

Climate Change, Inequality, and the Optimal Climate Policy

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Introduction

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- ▶ However, relatively little is known on how (and whether!) climate policy should depend on **inequality** across countries.
- ▶ Indeed, energy taxation and climate policy have strong **distributional** consequences:
 - (i) between higher income countries and developing economies
 - (ii) between warm and cold countries, and differential exposure to climate change
 - (iii) between energy importer and exporters, and dependence on energy consumption and rents

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⇒ What is the **optimal carbon policy** in the presence of climate externality and **inequality**?

- The optimal design of Pigouvian taxation crucially depends on **redistribution instruments** i.e. lump-sum transfers across countries

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- ▶ Study an Integrated Assessment Model (IAM) with heterogeneous countries to:
 - Evaluate the welfare costs of global warming and the Social Cost of Carbon
 - Solve for general *optimal Ramsey policies* for carbon taxation
 - Provide a novel numerical methodology for IAMs with *heterogeneity*

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- ▶ Preview of the results:
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 - ▶ Preview of the results:
 - Social Cost of Carbon and carbon tax needs to be adjusted for inequality levels
 - Energy taxation also accounts for supply and demand GE effects
- ⇒ **Theoretically:** effect *ambiguous*. **Quantitative:** carbon tax should be *lower* with inequality
- Country-specific taxes: poorer countries should pay relatively lower taxes

Roadmap

- ▶ Simplified set-up: Toy example
- ▶ First-best carbon taxation: all instruments available
- ▶ Second-best carbon taxation: Constrained efficient without lump-sum transfers
 - Uniform carbon taxation
 - Heterogeneous carbon prices across countries
- ▶ Quantitative model: Dynamic IAMs with heterogeneity & Extensions
- ▶ Optimal Climate Agreements

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Toy Model – Household & Firms

Static deterministic Neoclassical economy

- countries $i \in \mathbb{I}$, heterogeneous in productivity z_i , temperature T_i , energy extraction cost C_i
- In each country, five agents:
 - (i) Rep. household, (ii) final good firm, (iii-v) oil-gas (fossil), coal and renewable energy producers

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1. Representative household problem in each country i (passive):

$$\mathcal{V}_i = u(c_i) \qquad c_i = \underbrace{w_i \ell_i}_{\text{labor income}} + \underbrace{\pi_i^f}_{\text{fossil firm profit}} + \underbrace{t_i^{ls}}_{\text{lump-sum transfers}}$$

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2. Competitive final good producer in country i

$$\max_{\ell_i, e_i^f, e_i^c, e_i^r} \underbrace{D(T_i)}_{\text{climate damage}} \underbrace{z_i}_{TFP} f(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - \underbrace{(q^f + t_i^\varepsilon) e_i^f}_{\text{fossil (oil-gas)}} - \underbrace{(q_i^c + t_i^\varepsilon) e_i^c}_{\text{coal}} - \underbrace{q_i^r e_i^r}_{\text{non-carbon energy}}$$

- Climate policy: Carbon tax t_i^ε

Model – Energy markets & Emissions

3. Competitive fossil fuels energy producer, selling on international fossil market:

- Supply of fossil energy e_i^x by extraction at cost C_i^f

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - C_i^f(e_i^x)$$

$$E^f = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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4. Coal energy firm: elastic supply e_i^c at price $q_i^c = z_i^c$

5. Renewable energy firm: elastic supply e_i^r at price $q_i^r = z_i^r$

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► Climate system:

- GHGs from oil-gas and coal affect temperatures:

$$T_i = \bar{T}_{i0} + \Delta_i \mathcal{E} = \bar{T}_{i0} + \underbrace{\Delta_i}_{\text{pattern scaling}} \sum_{\mathbb{I}} \underbrace{(e_i^f + e_i^c)}_{\text{GHG from energy}}$$

Model – Equilibrium

- Given policies $\{t_i^f, t_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_i, e_i^f, e_i^r, e_i^x\}_i$, states $\{T_i\}_i$ and prices $\{q^f, q_i^c, q_i^r, w_i\}_i$ such that:
 - Households choose $\{c_i\}_i$ to max. utility s.t. budget constraint
 - Firm choose inputs $\{e_i^f, e_i^c, e_i^r\}_i$ to max. profit
 - Oil-gas firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e_i^c, e_i^r\}$
 - Emissions $\mathcal{E}$ affects climate $\{T_i\}_i$.
 - Government budget clear $\sum_i t_i^{ls} = \sum_i t_i^e (e_i^f + e_i^c)$
 - Prices $\{q^f, q_i^c, q_i^r\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_i^x = \sum_{\mathbb{I}} e_i^f$, and e_i^c, e_i^r
The good market clearing holds by Walras law.

Optimal world policy – Summary of results

- ▶ Competitive equilibrium Details eq 0
 - Passive policies $t^f = 0$, and large cost of climate change
- ▶ First-Best, with unlimited instruments Details eq 1
 - Welfare: $\mathcal{W} = \max_{\{t, c, e\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \omega_i \mathcal{V}_i$
 - Social Planner redistribute across countries with lump-sum transfers t_i^{ls}
 - Set the optimal Pigouvian carbon tax to $t^f = SCC$
- ▶ Second-best Ramsey policy, with limited instruments Details eq 2
 - Optimal carbon tax accounts for (i) inequality and local climate damage, (ii) energy supply elasticities, (iv) energy demand distortions
- ▶ Endogenous Participation? If countries can exit climate agreements Details eq 3
 - All formulas corrected for participation constraints (multipliers affect distribution weights)
 - Optimal design of climate agreements / climate clubs $\Rightarrow$ JMP

Competitive equilibrium

► Key objects:

- Marginal value of wealth $\lambda_i = u'(c_i)$
- Marginal value of carbon ψ_i^ε for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := -\frac{\partial \mathcal{V}_i / \partial \mathcal{S}}{\partial \mathcal{V}_i / \partial c_i} = \frac{\psi_i^\varepsilon}{\lambda_i} = \Delta_i \gamma (T_i - T_i^*) y_i > 0$$

- w/ Nordhaus-style damage $\mathcal{D}_i(T) = e^{-\gamma(T-T_i^*)^2}$
- Stationary eqbm closed-form formula, as in GHKT (2014)

Closed Form Solution

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First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{x}, \mathbf{c}, \mathbf{q}\}_i} \sum_{\mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \omega_i \mathcal{V}_i$$

- Full set of instruments $\mathbf{t} = \{\mathbf{t}_i^{\varepsilon}, \mathbf{t}_i^{ls}\}$, including transfers *across countries*

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- Full set of instruments $\mathbf{t} = \{\mathbf{t}_i^\varepsilon, \mathbf{t}_i^{ls}\}$, including transfers *across countries*
- Key objects: Local vs. Global Social Cost of Carbon,

$$scc^{fb} := -\frac{\partial \mathcal{W} / \partial \mathcal{S}}{\partial \mathcal{W} / \partial \bar{c}} = \frac{\psi^\varepsilon}{\bar{\lambda}} = \frac{\sum_{\mathbb{I}} \psi_i^\varepsilon}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_i}$$

$$LCC_i := \frac{\partial \mathcal{V}_i / \partial \mathcal{S}}{\partial \mathcal{V}_i / \partial c_i} = \frac{\psi_i^\varepsilon}{\lambda_i}$$

First-Best, Optimal policy with transfers

- Proposition 1: Optimal carbon tax:

$$t^{\varepsilon} = SCC^{fb}$$

- Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC^{fb} = \frac{\psi^{\varepsilon}}{\bar{\lambda}} = \sum_{\mathbb{I}} \frac{\psi_i^{\varepsilon}}{\lambda_i} = \sum_{\mathbb{I}} LCC_i$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_i) = \lambda_i = \bar{\lambda} = \lambda_j = \omega_j u'(c_j) \quad \forall i, j \in \mathbb{I}$$

- Imply cross-countries lump-sum transfers $\exists i$ s.t. $t_i^{ls} \geq 0$ or $\exists j$ s.t. $t_j^{ls} \leq 0$

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Ramsey policy with limited transfers

- Second best without access to lump-sum transfers: choice of a carbon tax $\{t^\varepsilon, t^r\}$

- Tax receipts redistributed lump-sum: $t_i^{ls} = t^\varepsilon e_i^f$
- Inequality across regions:

$$\hat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \leq 1$$

⇒ ceteris paribus, poorer countries have higher $\hat{\lambda}_i$

- Social Cost of Carbon integrates these inequalities:

$$SCC^{sb} = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i$$

$$SCC^{sb} = \sum_{\mathbb{I}} LCC_i + \text{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Ramsey Problem – Optimal Carbon and Energy Policy

► Tax on fossil energy has redistributive effects:

1. Redistributes rent through energy markets: lowers eq. fossil price, hurts exporters
2. Distorts energy demand, esp. countries that depend more on energy

$$\text{Supply Redistribution}^{sb} + \text{Demand Distortion}^{sb} = \underbrace{\mathcal{C}_{EE}^f}_{\text{agg. supply elasticity}} \underbrace{\text{Cov}_i(\hat{\lambda}_i, e_i^f - e_i^x)}_{\text{terms-of-trade redistribution}} - \underbrace{\text{Cov}_i\left(\hat{v}_i, \frac{q^f(1-s_i)}{\sigma_i e_i}\right)}_{\text{demand distortion}}$$

◦ Params: $\mathcal{C}_{EE}^f$ agg. fossil supply elasticity, s_i^e energy cost share and σ_i energy demand elasticity [Details](#)

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► Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad t^e = \text{SCC}^{sb} + \text{Supply Redistribution}^{sb} + \text{Demand Distortion}^{sb}$$

– Reexpressing demand terms:

$$t^e = \left(1 + \text{Cov}_i\left(\hat{\lambda}_i, \frac{\sigma_i e_i}{1-s_i^e}\right)\right)^{-1} \left[\sum_{\mathbb{I}} \text{LCC}_i + \text{Cov}_i(\hat{\lambda}_i, \text{LCC}_i) + \mathcal{C}_{EE}^f \text{Cov}_i(\hat{\lambda}_i, e_i^f - e_i^x)\right]$$

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Ramsey Problem – Country-specific carbon tax

- ▶ Suppose the planner has access to a *distribution* of carbon price.
- ▶ Proposition 3: Optimal country-specific fossil energy tax:

$$\Rightarrow \quad t_i^\varepsilon = \frac{1}{\widehat{\lambda}_i} \left[SCC^{sb} + \text{Supply Redistribution}^{sb} \right] \propto \frac{1}{\omega_i u'(c_i)}$$

- Social cost of carbon: $SCC^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_i LCC_i$
- Supply Redistribution^{sb} = $C_{EE}^f \text{Cov}_i \left(\widehat{\lambda}_i, e_i^f - e_i^x \right)$

$\Rightarrow$ Reduce the tax burden for poorer/more “valuable” countries

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Quantitative Dynamic Model

1. Climate system $\mathcal{S}_t = \mathcal{E} - \delta_s \mathcal{S}$, (multi-box climate system (WIP))
2. Firms:
 - Include capital to produce $f(\ell, k, e^f, e^c, e^r)$
 - Separate oil+gas e^f and coal e^c : coal rent = 0 vs. oil rent $\pi^f > 0$, higher emissions $\xi^c e^c$
 - Match the energy mix of each country $\{e^f, e^c, e^r\}$
3. Households:
 - Consumption / saving + capital $\Rightarrow$ Keynes-Ramsey rule
4. Energy markets
 - Fossil energy extraction/depleting reserves $\dot{\mathcal{R}}_{it} = -e_{it}^x \Rightarrow$ Hotelling problem (WIP)
 - Price of clean energy q_{it}^r trending down
5. Population $\dot{p}_i/p_i = n$ and TFP growth dynamics $\dot{z}_i/z_i = \bar{g}$
6. Energy-augmenting Directed TC (WIP)

Ramsey Problem – Theoretical extensions (in the papers)

- ▶ Trade: redistributive effect through leakage effect
 - Trade model à la Armington Details trade
 - Additional trade-off: account for distortions on goods central in trade networks
- ▶ Uncertainty:
 - Stochastic climate impact, e.g. climate sensitivity Δ_i or damage curvature γ_i , Details uncertainty
 - Interaction in SCC between uncertainty and inequalities: covariance between cost of climate change and social welfare weights
- ▶ Dynamic considerations
 - Reserves values ψ_{it}^R = Hotelling rent: carbon tax changes intertemp. subst^o of fossil prod. c.f. Heal, Schlenker (2019), Asker, Collard-Wexler, De Loecker (2019-2024),
 - Curb capital demand + distort consumption/saving decision c.f. optimal fiscal instruments in H.A. models, BEGS/LG-R/D-S

Dynamic models using the Pontryagin Max. Principle

- Every dynamic model can be expressed as a Forward Backward system of ODEs: states $\{k_{it}, \mathcal{S}_t\}$ and costates $\{\lambda_{it}^k, \psi_t^{\mathcal{S}}\}$. Ex NCG:

$$\begin{cases} \dot{k}_{it} &= f(k_{it}, e_{it}) - \delta k_{it} - q_{it}^e e_{it} - c_{it} \\ \dot{\lambda}_{it}^k &= -(f_k(k_{it}, e_{it}) - \delta - \rho) \lambda_{it}^k \end{cases}$$

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- Novelty:

- Heterogeneity: ex ante, arbitrary, ex post: number of state/costate
- Large state space: as a *sequential* approach, no need for grid, just follow the agents
- Optimal Ramsey (climate) policy:
policies usually expressed as fct of multipliers $\Leftrightarrow$ costates: $t^\varepsilon = SCC = -\psi_t^{\mathcal{S}} / \lambda_t^k$

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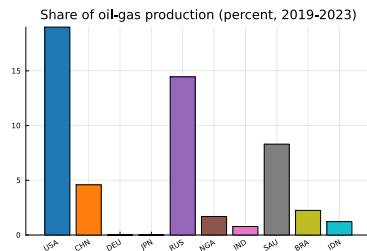
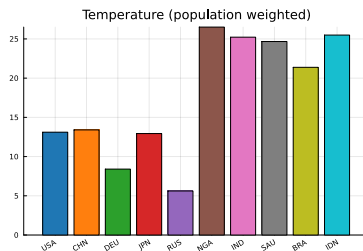
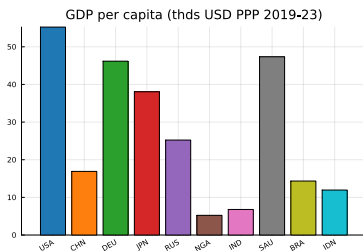
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- Global method $\rightarrow$ eqbm potentially far from steady state: non-linearity/tipping points?
- Caveat: Rely (for now) on non-linear solvers.

Quantification

- Parameters matching country-level data (avg. 2018-2023) Details Quantification
- TFP $z_i \Rightarrow$ GDP y_i , Population p_i , Temperature T_{i0} , Pattern scaling Δ_i
- Mix: oil-gas e_i^f , Coal e_i^c , Low-carbon e_i^r , energy share ε_i , oil-gas prod^o e_i^x , reserves $\mathcal{R}_i$, rents π_i^f



Inequality and Pareto weights

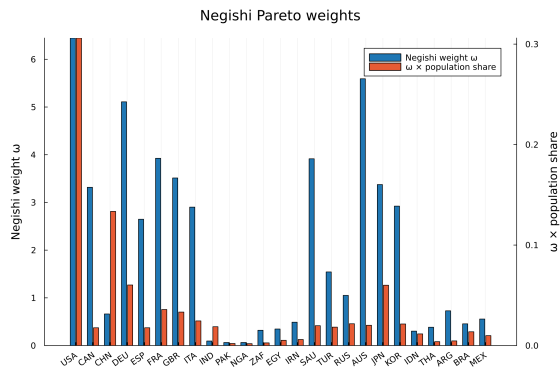
- ▶ Recall welfare objective: $\mathcal{W}_t = \sum_{i \in \mathbb{I}} p_i \omega_i \mathcal{V}_{it}$
- ▶ **Crucial:** how to choose ω_i ?

Inequality and Pareto weights

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- ▶ **Two benchmarks:**
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Inequality and Pareto weights

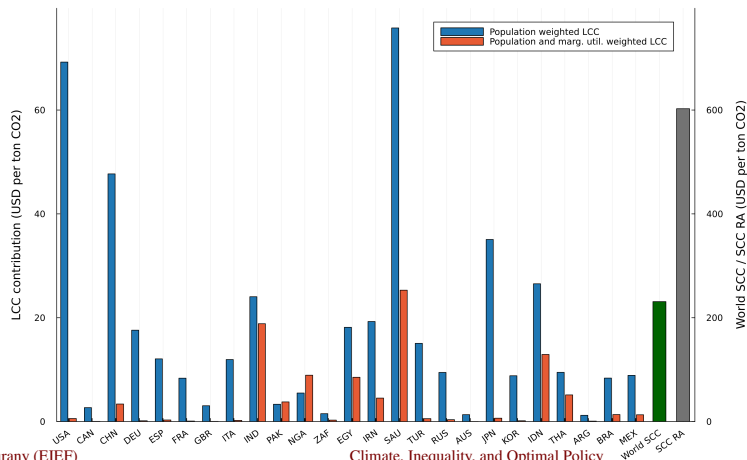
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- ▶ **Two benchmarks:**
 - ▷ Uniform Pareto weights $\omega_i = 1$
 - ▷ Negishi weights $\omega_i = \frac{1}{u'(\bar{c}_i)}$
 - with $\bar{c}_i$ conso in initial eqbm 2018-23 *without climate change*
 - implies no redistribution motive:
 $\omega_i u'(\bar{c}_i) = \omega_j u'(\bar{c}_j) \quad \forall i, j \in \mathbb{I}$
 - Climate change + carbon tax have redistributive effects $\Rightarrow$ change allocation c_i



Local Cost of Carbon & Social Cost of Carbon

- Recall: $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i$
- Difference: $LCC_i = \frac{\psi_i^S}{\lambda_i}$ (blue, left bars) vs. $\hat{\lambda}_i LCC_i = \frac{\psi_i^S}{\lambda}$ (red, right bars)

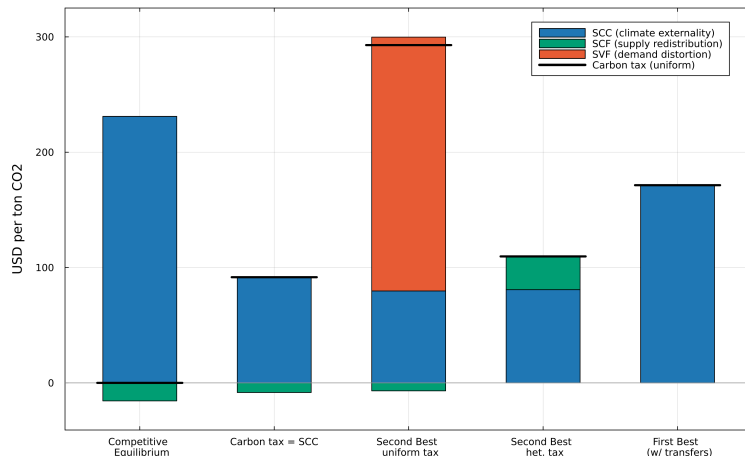
HA static competitive LCC weighting comparison



The Carbon Tax is not (only) the Social Cost of Carbon!

- ▶ First-best w/ transfers: $\Rightarrow t^E = SCC^{fb}$, and $|t_i^{ls}| \gg 0$
- ▶ Second-best carbon tax w/o transfers: $\Rightarrow t^E = SCC^{sb} + \text{Supply Redistrib.} + \text{Demand Distort}^o$
- ▶ Uniform weights $\omega_i = 1$

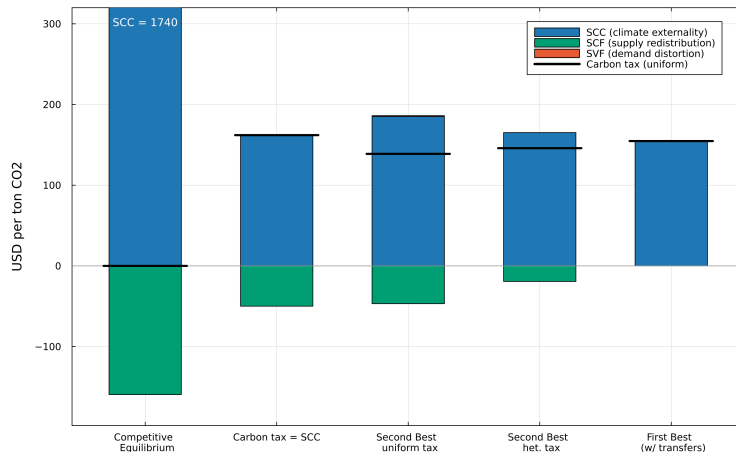
Carbon tax decomposition across policy regimes



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- ▶ First-best w/ transfers: $\Rightarrow t^E = SCC^{fb}$, and $|t_i^L| \gg 0$
- ▶ Second-best carbon tax w/o transfers: $\Rightarrow t^E = SCC^{sb} + \text{Supply Redistrib.} + \text{Demand Distort}^o$
- ▶ Negishi weights $\omega_i \propto 1/u'(\bar{c}_i)$

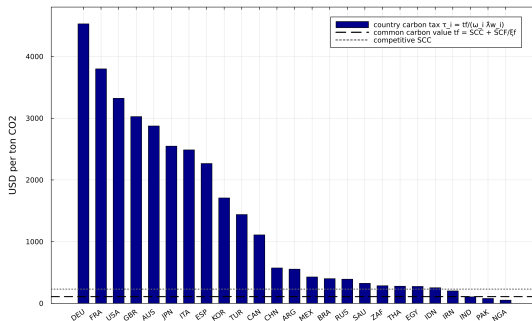
Carbon tax decomposition across policy regimes (Negishi weights)



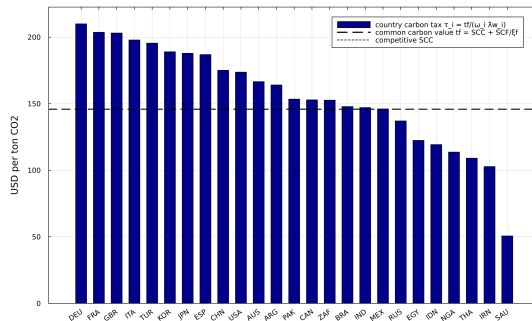
Heterogeneous Carbon taxes are not the Local Social Cost of Carbon!

- ▶ Het. carbon taxes: $t_i^\epsilon \propto \frac{1}{\omega_i u'(c_i)} [SCC^{sb} + \text{Supply Redistrib.}] \neq LCC_i$
- ▶ More results: Details Aggregates

Heterogeneous second-best carbon taxes



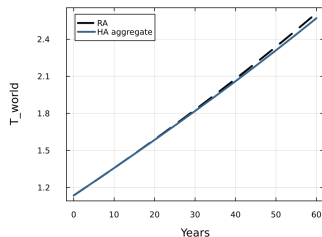
Heterogeneous second-best carbon taxes



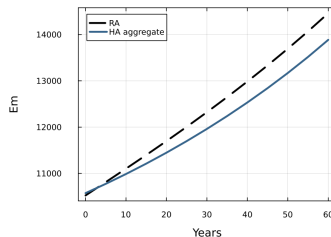
Climate dynamics (Business as Usual) with heterogeneity

Representative-agent vs heterogeneous-agent aggregate dynamics

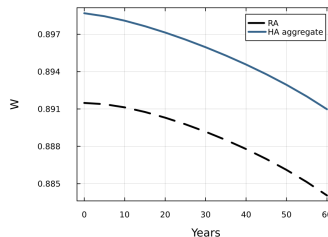
World temperature



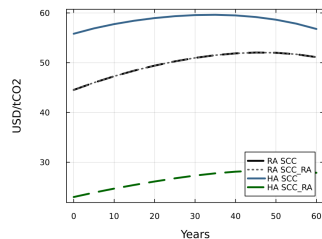
Aggregate emissions



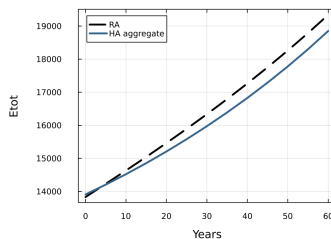
Welfare



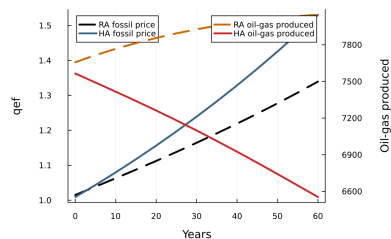
Social cost of carbon



World energy use



Fossil price and oil-gas production



Taking stocks + Going further?

► Optimal climate policy with inequality:

- Requires sizeable transfers. Otherwise, need large change in carbon tax level / trajectory
- Need to account for the distribution + reallocation/GE effects on energy and good markets

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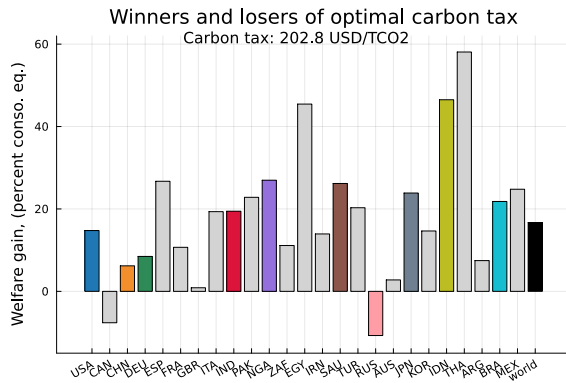
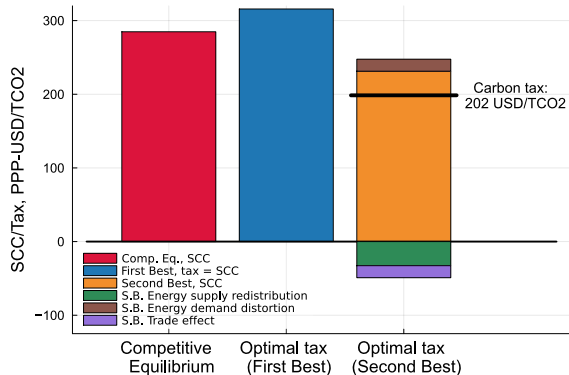
Taking stocks + Going further?

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 - ▶ Incentives and implementability:
 - What if some countries strategically deviate from applying the optimal carbon tax?
 - Game theoretical consideration $\Rightarrow$ participation constraints
 - Implementation of “climate clubs”:
Penalty tariffs or CBAM to non-participants crucial for enforcing carbon policy
- $\Rightarrow$ **Job Market Paper:** “The Optimal Design of Climate Agreements”

Roadmap

- ▶ Simplified set-up: Toy example
- ▶ First-best carbon taxation: all instruments available
- ▶ Second-best carbon taxation: Constrained efficient without lump-sum transfers
 - Uniform carbon taxation
 - Heterogeneous carbon prices across countries
- ▶ Quantitative model: Dynamic IAMs with heterogeneity & Extensions
- ▶ **Optimal Climate Agreements**

Second-Best climate policy and Gains from cooperation



Main result and Intuition

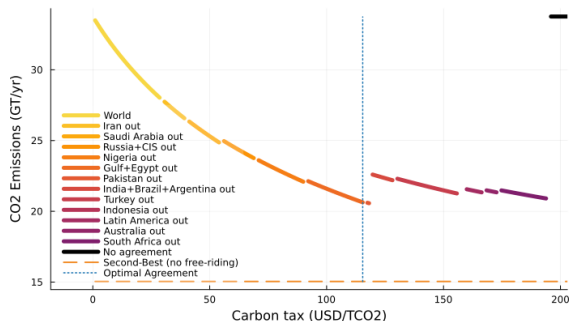
- ▶ The optimal climate agreement navigates the **intensive** and **extensive** margin tradeoff:
 - **Participation:** most countries in the world with the exception of: *Russia, former Soviet countries, Saudi Arabia, Gulf countries, Egypt, Iran, Nigeria*
 - **Carbon tax:** reduce carbon tax from \$200 to \$120/ tCO_2
 - **Trade tariffs:** impose substantial tariff 90% on the goods from non-members

Main result and Intuition

- ▶ The optimal climate agreement navigates the **intensive** and **extensive** margin tradeoff:
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 - **Carbon tax:** reduce carbon tax from \$200 to \$120/tCO₂
 - **Trade tariffs:** impose substantial tariff 90% on the goods from non-members
 - ▶ **Mechanism:**
 - Countries participate depending on $\left\{ \begin{array}{l} \text{(i) the cost of distortionary carbon taxation} \\ \text{(ii) the cost of tariffs (= the gains from trade)} \end{array} \right.$
 - Russia/Middle East/South Asia do not join the club for high carbon tax *for any tariffs* when cost of taxing fossil-fuels $\gg$ cost of tariffs / autarky
- ⇒ Result, we need to decrease the carbon tax: *reduce ambition*

Laffer curve for carbon taxation

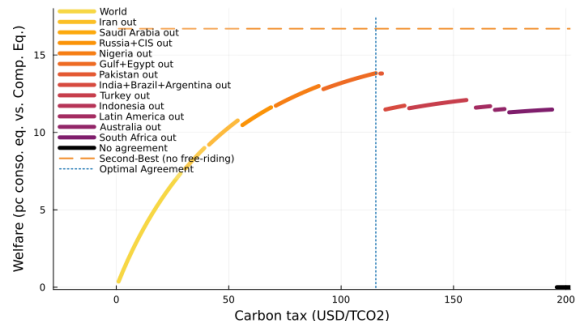
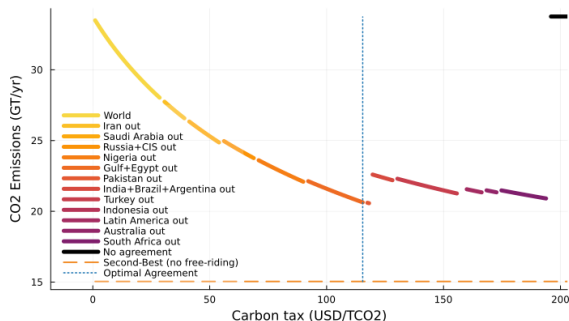
- Due to free-riding incentives, **cannot reach** globally optimal carbon tax $t^{\varepsilon,*} = \$200$



Emissions $\mathcal{E}$ (in $GtCO_2/yr$) and welfare $\mathcal{W}$ as function of the carbon tax t^{ε} , with tariff $t^b = 90\%$.

Laffer curve for carbon taxation

- Due to free-riding incentives, **cannot reach** globally optimal carbon tax $t^{\varepsilon,*} = \$200$
- Need to lower the carbon tax to \$120 to **increase participation**:
Improve welfare by sharing the costs of carbon mitigation with *more countries*



Emissions $\mathcal{E}$ (in $GtCO_2/yr$) and welfare $\mathcal{W}$ as function of the carbon tax t^{ε} , with tariff $t^b = 90\%$.

Climate Agreements: Intensive vs. Extensive Margin

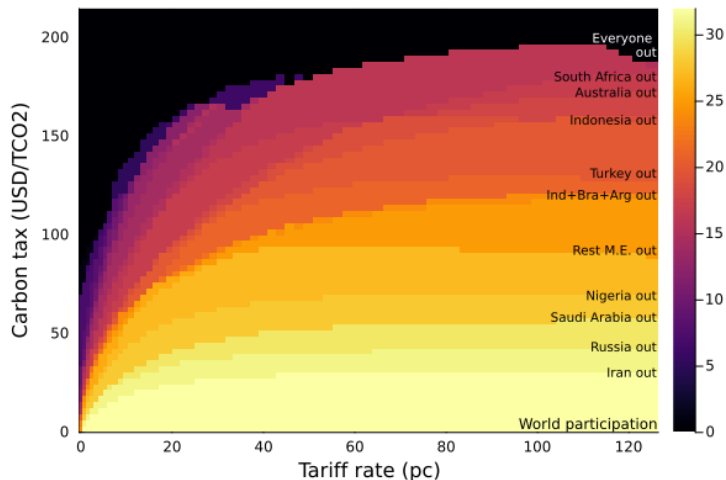
► Intensive margin:

given a coalition:

higher tax $t^{\mathcal{E}}$, emissions $\mathcal{E} \downarrow$,
improve welfare $\mathcal{W} \uparrow$

► Extensive margin:

carbon tax also deters
participation
individual countries free-ride
increasing emissions $\mathcal{E} \uparrow$

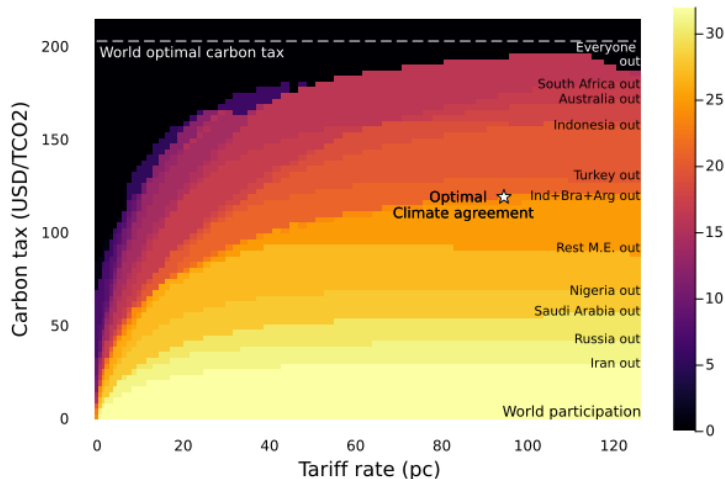


Optimal Climate Agreement

► Despite full discretion of instruments (t^e, t^b), we cannot sustain an agreement with Russia, Middle East, South-Asia & South America

⇒ need to **reduce carbon tax** from \$200 to \$120

⇒ Beneficial to **leave several fossil-fuel producers outside the agreement**
e.g. no incentive for Russia to join: cold, closed to trade, large fossil-fuel producer



Graph welfare

Mechanisms

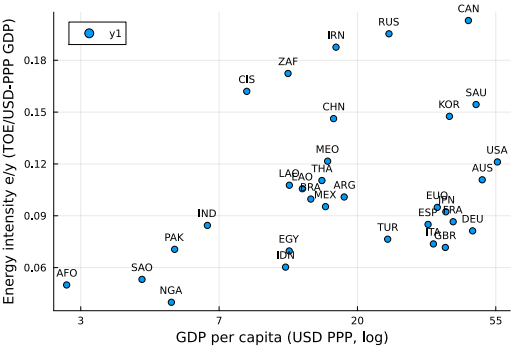
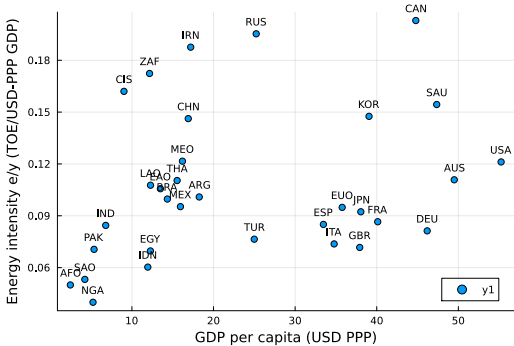
Conclusion

- ▶ In these projects, I solve for the optimal climate policy
 - Accounting for *inequality* + GE effects through energy markets and trade leakage
 - ... and strategic *free-riding incentives*
 - ▶ Additionally climate agreements design jointly solve for:
 - The optimal choice of countries participating
 - The carbon tax and tariff levels, accounting for participation constraints
 - ▶ The optimal climate club:
 - *trade-off*: gains of cooperation / free-riding incentives
vs. gains from trade, i.e. cost of retaliation tariffs
- ⇒ large coalition, lower the carbon tax from \$250 to \$200 to \$120

Appendices

Energy intensity

► Energy intensity e_i/y_i vs. GDP per capita y_i



Step 0: Competitive equilibrium & Trade

- ▶ Each household in country i maximize utility and firms maximize profit
- ▶ Standard trade model results:
 - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i p_i} = a_{ij} \frac{(d_{ij}(1+t_{ij}^b)p_j)^{1-\theta}}{\sum_k a_{ik}(d_{ik}(1+t_{ik}^b)p_k)^{1-\theta}} \quad \& \quad p_i = \left(\sum_j a_{ij}(d_{ij}p_j)^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Energy consumption doesn't internalize climate damage:

$$p_i M P e_i = q^e$$

- Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

- “Local Social Cost of Carbon”, for region i

$$LCC_i = \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \frac{p_i}{p_i} \quad (> 0 \text{ if heat causes losses})$$

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{\mathbb{I}} \mathcal{V}_i$$

- Full array of instruments: cross-countries lump-sum transfers $\mathbf{t}_i^{ls}$, individual carbon taxes $\mathbf{t}_i^e$ on energy e_i^f , bilateral tariffs $\mathbf{t}_{ij}^b$
 - Budget constraint: $\sum_i \mathbf{t}_i^{ls} = \sum_i \mathbf{t}_i^f e_i^f + \sum_{i,j} \mathbf{t}_{ij}^b c_{ij} d_{ij} p_j$
- Maximize welfare subject to
- Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

► Social planner results:

- Consumption:

$$\omega_i u'(c_i) = \left[\sum_j a_{ij} (d_{ij} \omega_j \mu_j)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

- Energy use:

$$\omega_i \mu_i MPe_i = \mu^e + SCC$$

- Social cost of carbon:

$$SCC = - \frac{\sum_j \Delta_j \omega_j \mu_j \mathcal{D}'_j(T_j) \bar{y}_j}{\frac{1}{I} \sum_j \omega_j \mu_j}$$

[back](#)

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \mathbf{e}, \mathbf{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i u(c_i) = \sum_{i \in \mathbb{I}} \mathcal{V}_i$$

- One single instrument: uniform carbon tax $\mathbf{t}^e$ on energy e_i^f
 - Rebate tax lump-sum to HHs $\mathbf{t}_i^{ls} = \mathbf{t}^e e_i^f$
- Ramsey policy: Primal approach, maximize welfare subject to
 - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
 - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
 - Trade-off faced by the planner:
 - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

[back](#)

Step 2: World optimal Ramsey policy

- The planner takes into account
 - (i) the **marginal value of wealth** λ_i
 - (ii) the **shadow value of good i** , from market clearing, μ_i :

$$\text{w/o trade} \quad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \quad \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} p_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right] \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade:} \quad \hat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{j \in \mathbb{I}} \omega_j u'(c_j)} \leq 1 \quad \Rightarrow \quad \text{ceteris paribus, poorer countries have higher } \hat{\lambda}_i$$

$$\text{vs. w/ trade:} \quad \hat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{j \in \mathbb{I}} \omega_j (\lambda_j + \mu_j)} \leq 1$$

Step 2: Optimal policy – Social Cost of Carbon

► Key objects: Local vs. Global Social Cost of Carbon:

- Marginal cost of carbon $\psi_i^\mathcal{E}$ for country i
- “Local social cost of carbon” (LCC) for region i :

$$LCC_i := \frac{\partial \mathcal{V}_i / \partial \mathcal{E}}{\partial \mathcal{V}_i / \partial w_i} = \frac{\psi_i^\mathcal{E}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_{if}(e_i^f) p_i \quad (> 0 \text{ if heat causes losses})$$

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- Social Cost of Carbon for the planner:

$$SCC := \frac{\partial W / \partial \mathcal{E}}{\partial W / \partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)}$$

- Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{Cov}_i(\hat{\lambda}_i, LCC_i)$$

Step 2: Optimal policy – Other motives

- ▶ Taxing fossil energy has additional redistributive effects:
 1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
 2. Distort energy demand, of countries that need more or less energy
- ▶ New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W} / \partial E}{\partial \mathcal{W} / \partial w} = \mathcal{C}_{EE}^f \text{Cov}_i \left(\hat{\lambda}_i, e_i^f - e_i^x \right) - \text{Cov}_i \left(\hat{\lambda}_i, \frac{q^f (1 - s_i^f)}{\sigma} \right)$$

- Params: $\mathcal{C}_{EE}^f$ agg. fossil supply elasticity, s_i^f energy cost share and σ energy demand elasticity

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- ▶ Proposition 2: Optimal fossil energy tax:

$$\Rightarrow \quad t^\epsilon = SCC + SVF$$

– Social cost of carbon: $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i LCC_i$

Step 2: Optimal policy – Details

► Taxing fossil energy has additional redistributive effects:

1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers

$$\begin{aligned}\text{Supply Redistribution} &= \left(\sum_i \frac{1}{C_{ee}^i(e_i^x)} \right)^{-1} \sum_i \hat{\lambda}_i (e_i^f - e_i^x) \\ &= q^f \left(\sum_i \frac{e_i^x}{\nu_i} \right)^{-1} \sum_i \hat{\lambda}_i (e_i^f - e_i^x)\end{aligned}$$

2. Distort energy demand, of countries that need more or less energy:

FOC $MPe_i^f = q^f$, multiplier $\propto \hat{v}_i^f$

$$\begin{aligned}\text{Demand Distortion} &= \frac{1}{I} \sum_i \hat{v}_i^f \partial_{e^f} MPe_i^f + \hat{v}_i^r \partial_{e^f} MPe_i^f \\ &= -(q^f + t^f) \frac{1}{I} \sum_i \hat{v}_i^f \frac{1}{e_i^f} \left[\frac{1-s_i^f}{\sigma^e} + s_i^f \frac{1-s_i^e}{\sigma^y} \right] + \frac{1}{I} \sum_i \hat{v}_i^r \frac{q_i^r}{e_i^f} s_i^f \left[\frac{1}{\sigma^e} - \frac{1-s_i^e}{\sigma^y} \right]\end{aligned}$$

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► Proposition 2: Optimal fossil energy tax: [back](#)

$$\Rightarrow \quad t^e = \text{SCC} + \text{Supply Redistrib.} + \text{Demand Distortion}$$

– Social cost of carbon: $\text{SCC} = \sum_{\mathbb{I}} \hat{\lambda}_i \text{LCC}_i$

Step 3: Ramsey Problem with participation constraints

- ▶ Consider that countries can “exit” climate agreement.
- ▶ For a climate “club” of $\mathbb{J} \subset \mathbb{I}$ countries:
 - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^e
 - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $t^{b,r}$ on goods and get consumption $\tilde{c}_i$
 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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 - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f
- ▶ Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- ▶ Welfare:

$$\mathcal{W} = \max_{\{t, e, q\}_i} \sum_{\mathbb{J}} \omega_i u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i u(c_i)$$

Step 3: Ramsey Problem with participation constraints

► Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

► Proposition 3.1: Second-Best social valuation with participation constraints

- Participation incentives change our measure of inequality

$$\text{w/ trade:} \quad \omega_i(1+\nu_i)u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij}(d_{ij}p_j)^{1-\theta} \left[\omega_i \tilde{\lambda}_i + \omega_j \tilde{\mu}_j + \tilde{\eta}_{ij}(1-s_{ij}) \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(\tilde{\lambda}_i + \tilde{\mu}_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(\tilde{\lambda}_i + \tilde{\mu}_i)} \neq \hat{\lambda}_i$$

$$\text{vs. w/o trade} \quad \hat{\tilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J} \sum_{j \in \mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \hat{\lambda}_i$$

- Result: $\omega_i(1+\nu_i)$ are the “endogenous Pareto weights”

Step 3: Participation constraints & Optimal policy

► Proposition 3.2: Second-Best taxes:

- Taxation with imperfect instruments:
 - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
 - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu \sigma}{q^f (1-s^f)}$
with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
- Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\Rightarrow t^e(\mathbb{J}) = \text{SCC} + \text{SVF}$$

$$= \frac{1}{1 - \vartheta_{\mathbb{J}^c}} \sum_{i \in \mathbb{I}} \tilde{\lambda}_i \text{LCC}_i + \frac{1}{1 - \vartheta_{\mathbb{J}^c}} c_{EE}^f \sum_{i \in \mathbb{I}} \tilde{\lambda}_i (e_i^f - e_i^x) - \sum_{i \in \mathbb{J}} \tilde{\lambda}_i \frac{q^f (1-s_i^f)}{\sigma}$$

- Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
As of now, only opaque system of equations (fixed point w/ demand/multipliers)

Climate uncertainty and the Cost of Carbon:

- Stochastics: for any shock ϵ with distribution $\epsilon \sim \varphi(\epsilon)$
- New measure for **inequalities**:

$$\hat{\lambda}_{it}^w(\epsilon) = \frac{\lambda_{it}^k(\epsilon)}{\mathbb{E}_{i,\epsilon}[\lambda_{it}^w(\epsilon)]} = \frac{\omega_i u'(c_{it}(\epsilon))}{\int_{\epsilon} \int_j \omega_j u'(c_{j,t}(\epsilon)) dj d\varphi(\epsilon)}$$

- Uncertainty-adjusted SCC writes:

$$\begin{aligned} \mathbb{E}_{\epsilon}[SCC] &= \int_{\mathcal{E}} \int_{\mathbb{I}} \hat{\lambda}_{it}^w(\epsilon) LCC_{it}(\epsilon) d\varphi(\epsilon) \\ &= \underbrace{\mathbb{E}_j \left[\text{Cov}_{\epsilon} \left(\hat{\lambda}_{it}^w(\epsilon), LCC_{jt}(\epsilon) \right) \right]}_{=\text{effect of aggregate risk } \epsilon} + \underbrace{\text{Cov}_j \left[\mathbb{E}_{\epsilon} \left(\hat{\lambda}_{it}^w(\epsilon) \right), \mathbb{E}_{\epsilon} \left(LCC_{jt}(\epsilon) \right) \right]}_{=\text{effect of heterogeneity across } j} + \underbrace{\mathbb{E}_{j,\epsilon} [LCC_{jt}(\epsilon)]}_{=\text{average exp. damage}} \\ &> \mathbb{E}_{\epsilon} [\overline{SCC}(\epsilon)] \quad \& \quad > SCC_t \end{aligned}$$

⇒ Climate uncertainty reinforces the unequal costs of climate change! [back](#)

Sequential solution method

► Summary of the dynamic model:

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

Sequential solution method

► Summary of the dynamic model:

- ODEs for states $\{\mathbf{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
- Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda_{it}^w, \lambda_{it}^\tau, \lambda_t^S, \lambda_{it}^{\mathcal{R}}\}_{it}$
- Non-linear equations (FOCs) for household controls $\{\mathbf{c}_1\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\mathbf{c}_2\} = \{e_{it}^f, e_{it}^r, k_{it}\}_{it}$ and static supplies $\{\mathbf{c}_3\} = \{e_{it}^x, \bar{e}_{it}^r\}_{it}$.
- Market clearing as equation for prices $\{\mathbf{q}\} = \{q_t^f, r_t^*\}_t$
- Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

► Global Numerical solution:

- Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
- Express as a large vector $\mathbf{y} = \{\mathbf{x}, \boldsymbol{\lambda}, \mathbf{c}, \mathbf{q}\}$ in a large non-linear function

$$F(\mathbf{y}) = \mathbf{0}$$

- Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Impact of increase in temperature

- Marginal values of the climate variables: λ_{it}^s and λ_{it}^τ

$$\dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho + \zeta) + \overbrace{\gamma_i(\tau_{it} - \tau_i^*) \mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it}, e_{it}) \lambda_{it}^k + \overbrace{\phi_i(\tau_{it} - \tau_i^*) \mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c, \tau)}$$

$$\dot{\lambda}_{it}^s = \lambda_{it}^s(\rho + \delta^s) - \zeta \chi \Delta_i \lambda_{it}^\tau$$

- Costate λ_{it}^s : marg. cost of 1Mt carbon in atmosphere, for country i . Increases with:
- Temperature gaps $\tau_{it} - \tau_i^*$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
 - Development level $f(k_{it}, e_{it})$ and c_{it}
 - Climate params: χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
 - [back](#)

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature $\Leftrightarrow$ solving ODE

$$\dot{\lambda}_{it}^{\tau} = \lambda_t^{\tau}(\tilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^*)\mathcal{D}^y(\tau)f(k, e)\lambda_t^k + \phi(\tau - \tau^*)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda_t^S(\tilde{\rho} + \delta^S) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\tau}$$

- Solving for λ_t^{τ} and λ_t^S , in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^{\tau} = 0$

$$\lambda_{it}^{\tau} = - \int_t^{\infty} e^{-(\tilde{\rho} + \zeta)u} (\tau_u - \tau^*) \left(\gamma \mathcal{D}^y(\tau_u) y_{\tau} \lambda_u^k + \phi \mathcal{D}^u(\tau_u) u(c_u) \right) du$$

$$\lambda_{it}^{\tau} = - \frac{1}{\tilde{\rho} + \Delta\zeta} (\tau_{\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{\infty}) y_{\infty} \lambda_{\infty}^k + \phi \mathcal{D}^u(\tau_{\infty}) u(c_{\infty}) \right)$$

$$\lambda_t^S = - \int_t^{\infty} e^{-(\tilde{\rho} + \delta^S)u} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,u}^{\tau} dj du$$

$$= \frac{1}{\tilde{\rho} + \delta^S} \zeta \chi \int_{\mathbb{I}} \Delta_j \lambda_{j,\infty}^{\tau}$$

$$= - \frac{\chi}{\tilde{\rho} + \delta^S} \frac{\zeta}{\tilde{\rho} + \zeta} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{\infty} \lambda_{j,\infty}^k + \phi \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

$$\lambda_t^S \xrightarrow{\zeta \rightarrow \infty} - \frac{\chi}{\tilde{\rho} + \delta^S} \int_{\mathbb{I}} \Delta_j (\tau_{j,\infty} - \tau^*) \left(\gamma \mathcal{D}^y(\tau_{j,\infty}) y_{j,\infty} \lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty}) u(c_{j,\infty}) \right) dj$$

Cost of carbon / Marginal value of temperature

► *Proposition (Stationary LCC):*

When $t \rightarrow \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \rightarrow \tau_\infty$, the LCC is *proportional* to climate sensitivity χ , **marg. damage** γ_i^y , γ_i^u , **temperature**, and **output, consumption**.

$$LCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^*) \left(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \right)$$

- Stationary equilibrium: $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
- Fast temperature adjustment $\zeta \rightarrow \infty$
- [Back](#)

Quantification: Climate and parametrization

► Climate block: Climate System

- Static economic model: decisions taken “once and for all”, e.g. $\mathcal{E} = \sum_i e_i^f + e_i^c$.
- “Dynamic” climate system: $\dot{S}_t = \mathcal{E} - \delta_s S_t$, $T_{it} = \bar{T}_{i0} + \Delta_i S_t$
- Quadratic damage functions as in Nordhaus-DICE: $\mathcal{D}(T_{it} - T_i^*) = e^{-\gamma(T_{it} - T_i^*)^2}$
- Feedback in Present discounted value: $\mathcal{D}_i^y(\mathcal{E}) = \bar{\rho} \int_0^\infty e^{-(\rho - n + (1 - \eta)\bar{g})t} \mathcal{D}(T_{it} - T_i^*) dt$

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- **Estimate** γ from trade data (as Rudik, Lyn, Tan, and Ortiz-Bobea (2021)) Damage, Gravity

$$\begin{aligned} \log \mathbb{E}[X_{ij}/X_{ii}] &= \beta_0(T_{jt} - T_{it}) + \beta_1(T_{jt}^2 - T_{it}^2) + \varsigma_{ij} + \varsigma_t + \Gamma' W_{it} + \Omega' W_{jt} \\ \Rightarrow \gamma &= 0.012 \approx 3 \times \gamma_{DICE} \end{aligned}$$

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► Standard functional forms:

- CRRA utility, Nested CES production, Iso-elastic fossil fuel extraction back

Quantification – Firms

- Production function $y_i = \mathcal{D}_i^y(T_i) z_i F(k, \varepsilon(e^f, e^r))$

$$F_i(\varepsilon(e^f, e^c, e^r), \ell) = \left[(1 - \epsilon) \frac{1}{\sigma_y} (\bar{k}^\alpha \ell^{1-\alpha})^{\frac{\sigma_y-1}{\sigma_y}} + \epsilon \frac{1}{\sigma_y} \left(z_i^e \varepsilon_i(e^f, e^c, e^r) \right)^{\frac{\sigma_y-1}{\sigma_y}} \right]^{\frac{\sigma_y}{\sigma_y-1}}$$

$$\varepsilon_i(e^f, e^c, e^r) = \left[(\omega^f)^{\frac{1}{\sigma_e}} (e^f)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^c)^{\frac{1}{\sigma_e}} (e^c)^{\frac{\sigma_e-1}{\sigma_e}} + (\omega^r)^{\frac{1}{\sigma_e}} (e^r)^{\frac{\sigma_e-1}{\sigma_e}} \right]^{\frac{\sigma_e}{\sigma_e-1}}$$

- Calibrate TFP z_i to match $y_i = GDP_i$ per capita in 2019-23 (avg. PPP).
- Technology: $\omega^f = 56\%$, $\omega^c = 27\%$, $\omega^r = 17\%$, $\epsilon = 12\%$ for all i
- Calibrate (z_i^e) to match Energy/GDP $q^e e_i / p_i y_i$

- Damage functions in production function y :

$$\mathcal{D}_i^y(T) = e^{-\gamma_i^{\pm, y} (T - T_i^*)^2}$$

- Asymmetry in damage to match empirics with $\gamma^y = \gamma^{+, y} \mathbb{1}_{\{T > T_i^*\}} + \gamma^{-, y} \mathbb{1}_{\{T < T_i^*\}}$
- Symmetric damage: $\gamma_i^{\pm, y} = \bar{\gamma}^{\pm, y}$ & $T_i^* = \bar{\alpha} T_{it_0} + (1 - \bar{\alpha}) T^*$

Quantification – Energy markets

- ▶ Fossil production e_{it}^x and reserve $\mathcal{R}_{it}$
 - Cost $\mathcal{C}_i(e^x, \mathcal{R}) = \frac{\bar{\nu}_i}{1+\nu_i} \left(\frac{e^x}{\mathcal{R}} \right)^{1+\nu_i} \mathcal{R}$
 - Now: $\bar{\nu}_i$ to match extraction data e_i^x , $\mathcal{R}_{it}$ calibrated to *proven reserves* data from BP. ν_i extraction cost curvature to match profit $\pi_i^f = \frac{\bar{\nu}_i \nu_i}{1+\nu_i} \left(\frac{e_i^x}{\mathcal{R}_i} \right)^{\nu_i} \mathcal{R}_i \mathbb{P}_i$
 - Future: Choose $(\bar{\nu}_i, \nu_i, \mathcal{R}_i)$ to match marginal cost $\mathcal{C}_e$ & extraction data e_i^x (BP, IEA)
- ▶ Coal and Renewable: Production $\bar{e}_i^r, \bar{e}_i^x$ and price q_i^c, q_i^r
 - Calibrate $q_i^c = z^c \mathbb{P}_i, q_i^r = z^r \mathbb{P}_i$
Choose z_i^c, z_i^r to match the energy mix (e_i^f, e_i^c, e_i^r)
- ▶ Population dynamics
 - Match UN forecast for growth rate / fertility

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Quantification – Climate system and damage

► Static economic model:

decisions $e_i^f + e_i^c$ taken “once and for all”, $\mathcal{E} = \sum_i e_i^f + e_i^c$

- Climate system:

$$\dot{S}_t = \mathcal{E} - \delta_s S_t$$

$$T_{it} = \bar{T}_{i0} + \Delta_i S_t$$

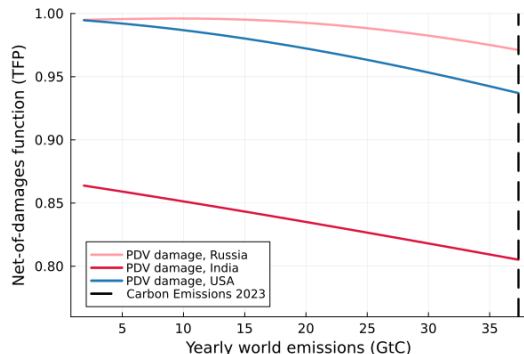
- Path damages heterogeneous across countries
Quadratic, c.f. Nordhaus-DICE / IAM

$$\mathcal{D}(T_{it} - T_i^*) = e^{-\gamma(T_{it} - T_i^*)^2}$$

- Economic feedback in Present discounted value

$$\mathcal{D}_i^y(\mathcal{E}) = \bar{\rho} \int_0^\infty e^{-\overbrace{(\rho - n + (1-\eta)\bar{g})}^{\equiv \bar{\rho}} t} \mathcal{D}(T_{it} - T_i^*) dt$$

- Similarly for $\mathcal{D}_i^u(\mathcal{E})$, SCC and $LCC_i \dots$

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Calibration

Table: Baseline calibration (* = subject to future changes) [back](#)

Technology & Energy markets

α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ_e	2.0	Elasticity fossil-renewable	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	0.01*	Long run TFP growth	Conservative estimate for growth

Preferences & Time horizon

ρ	0.015	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	Risk aversion	Standard Calibration
n	0.0035	Long run population growth	Average world population growth

Climate parameters

ξ^f, ξ^c	2.761 & 3.961	Emission factor – Oil+nat. gas vs. Coal	Conversion 1 MTOE $\Rightarrow$ 1 MT CO ₂
χ	2.3/1e6	Climate sensitivity	Pulse experiment: 100 GtC $\equiv$ 0.23°C medium-term warming
δ_s	0.0004	Carbon exit from atmosphere	Pulse experiment: 100 GtC $\equiv$ 0.15°C long-term warming
$\gamma^\oplus$	0.003406	Damage sensitivity	Nordhaus, Barrage (2023)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
T^*	14.5	Optimal yearly temperature	Average yearly temperature/Developed economies

Matching country-level moments

Table: Heterogeneity across countries

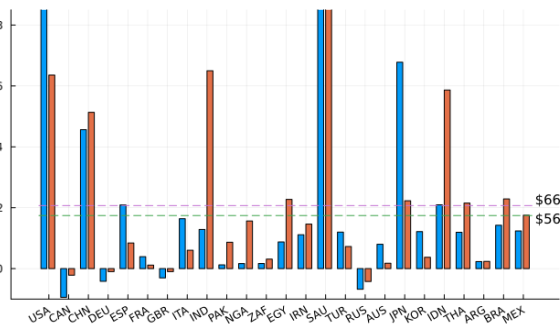
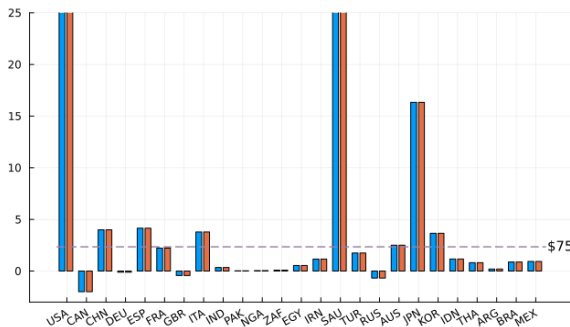
Dimension of heterogeneity	Model parameter	Matched variable from the data	Source
Population	Country size $\mathcal{P}_i$	Population	UN
TFP/technology/institutions	Firm productivity z_i	GDP per capita (2019-PPP)	WDI
Productivity in energy	Energy-augmenting productivity z_i^e	Energy cost share	SRE
Cost of coal energy	Cost of coal production C_i^c	Energy mix/coal share e_i^c/e_i	SRE
Cost of non-carbon energy	Cost of non-carbon production C_i^r	Energy mix/coal share e_i^r/e_i	SRE
Local temperature	Initial temperature T_{it_0}	Pop-weighted yearly temperature	Burke et al
Pattern scaling	Pattern scaling Δ_i	Sensitivity of T_{it} to world $\bar{T}_t$	Burke et al
Oil-gas reserves	Reserves $\mathcal{R}_i$	Proved Oil-gas reserves	SRE
Cost of oil-gas extraction	Slope of extraction cost $\bar{\nu}_i$	Oil-gas extracted/produced e_i^x	SRE
Cost of oil-gas extraction	Curvature of extraction cost ν_i	Profit π_i^f / energy rent	WDI
Trade costs	Distance iceberg costs τ_{ij}	Geographical distance $\tau_{ij} = d_{ij}^\beta$	CEPII
Armington preferences	CES preferences a_{ij}	Trade flows	CEPII

Local Cost of Carbon & Carbon Tax – First and Second Best

► Recall: $SCC = \sum_{\mathbb{I}} \hat{\lambda}_i^w LCC_i$

► Difference: $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$ vs.

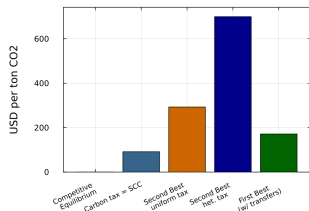
$$\hat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\lambda_i^w}$$



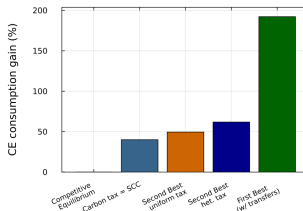
Allocations under different policy regimes

Static policy regimes: aggregates

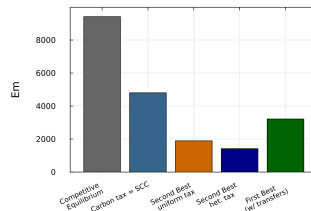
Carbon tax (pop-weighted mean)



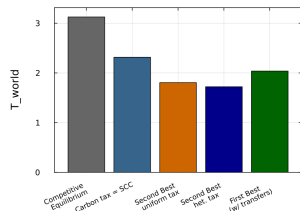
Welfare gain vs competitive



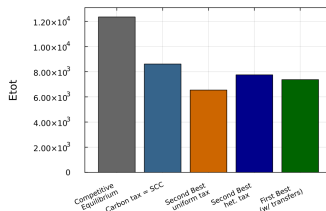
Aggregate emissions



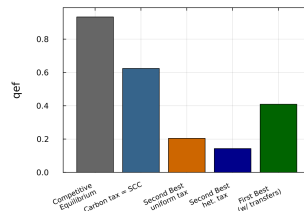
World temperature



World energy use



Producer fossil price



Allocations under different policy regimes

