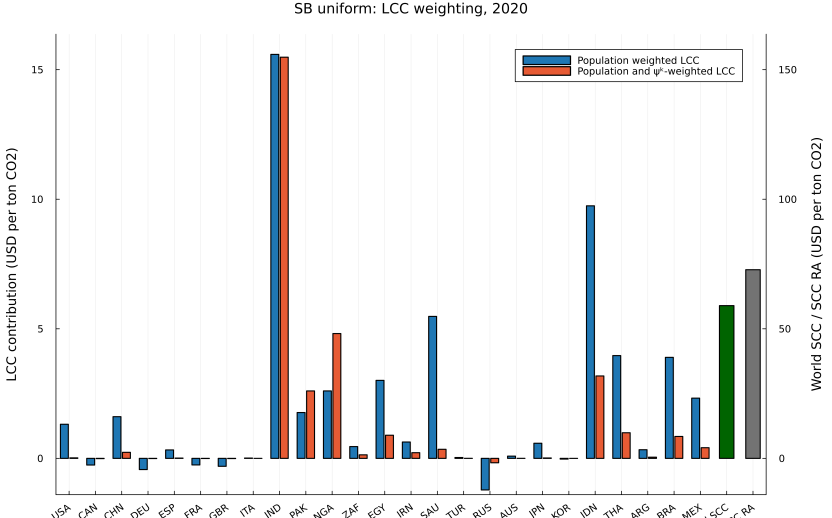
$$SCC_t^{sb} = \textstyle\sum_{\mathbb{I}} \omega_i LCC_{it} + I \, \mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}, LCC_{it}\right)$$

- To summarize:
  - First-Best: $SCC_t^{fb} = \sum_{\mathbb{I}} \overbrace{\widehat{\phi}_{it}}^{=1} \, LCC_{it} = \sum_{\mathbb{I}} LCC_{it}$
  - Second-Best: $SCC_t^{sb} = \sum_{\mathbb{I}} \widehat{\psi}_{it} \, LCC_{it} \lessgtr \sum_{\mathbb{I}} \omega_i LCC_{it}$
  - Comp. Eqblm: $SCC_t^{ce} = \sum_{\mathbb{I}} \widehat{\lambda}_{it} \, LCC_{it} \lessgtr \sum_{\mathbb{I}} \omega_i LCC_{it}$

  $\widehat{\lambda}_{it} := \omega_i \lambda_{it}^k / \bar{\lambda}_t^k = \omega_i u'(c_i) / \frac{1}{I} \sum_i \omega_j u'(c_j)$ and $\widehat{\phi}_{it} := \omega_i u'(c_i) / \bar{\phi}_t^k = 1$

Local Cost of Carbon & Social Cost of Carbon

▷ $SCC = \sum_{\mathbb{I}} \widehat{\psi}_i \, LCC_i$ ▷ Difference: $LCC_{it} = \psi_i^{\mathcal{S}} / \psi_i^k$ (blue, left bars) vs. $\widehat{\psi}_i \, LCC_i = \psi_i^{\mathcal{S}} / \bar{\psi}^k$ (orange, right bars)



Carbon tax and energy markets

- Tax on fossil energy has redistributive effects:
  1. Redistributes rent through energy markets: lowers eq. fossil price, hurts exporters
  2. Distorts firms' energy demand, esp. countries that depend more on energy

◦ Params: $\mathcal{C}^f_{EE}$ agg. fossil inv. supply elasticity, s^f_i energy cost share and σ_i energy demand elasticity Details

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  2. Distorts energy demand, and its impact on the firm' choice of input x

$$\text{Demand Distortion}_t^{sb\,f} = \sum_i \sum_{x \in \{e^f, e^c, e^r, k\}} \widehat{\upsilon}_i^x \frac{d}{de_{it}^f} MPx_{it}$$

  - Params: $\mathcal{C}_{EE}^f$ agg. fossil inv. supply elasticity, s_i^f energy cost share and σ_i energy demand elasticity Details

Carbon tax and energy markets

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  2. Distorts energy demand, and its impact on the firm' choice of input x

$$\text{Demand Distortion}_t^{sb\,f} = -\mathbb{C}\text{ov}_i\left(\widehat{v}_i^f, \frac{q^f(1-s_i^f)}{\sigma_i e_{it}^f}\right)$$

  - Params: $\mathcal{C}_{EE}^f$ agg. fossil inv. supply elasticity, s_i^f energy cost share and σ_i energy demand elasticity Details

Ramsey Problem – Optimal Carbon Tax

- Carbon tax redistributive effects through energy markets:
  [1.] Redistributes energy rent, [2.] Distorts energy demand

$$\text{Supply Redistribution}_t^{sb\,f} + \text{Demand Distortion}_t^{sb\,f} = \underbrace{\mathcal{C}^f_{EE\,t}}_{\substack{\text{agg. supply}\\ \text{inv. elasticity}}} \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}, e^f_i - e^x_i\right)}_{\substack{\text{terms-of-trade}\\ \text{redistribution}}} - \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\upsilon}_{it}, \frac{q^f_t(1-s^f_{it})}{\sigma_i e^f_{it}}\right)}_{\substack{\text{energy}\\ \text{distortion}}}$$

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  - Params: $\mathcal{C}^f_{EE\,t}$ agg. fossil inv. supply elasticity, s^e_i energy cost share and σ_i energy demand elasticity Details

- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \quad \mathrm{t}^{\varepsilon}_t = SCC^{sb}_t + \text{Supply Redistribution}^{sb}_t + \text{Demand Distortion}^{sb}_t$$

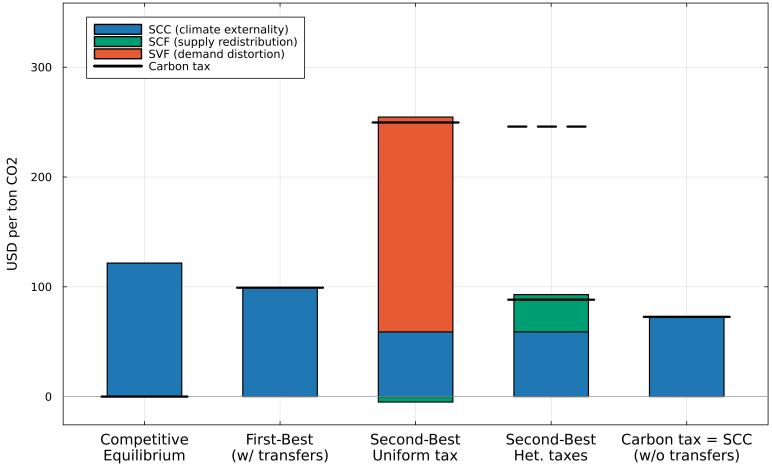
  - Reexpressing demand terms:

$$\mathrm{t}^{\varepsilon}_t = \left(1 + \mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it},\, \widehat{\frac{\sigma_i e^f_{it}}{1-s^f_{it}}}\right)\right)^{-1} \left[\sum\nolimits_{\mathbb{I}} \omega_i LCC_{it} + I\mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}, LCC_{it}\right) + \mathcal{C}^f_{EE}\, \mathbb{C}\text{ov}_i\left(\widehat{\psi}_{it}, e^f_i - e^x_i\right)\right]$$

The Carbon Tax is not (only) the Social Cost of Carbon!

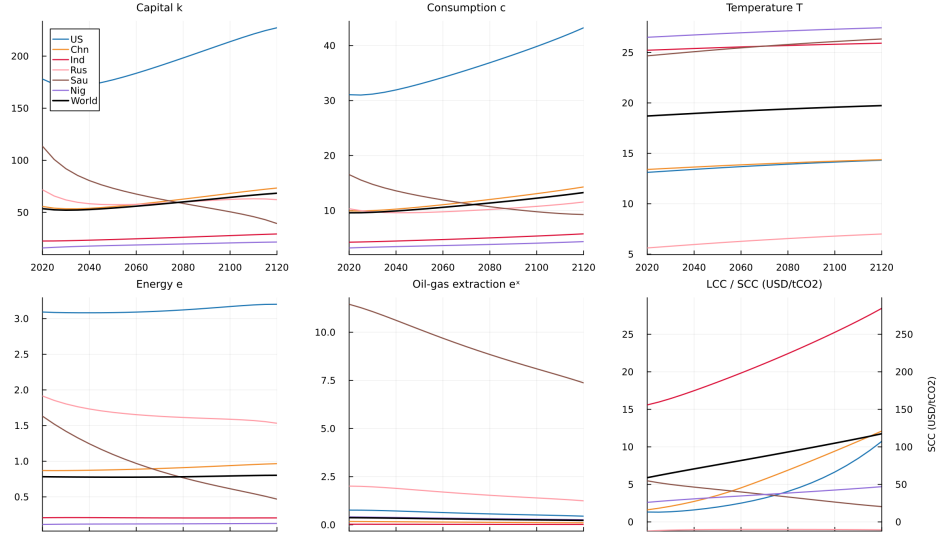
- First-best w/ transfers: $\Rightarrow \quad t^{\varepsilon} = SCC^{fb}$, and $|t_i^{ls}| \gg 0$
- Second-best carbon tax w/o transfers: $\Rightarrow \quad t^{\varepsilon} = SCC^{sb}$+Supply Redistrib.+Demand Distort°

Carbon tax decomposition across policy regimes (2020)



First-Best Allocation

$\triangleright\ c_{it} \approx$ unequal, $\triangleright$ fossil fuels / emissions: -60%, $\triangleright\ \mathcal{T}_{2100} = 2.1°C$



Roadmap

- Simplified set-up: Toy example
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  - Uniform carbon taxation
  - **Heterogeneous carbon prices across countries**
- Extensions
- Optimal Climate Agreements

Ramsey Problem – Country-specific carbon tax

- Suppose the planner has access to a *distribution* of carbon price.

$$\mathcal{W}_t^{sb} = \max_{\{\mathrm{t}_{is}^{\varepsilon}\}_{i,s\geq t}} \sum_i \omega_i \mathcal{V}_{it}$$

- *Proposition 3:* Optimal country-specific fossil energy tax:

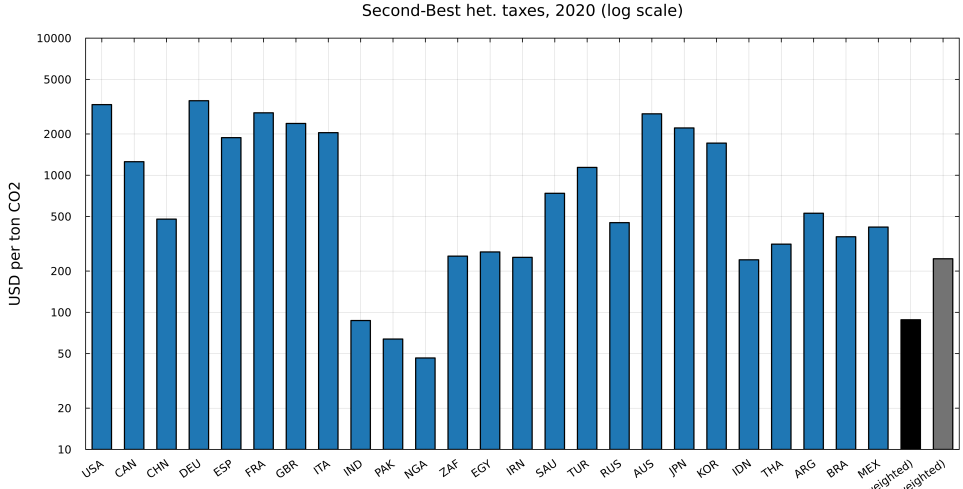
$$\Rightarrow \quad \mathrm{t}_{it}^{\varepsilon} = \frac{1}{\widehat{\psi}_{it}} \left[SCC_t^{sb} + \text{Supply Redistribution}_t^{sb} \right] \propto \frac{1}{\omega_i u'(c_{it})}$$

  - Social Cost of Carbon: $SCC_t^{sb} = \sum_{\mathbb{I}} \widehat{\psi}_{it} \, LCC_{it}$
  - Supply Redistribution$_t^{sb} = \mathcal{C}_{EE\,t}^{f} \mathbb{C}\mathrm{ov}_i \left(\widehat{\psi}_{it}, e_{it}^{f} - e_{it}^{x} \right)$

$\Rightarrow$ Reduce the tax burden for poorer/more "valuable" countries

Heterogeneous Carbon taxes are not the Local Social Cost of Carbon!

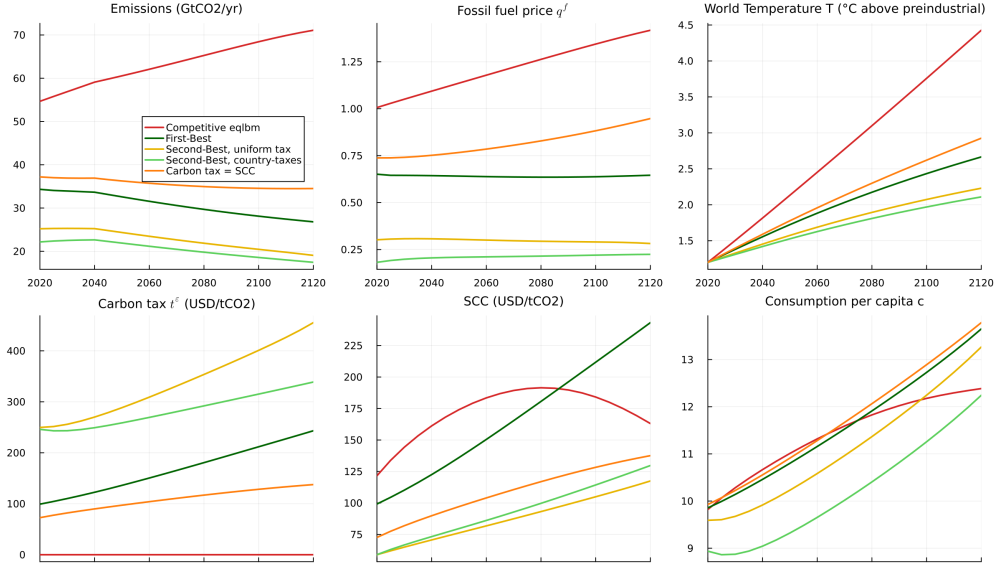
Country-specific carbon taxes: $t_i^{\varepsilon} \propto \frac{1}{\omega_i u'(c_i)}[SCC^{sb} + \text{Supply Redistrib.}] \neq LCC_i$



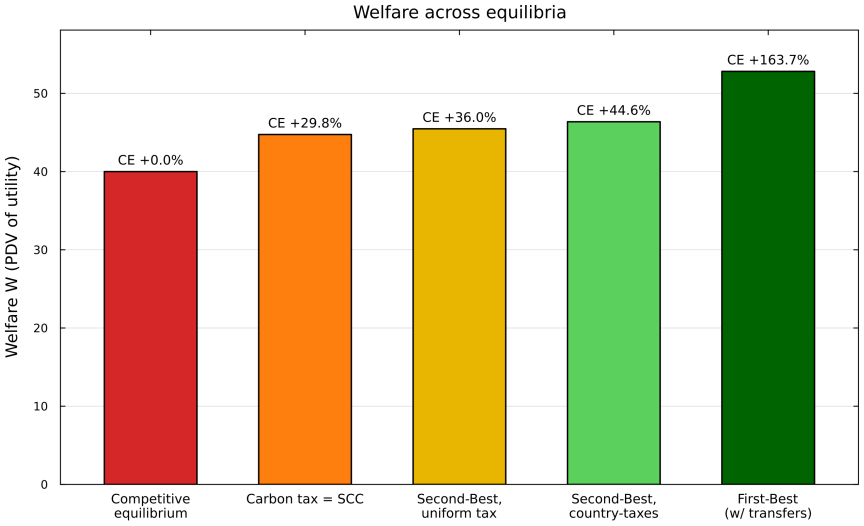
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- **Summary & Influence of Pareto weights**
- Extensions
- Optimal Climate Agreements

Allocation & Aggregates

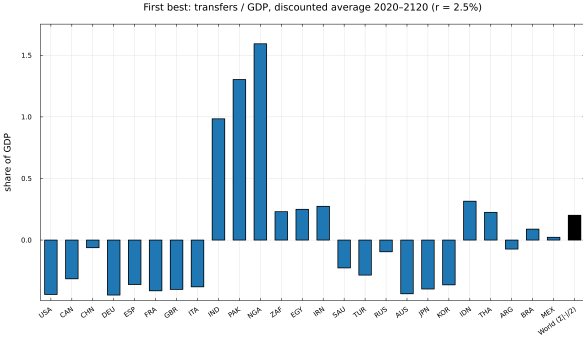


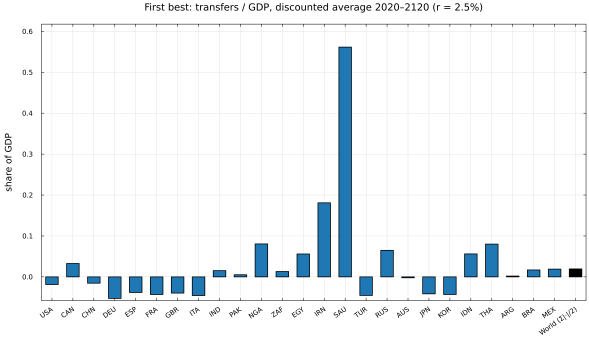
Welfare gains from climate policy



First-Best and Transfers – Pareto Weights

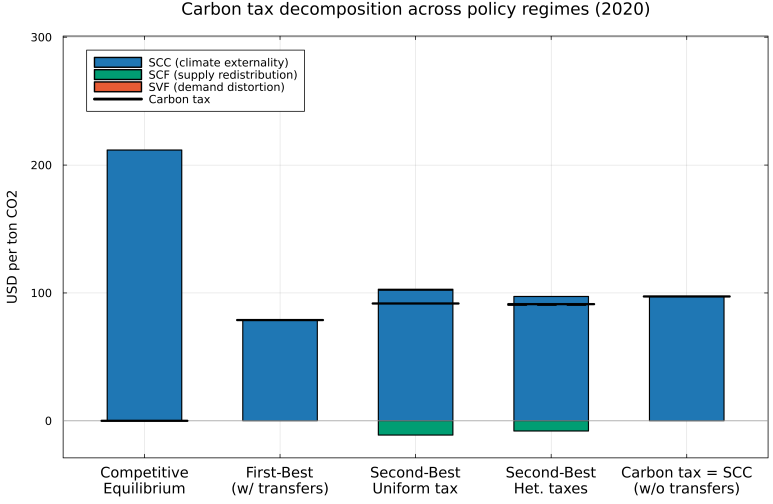
- Transfers t_{it}^{ls} for consumption equalization $\omega_i u'(c_{it}) = \omega_j u'(c_{jt}) = \phi_t$





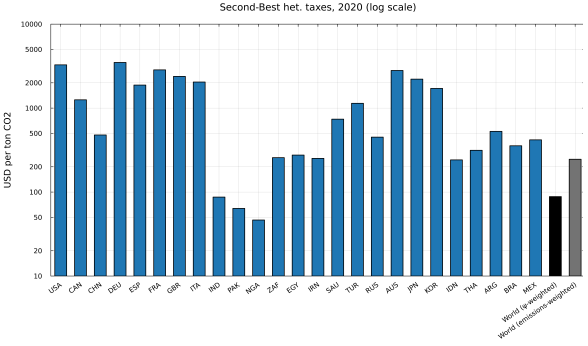
The Carbon Tax and Social Cost of Carbon – Pareto Weights

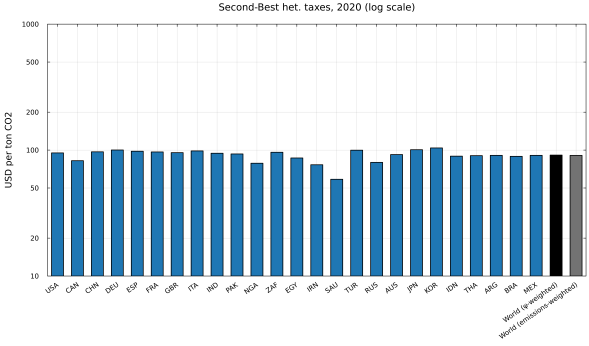
- First-best w/ transfers: $\Rightarrow \quad t^{\varepsilon} = SCC^{fb}$, and $|t_i^{ls}| \gg 0$
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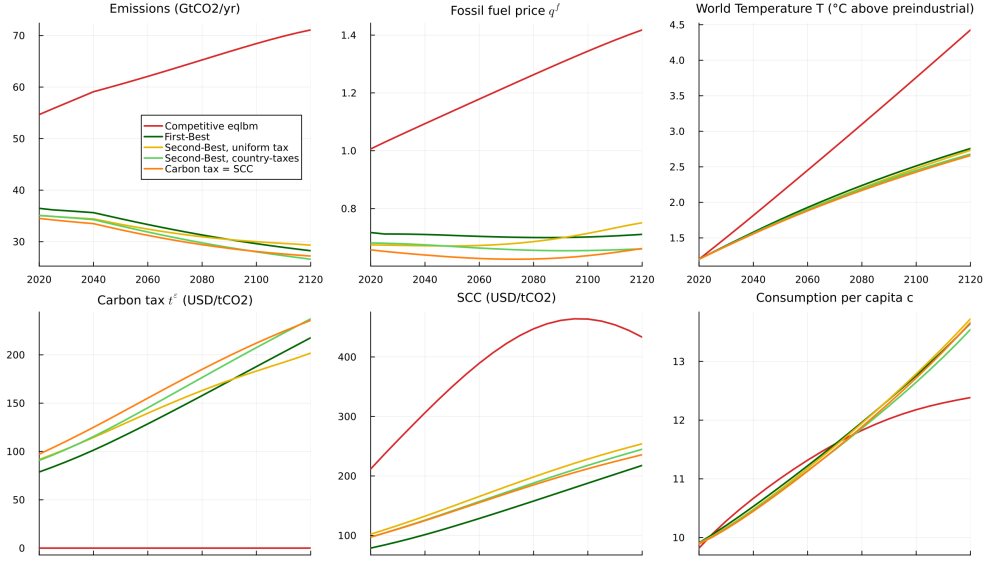
Heterogeneous Carbon taxes – Pareto Weights

- Country-specific carbon taxes: $t_i^{\varepsilon} \propto \frac{1}{\omega_i u'(c_i)}[SCC^{sb} + \text{Supply Redistrib.}] \neq LCC_i$





Allocation & Aggregates – Pareto Weights



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- **Extensions**
- Optimal Climate Agreements

Ramsey Problem – Theoretical extensions (in the papers)

- Time-inconsistency of optimal policy with imperfect instruments with heterogeneity
  - Carbon policy trajectory with full-commitment: distort dynamic capital/energy choices
  - Discretion would differ: no promise keeping
  - In practice: not important quantitatively
- Trade: redistributive effects through leakage effect
  - Trade model à la Armington Details trade
  - Additional trade-off: account for distortions on goods central in trade networks
- Uncertainty:
  - Stochastic climate impact, e.g. climate sensitivity Δ_i or damage curvature γ_i, Details uncertainty
  - Interaction in SCC between uncertainty and inequalities: covariance between cost of climate change and social welfare weights
- Dynamic considerations in energy markets:
  - Reserves values ψ_{it}^R = Hotelling rent: carbon tax changes intertemp. subst° of fossil prod. c.f. Heal, Schlenker (2019), Asker, Collard-Wexler, De Loecker (2019-2024),

Conclusion

- In this projects, I solve for the optimal climate policy
  - Accounting for multi-dimensional ***inequality***
  - GE effects through energy markets
- Main result, with uniform Pareto weights
  - First-Best allocation implies very large transfers
  - Second-Best overshoot $\$250/tCO_2$ instead of $\sim \$100$
    $\Rightarrow$ more aggressive on climate and cause "energy-repression" to help poorer countries
  - Country-specific taxes "compensate loosers" of climate change and climate policy
- With Negishi-weights:
  climate policy is less agressive not to hurt countries exposed to energy markets.

Conclusion + Going further?

- Optimal climate policy with inequality:
  - First-Best: requires sizeable transfers.
  - Second-Best: Otherwise, need large change in carbon tax level

  $\Rightarrow$ Need to account for the distribution + reallocative/GE effects on energy and good markets

Conclusion + Going further?

- Optimal climate policy with inequality:
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- Incentives and implementability:
  - What if some countries strategically deviate from applying the optimal carbon tax?

Conclusion + Going further?

- Optimal climate policy with inequality:
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  $\Rightarrow$ Need to account for the distribution + reallocative/GE effects on energy and good markets
- Incentives and implementability:
  - What if some countries strategically deviate from applying the optimal carbon tax?
  - Game theoretical consideration $\Rightarrow$ participation constraints
  - Implementation of "climate clubs":
    Penalty tariffs or CBAM to non-participants crucial for enforcing carbon policy

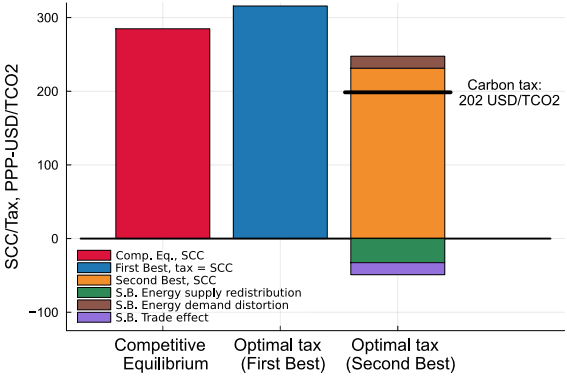
  $\Rightarrow$ Job Market Paper: "The Optimal Design of Climate Agreements"

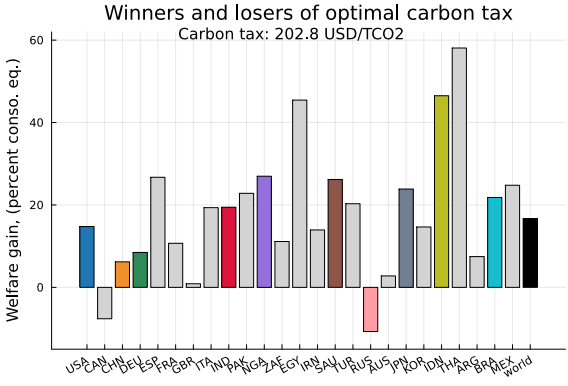
Appendices

Roadmap

- Simplified set-up: Toy example
- First-best carbon taxation: all instruments available
- Second-best carbon taxation: Constrained efficient without lump-sum transfers
  - Uniform carbon taxation
  - Heterogeneous carbon prices across countries
- Quantitative model: Dynamic IAMs with heterogeneity & Extensions
- **Optimal Climate Agreements**

Second-Best climate policy and Gains from cooperation





Main result and Intuition

- The optimal climate agreement navigates the intensive and extensive margin tradeoff:
  - ***Participation:*** most countries in the world with the exception of: *Russia, former Soviet countries, Saudi Arabia, Gulf countries, Egypt, Iran, Nigeria*
  - ***Carbon tax:*** reduce carbon tax from \$200 to $\$120/tCO_2$
  - ***Trade tariffs:*** impose substantial tariff 90% on the goods from non-members

Main result and Intuition

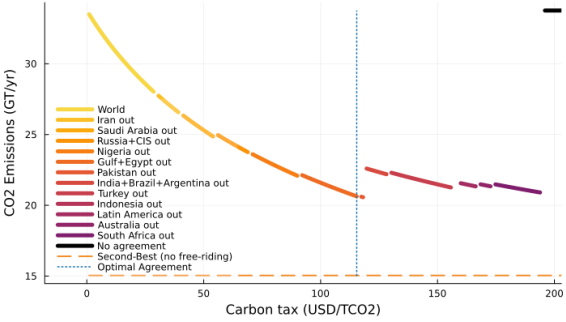
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  - ***Carbon tax:*** reduce carbon tax from \$200 to $\$120/tCO_2$
  - ***Trade tariffs:*** impose substantial tariff 90% on the goods from non-members

- Mechanism:
  - Countries participate depending on $\begin{cases} \text{(i) the cost of distortionary carbon taxation} \\ \text{(ii) the cost of tariffs (= the gains from trade)} \end{cases}$
  - Russia/Middle East/South Asia do not join the club for high carbon tax *for any tariffs* when cost of taxing fossil-fuels $\gg$ cost of tariffs / autarky

  $\Rightarrow$ Result, we need to decrease the carbon tax: *reduce ambition*

Laffer curve for carbon taxation

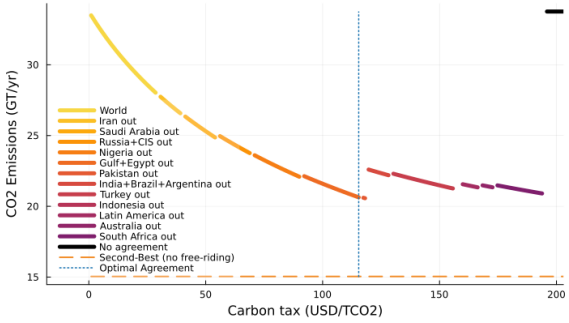
– Due to free-riding incentives, cannot reach globally optimal carbon tax $t^{\varepsilon,\star} = \$200$

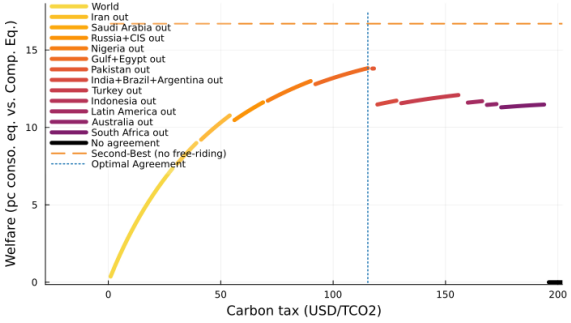


Emissions $\mathcal{E}$ (in $GtCO_2/yr$) and welfare $\mathcal{W}$ as function of the carbon tax t^{ε}, with tariff $t^{b} = 90\%$.

Laffer curve for carbon taxation

- Due to free-riding incentives, cannot reach globally optimal carbon tax $t^{\varepsilon,\star} = \$200$
- Need to lower the carbon tax to \$120 to increase participation:
  Improve welfare by sharing the costs of carbon mitigation with *more countries*





Emissions $\mathcal{E}$ (in $GtCO_2/yr$) and welfare $\mathcal{W}$ as function of the carbon tax t^{ε}, with tariff $t^{b} = 90\%$.

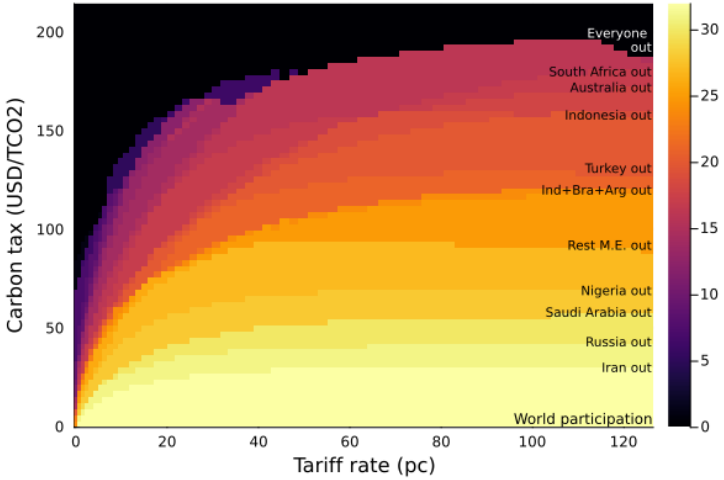
Climate Agreements: Intensive vs. Extensive Margin

- Intensive margin:

  given a coalition:
  higher tax t^{ε}, emissions $\mathcal{E} \downarrow$,
  improve welfare $\mathcal{W} \uparrow$

- Extensive margin:

  carbon tax also deters participation
  individual countries free-ride
  increasing emissions $\mathcal{E} \uparrow$

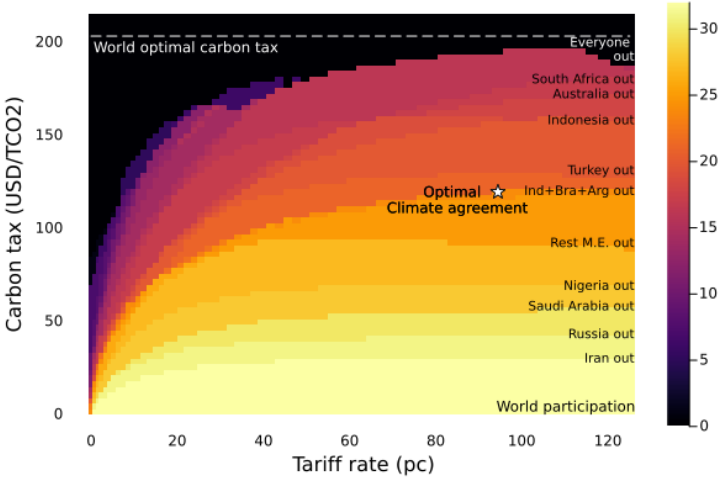


Optimal Climate Agreement

- Despite full discretion of instruments (t^{ε}, t^{b}), we cannot sustain an agreement with Russia, Middle East, South-Asia & South America

⇒ need to reduce carbon tax from \$200 to \$120

⇒ Beneficial to leave several fossil-fuel producers outside the agreement
e.g. no incentive for Russia to join: cold, closed to trade, large fossil-fuel producer



Graph welfare | Mechanisms

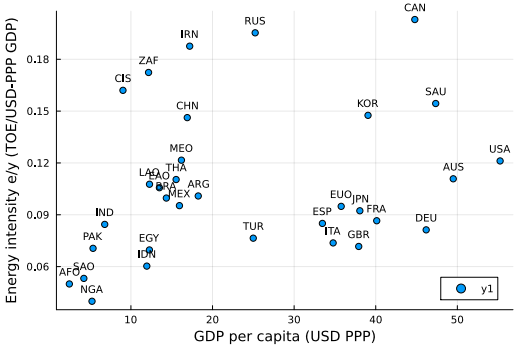
Conclusion

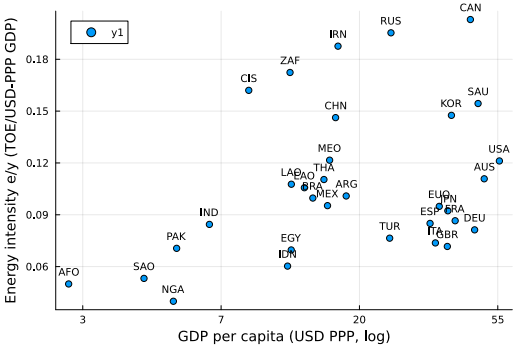
- In these projects, I solve for the optimal climate policy
  - Accounting for ***inequality*** + GE effects through energy markets and trade leakage
  - ... and strategic ***free-riding incentives***
- Additionaly climate agreements design jointly solve for:
  - The optimal choice of countries participating
  - The carbon tax and tariff levels, accounting for participation constraints
- The optimal climate club:
  - ***trade-off:*** gains of cooperation / free-riding incentives
    vs. gains from trade, i.e. cost of retaliation tariffs

  $\Rightarrow$ large coalition, lower the carbon tax from \$250 to \$200 to \$120

Energy intensity

- Energy intensity e_i/y_i vs. GDP per capita y_i





Step 0: Competitive equilibrium & Trade

- Each household in country i maximize utility and firms maximize profit
- Standard trade model results:
  - Consumption and trade:

$$s_{ij} = \frac{c_{ij} p_{ij}}{c_i \mathbb{P}_i} = a_{ij} \frac{(d_{ij}(1+\mathrm{t}^b_{ij})\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(d_{ik}(1+\mathrm{t}^b_{ik})\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \left(\sum_j a_{ij} (d_{ij}\mathrm{p}_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region i

$$LCC_i = \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi^{\mathcal{E}}_i}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e^f_i) \frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i\in\mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{V}_i$$

  - Full array of instruments: cross-countries lump-sum transfers t_i^{ls}, individual carbon taxes t_i^{e} on energy e_i^f, bilateral tariffs t_{ij}^b
  - Budget constraint: $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} d_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

- Social planner results:
  - Consumption:

$$\omega_i u'(c_i) = \Big[\sum_j a_{ij} (d_{ij} \omega_j \mu_j)^{1-\theta} \Big]^{\frac{1}{1-\theta}}$$

  - Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon:

$$SCC = -\frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{i\in\mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{V}_i$$

  - One single instrument: uniform carbon tax t^{ε} on energy e_i^f
  - Rebate tax lump-sum to HHs $t_i^{ls} = t^{\varepsilon} e_i^f$

- Ramsey policy: Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
  - Trade-off faced by the planner:
    - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

- The planner takes into account
  - (i) the marginal value of wealth λ_i
  - (ii) the shadow value of good i, from market clearing, μ_i:

w/o trade $\quad \omega_i u'(c_i) = \omega_i \lambda_i$

vs. w/ trade in goods: $\quad \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} \mathrm{p}_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$

- Relative welfare weights, representing inequality

w/o trade: $\quad \widehat{\lambda}_i = \dfrac{\omega_i \lambda_i}{\bar{\lambda}} = \dfrac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lesseqgtr 1 \quad \Rightarrow$ ceteris paribus, poorer countries have higher $\widehat{\lambda}_i$

vs. w/ trade: $\quad \widehat{\lambda}_i = \dfrac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \lesseqgtr 1$

Step 2: Optimal policy – Social Cost of Carbon

- Key objects: Local vs. Global Social Cost of Carbon:
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
  - "Local social cost of carbon" (LCC) for region i:

$$LCC_i := \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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  - Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I}\sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

  - Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\big(\widehat{\lambda}_i, LCC_i\big)$$

Step 2: Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}^f_{EE}\, \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right) \; - \; \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)$$

  - Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^f_i energy cost share and σ energy demand elasticity

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- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \qquad \mathrm{t}^{\varepsilon} = SCC + SVF$$

  - Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i$

back

Step 2: Optimal policy – Details

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers

$$\begin{aligned}\text{Supply Redistribution} &= \Big(\sum_i \frac{1}{\mathcal{C}^i_{ee}(e^x_i)}\Big)^{-1} \sum_i \widehat{\lambda}_i (e^f_i - e^x_i) \\ &= q^f \Big(\sum_i \frac{e^x_i}{\nu_i}\Big)^{-1} \sum_i \widehat{\lambda}_i (e^f_i - e^x_i)\end{aligned}$$

  2. Distort energy demand, of countries that need more or less energy:
     FOC $MPe^f_i = q^f$, multiplier $\propto \widehat{v}^f_i$

$$\begin{aligned}\text{Demand Distortion} &= \frac{1}{I}\sum_i \widehat{v}^f_i \partial_{e^f} MPe^f_i + \widehat{v}^r_i \partial_{e^f} MPe^f_i \\ &= -(q^f + \mathfrak{t}^f)\frac{1}{I}\sum_i \widehat{v}^f_i \frac{1}{e^f_i}\Big[\frac{1-s^f_i}{\sigma^e} + s^f_i \frac{1-s^e_i}{\sigma^y}\Big] + \frac{1}{I}\sum_i \widehat{v}^r_i \frac{q^r_i}{e^f_i} s^f_i \Big[\frac{1}{\sigma^e} - \frac{1-s^e_i}{\sigma^y}\Big]\end{aligned}$$

Step 2: Optimal policy – Details

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- *Proposition 2:* Optimal fossil energy tax: back

$$\Rightarrow \quad \mathrm{t}^e = SCC + \text{Supply Redistrib.} + \text{Demand Distortion}$$

  – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i \, LCC_i$

Step 3: Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^{ε}
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^{b} with club members and countries outside the club. They still trade with the club members in energy at price q^{f}

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  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^{b} with club members and countries outside the club. They still trade with the club members in energy at price q^f
- Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{e},\boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha\omega_i \, u(c_i)$$

Step 3: Ramsey Problem with participation constraints

- Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- *Proposition 3.1:* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade:} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j \in \mathbb{I}} a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta} \Big[\omega_i \widetilde{\lambda}_i + \omega_j \widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij}) \Big]^{1-\theta} \Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i + \widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}} \omega_i(\widetilde{\lambda}_i + \widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Result: $\omega_i(1+\nu_i)$ are the "endogenous Pareto weights"

Step 3: Participation constraints & Optimal policy

- *Proposition 3.2:* Second-Best taxes:
  - Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$

      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
  - Optimal fossil energy tax $\mathrm{t}^f(\mathbb{J})$:

$$\Rightarrow \quad \mathrm{t}^{\varepsilon}(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i \, LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \, \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $\mathrm{t}^{b,r}(\mathbb{J})$ and $\mathrm{t}^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

Climate uncertainty and the Cost of Carbon:

- Stochastics: for any shock ϵ with distribution $\epsilon \sim \varphi(\epsilon)$
- New measure for inequalities:

$$\widehat{\lambda}^w_{it}(\epsilon) = \frac{\lambda^k_{it}(\epsilon)}{\mathbb{E}_{i,\epsilon}[\lambda^w_{it}(\epsilon)]} = \frac{\omega_i u'(c_{it}(\epsilon))}{\int_\epsilon \int_j \omega_j u'(c_{j,t}(\epsilon))\, dj\, d\varphi(\epsilon)}$$

- Uncertainty-adjusted SCC writes:

$$\mathbb{E}_\epsilon[SCC] = \int_{\mathcal{E}} \int_{\mathbb{I}} \widehat{\lambda}^w_{it}(\epsilon) LCC_{it}(\epsilon) d\varphi(\epsilon)$$

$$= \mathbb{E}_j\Big[\underbrace{\mathbb{C}\text{ov}_\epsilon\Big(\widehat{\lambda}^w_{it}(\epsilon), LCC_{jt}(\epsilon)\Big)}_{=\text{effect of aggregate risk } \epsilon}\Big] + \underbrace{\mathbb{C}\text{ov}_j\Big[\mathbb{E}_\epsilon\big(\widehat{\lambda}^w_{it}(\epsilon)\big), \mathbb{E}_\epsilon\big(LCC_{jt}(\epsilon)\big)\Big]}_{=\text{effect of heterogeneity across } j} + \underbrace{\mathbb{E}_{j,\epsilon}[LCC_{jt}(\epsilon)]}_{=\text{average exp. damage}}$$

$$> \mathbb{E}_\epsilon[\overline{SCC}(\epsilon)] \qquad \& \qquad > SCC_t$$

$\Rightarrow$ Climate uncertainty reinforces the unequal costs of climate change! back

Sequential solution method

- Summary of the dynamic model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution:
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Impact of increase in temperature

- Marginal values of the climate variables: λ^s_{it} and λ^τ_{it}

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau^\star_i)\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda^k_{it} + \overbrace{\phi_i(\tau_{it}-\tau^\star_i)\mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

- Costate λ^S_{it}: marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with:
  - Temperature gaps $\tau_{it}-\tau^\star_i$ & damage sensitivity of TFP γ^y_i and utility γ^u_i
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params: χ climate sensitivity, Δ_i "catching up" of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^{\star})\mathcal{D}^{y}(\tau)f(k,e)\lambda_t^{k} + \phi(\tau - \tau^{\star})\mathcal{D}^{u}(\tau)u(c)$$

$$\dot{\lambda}_t^{S} = \lambda^{S}{}_t(\widetilde{\rho} + \delta^{s}) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for λ_t^{τ} and $\lambda_t^{\mathcal{S}}$, in stationary equilibrium $\dot{\lambda}_t^{S} = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = -\int_t^{\infty} e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^{\star})\Big(\gamma\mathcal{D}^{y}(\tau_u)y_{\tau}\lambda_u^{k} + \phi\mathcal{D}^{u}(\tau_u)u(c_u)\Big)du$$

$$\lambda_{it}^{\tau} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_{\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^{y}(\tau_{\infty})y_{\infty}\lambda_{\infty}^{k} + \phi\mathcal{D}^{u}(\tau_{\infty})u(c_{\infty})\Big)$$

$$\begin{aligned}
\lambda_t^{S} &= -\int_t^{\infty} e^{-(\widetilde{\rho}+\delta^{s})u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,u}^{\tau}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^{s}}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,\infty}^{\tau} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^{s}}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^{y}(\tau_{j,\infty})y_{\infty}\lambda_{j,\infty}^{k} + \phi\mathcal{D}^{u}(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda_t^{S} &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^{s}}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^{\star})\Big(\gamma\mathcal{D}^{y}(\tau_{j,\infty})y_{j,\infty}\lambda_{j,\infty}^{k} + \mathcal{D}^{u}(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

- ***Proposition (Stationary LCC)***:
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

$$LCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^\star) \Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \Big)$$

  - Stationary equilibrium: $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Quantification: Climate and parametrization

- Climate block: Climate System
  - Static economic model: decisions taken "once and for all", e.g. $\mathcal{E} = \sum_i e_i^f + e_i^c$.
  - "Dynamic" climate system: $\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t, \quad T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$
  - Quadratic damage functions as in Nordhaus-DICE: $\mathcal{D}(T_{it} - T_i^\star) = e^{-\gamma(T_{it} - T_i^\star)^2}$
  - Feedback in Present discounted value: $\mathcal{D}_i^y(\mathcal{E}) = \bar{\rho} \int_0^\infty e^{-(\rho - n + (1-\eta)\bar{g})t} \mathcal{D}\left(T_{it} - T_i^\star\right) dt$

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  - "Dynamic" climate system: $\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t, \quad T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$
  - Quadratic damage functions as in Nordhaus-DICE: $\mathcal{D}(T_{it} - T_i^\star) = e^{-\gamma(T_{it}-T_i^\star)^2}$
  - Feedback in Present discounted value: $\mathcal{D}_i^y(\mathcal{E}) = \bar{\rho}\int_0^\infty e^{-(\rho - n + (1-\eta)\bar{g})t}\,\mathcal{D}\left(T_{it} - T_i^\star\right)dt$
  - Estimate γ from trade data (as Rudik, Lyn, Tan, and Ortiz-Bobea (2021)) Damage, Gravity

$$\log \mathbb{E}[X_{ij}/X_{ii}] = \beta_0(T_{jt} - T_{it}) + \beta_1(T_{jt}^2 - T_{it}^2) + \varsigma_{ij} + \varsigma_t + \Gamma' W_{it} + \Omega' W_{jt}$$
$$\Rightarrow \quad \gamma = 0.012 \approx 3 \times \gamma_{DICE}$$

Quantification: Climate and parametrization

- Climate block: Climate System
  - Static economic model: decisions taken "once and for all", e.g. $\mathcal{E} = \sum_i e_i^f + e_i^c$.
  - "Dynamic" climate system: $\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t, \quad T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$
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$$\Rightarrow \quad \gamma = 0.012 \approx 3 \times \gamma_{DICE}$$

- Standard functional forms:
  - CRRA utility, Nested CES production, Iso-elastic fossil fuel extraction back

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i F(k, \varepsilon(e^f, e^r))$

$$F_i(\varepsilon(e^f,e^c,e^r),\ell) = \left[(1-\epsilon)^{\frac{1}{\sigma_y}} \left(\bar{k}^{\alpha}\ell^{1-\alpha}\right)^{\frac{\sigma_y-1}{\sigma_y}} + \epsilon^{\frac{1}{\sigma_y}} \left(z_i^e\ \varepsilon_i(e^f,e^c,e^r)\right)^{\frac{\sigma_y-1}{\sigma_y}}\right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon_i(e^f,e^c,e^r) = \left[(\omega^f)^{\frac{1}{\sigma_e}} \left(e^f\right)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^c)^{\frac{1}{\sigma_e}} \left(e^c\right)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^r)^{\frac{1}{\sigma_e}} \left(e^r\right)^{\frac{\sigma_e-1}{\sigma_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

  - Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2019-23 (avg. PPP).
  - Technology: $\omega^f = 56\%, \omega^c = 27\%, \omega^f = 17\%, \epsilon = 12\%$ for all i
  - Calibrate (z_i^e) to match Energy/GDP $q^e e_i / \mathrm{p}_i y_i$

- Damage functions in production function y:

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm,y}(T-T_i^\star)^2}$$

  - Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+,y}\mathbb{1}_{\{T>T_i^\star\}} + \gamma^{-,y}\mathbb{1}_{\{T<T_i^\star\}}$
  - Symmetric damage: $\gamma_i^{\pm,y} = \bar{\gamma}^{\pm,y}$ & $T_i^\star = \bar{\alpha}T_{it_0} + (1-\bar{\alpha})T^\star$

Quantification – Energy markets

- Fossil production e^x_{it} and reserve $\mathcal{R}_{it}$
  - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i}\left(\frac{e^x}{\mathcal{R}}\right)^{1+\nu_i}\mathcal{R}$
  - Now: $\bar{\nu}_i$ to match extraction data e^x_i, $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. ν_i extraction cost curvature to match profit $\pi^f_i = \frac{\bar{\nu}_i \nu_i}{1+\nu_i}\left(\frac{e^x_i}{\mathcal{R}_i}\right)^{\nu_i}\mathcal{R}_i \mathbb{P}_i$
  - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e^x_i (BP, IEA)
- Coal and Renewable: Production $\bar{e}^r_i, \bar{e}^x_i$ and price q^c_i, q^r_i
  - Calibrate $q^c_i = z^c \mathbb{P}_i$, $q^r_{it} = z^r \mathbb{P}_i$
    Choose z^c_i, z^r_i to match the energy mix (e^f_i, e^c_i, e^r_i)
- Population dynamics
  - Match UN forecast for growth rate / fertility

back

Quantification – Climate system and damage

- Static economic model:
  decisions $e_i^f + e_i^c$ taken "once and for all", $\mathcal{E} = \sum_i e_i^f + e_i^c$
  - Climate system:

$$\dot{\mathcal{S}}_t = \mathcal{E} - \delta_s \mathcal{S}_t$$
$$T_{it} = \bar{T}_{i0} + \Delta_i \mathcal{S}_t$$

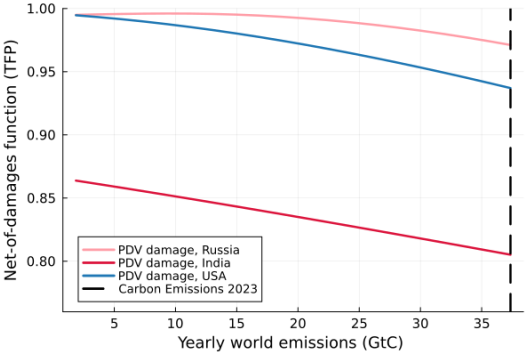
  - Path damages heterogeneous across countries
    Quadratic, c.f. Nordhaus-DICE / IAM

$$\mathcal{D}(T_{it} - T_i^\star) = e^{-\gamma (T_{it} - T_i^\star)^2}$$

  - Economic feedback in Present discounted value

$$\mathcal{D}_i^y(\mathcal{E}) = \bar{\rho} \int_0^\infty e^{-\overbrace{(\rho - n + (1-\eta)\bar{g})}^{\equiv \bar{\rho}} t} \mathcal{D}\left(T_{it} - T_i^\star\right) dt$$

  - Similarly for $\mathcal{D}_i^u(\mathcal{E})$, SCC and LCC_i . . . back



Calibration

Table: Baseline calibration ($\star$ = subject to future changes) back

Technology & Energy markets			
α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
Preferences & Time horizon			
ρ	0.015	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	Risk aversion	Standard Calibration
n	0.0035	Long run population growth	Average world population growth
Climate parameters			
ξ^f, ξ^c	2.761 & 3.961	Emission factor – Oil+nat. gas vs. Coal	Conversion 1 $MTOE \Rightarrow 1\ MT\ CO_2$
χ	$2.3/1e6$	Climate sensitivity	Pulse experiment: $100\,GtC \equiv 0.23°C$ medium-term warming
δ_s	0.0004	Carbon exit from atmosphere	Pulse experiment: $100\,GtC \equiv 0.15°C$ long-term warming
$\gamma^\oplus$	0.003406	Damage sensitivity	Nordhaus, Barrage (2023)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
$T^\star$	14.5	Optimal yearly temperature	Average yearly temperature/Developed economies

Matching country-level moments

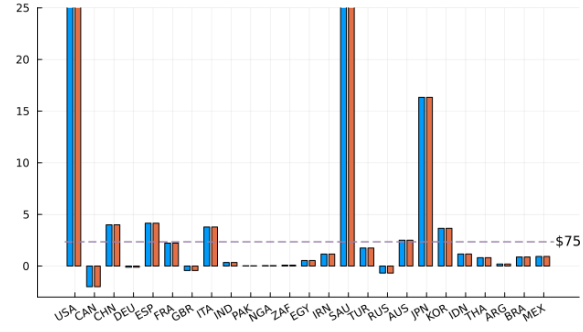
Table: Heterogeneity across countries

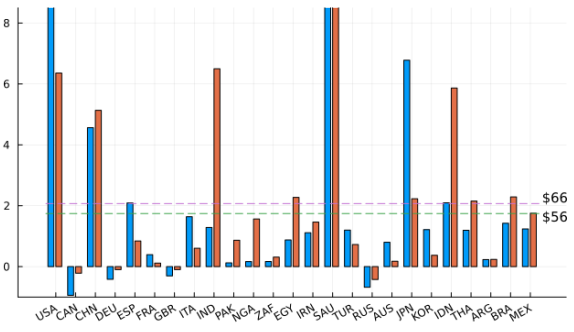
Dimension of heterogeneity	Model parameter	Matched variable from the data	Source
Population	Country size $\mathcal{P}_i$	Population	UN
TFP/technology/institutions	Firm productivity z_i	GDP per capita (2019-PPP)	WDI
Productivity in energy	Energy-augmenting productivity z_i^e	Energy cost share	SRE
Cost of coal energy	Cost of coal production $\mathcal{C}_i^c$	Energy mix/coal share e_i^c/e_i	SRE
Cost of non-carbon energy	Cost of non-carbon production $\mathcal{C}_i^r$	Energy mix/coal share e_i^r/e_i	SRE
Local temperature	Initial temperature T_{it_0}	Pop-weighted yearly temperature	Burke et al
Pattern scaling	Pattern scaling Δ_i	Sensitivity of T_{it} to world $\mathcal{T}_t$	Burke et al
Oil-gas reserves	Reserves $\mathcal{R}_i$	Proved Oil-gas reserves	SRE
Cost of oil-gas extraction	Slope of extraction cost $\bar{\nu}_i$	Oil-gas extracted/produced e_i^x	SRE
Cost of oil-gas extraction	Curvature of extraction cost ν_i	Profit π_i^f / energy rent	WDI
Trade costs	Distance iceberg costs τ_{ij}	Geographical distance $\tau_{ij} = d_{ij}^{\beta}$	CEPII
Armington preferences	CES preferences a_{ij}	Trade flows	CEPII

back

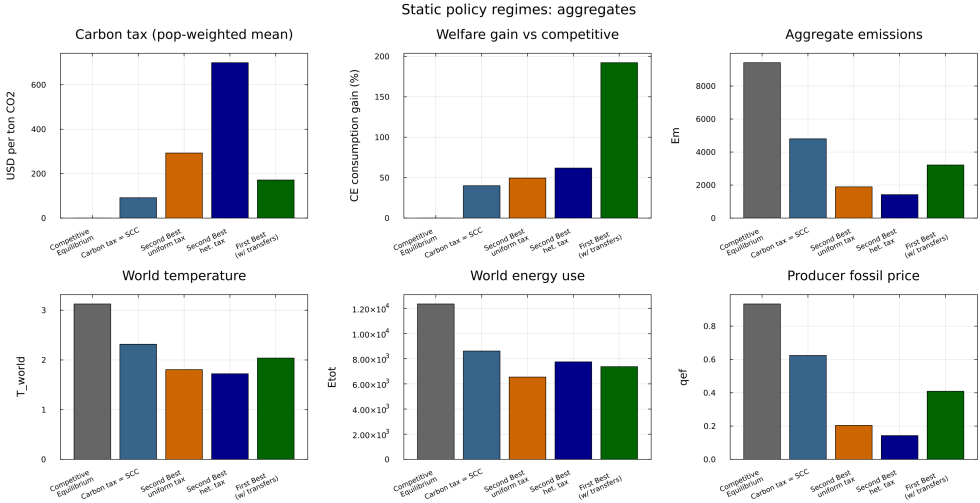
Local Cost of Carbon & Carbon Tax – First and Second Best

- Recall: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$
- Difference: $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$ vs. $\widehat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\overline{\lambda}^w}$





Allocations under different policy regimes



Allocations under different policy regimes

