

Climate Change, Inequality and Optimal Climate Policy

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Abstract

What is the optimal policy to fight climate change, and should it account for inequality across countries? Distortive taxation of carbon and fossil fuels has strong redistributive effects, hurting developing economies already affected by climate change, and countries exposed to energy markets through prices and energy rents. Through the lens of an Integrated Assessment Model (IAM) with heterogeneous countries, I solve for the optimal taxation of fossil fuels: it changes with inequality depending crucially on the availability of other redistribution instruments. I characterize the Social Cost of Carbon (SCC) and the First-Best allocation, before solving for the Second-Best Climate policy when cross-country transfers are constrained. With uniform Pareto weights, the redistributive and distortionary effects on energy markets are substantial, and carbon taxation should be increased twofold compared to the First-Best with bilateral transfers. Climate policy is more aggressive to help warmer and poorer countries and acts as a form of “energy repression” by lowering oil and gas prices. If country-specific carbon taxes are available, poorer, warmer, and more energy-dependent countries should pay lower carbon taxes. These qualitative results hold in a large class of IAMs, and I propose a novel global solution method for designing optimal policy in dynamic models with heterogeneity.

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1 Introduction

The economic consequences of climate change are expected to fall largely on developing countries. An increase in global temperature will disproportionately affect regions closer to the tropics and equator, where the climate is already warm, and countries tend to have lower income, see [Dell et al. \(2012\)](#); [Burke et al. \(2015\)](#); [IPCC et al. \(2022\)](#). In addition, those countries have contributed the least to the stock of greenhouse gas (GHG) emissions – generated by energy use from economic activity – while bearing most of the cost of climate change. These emissions are unequally distributed, as the lion’s share of energy consumption comes from developed economies, which have much larger carbon footprints per capita, see [Holtz-Eakin and Selden \(1995\)](#); [Chancel \(2022\)](#). Moreover, fighting global warming through climate policy implies additional redistributive effects through energy markets. Distortionary carbon taxation raises the cost of energy from fossil fuels – oil, gas, and coal – hurting the economies most reliant on them for their development. Phasing out fossil fuels also reduces their prices and redistributes rents from exporters to importers.

Should the conduct of climate policy account for the presence of these different layers of inequalities? What is the optimal carbon taxation against climate externalities in the presence of these distributional concerns? Should the level and the path of the carbon tax be changed when uncovering the redistributive – general equilibrium – effects on energy markets? This article studies these questions.

In the textbook case, the carbon tax follows Pigou’s prescription and should equal the sum of marginal climate damages, which defines the *Social Cost of Carbon (SCC)*. That prescription is typically valid in representative-agent economies – where the world is a region with a single household and firm, as in [Nordhaus \(2017\)](#); [Golosov et al. \(2014\)](#). However, would that result hold when regions are unequal in these different dimensions? Climate-economics frameworks have been extended to understand the heterogeneous impacts of global warming, e.g. [Nordhaus and Yang \(1996\)](#); [Hillebrand and Hillebrand \(2019\)](#); [Kotlikoff, Kubler, Polbin and Scheidegger \(2021b\)](#); [Krusell and Smith \(2022\)](#); [Cruz and Rossi-Hansberg \(2024a\)](#); [Rudik et al. \(2021\)](#) among many others. However, those studies have not investigated whether the optimal design of climate policy should change when considering heterogeneity and redistributive effects of climate change and energy taxation.

Therefore, the contribution of this article is to study how inequality affects the characterization of the Social Cost of Carbon and whether the optimal carbon tax aligns with the Pigouvian benchmark. Crucially, we will see that the optimal taxation of fossil fuels depends on the availability of other redistribution instruments. Without cross-country transfers, carbon taxes should be adjusted for their distortive and redistributive effects, and I will study the conditions under which a global carbon tax should differ from the Social Cost of Carbon, both qualitatively and quantitatively.

To analyze these questions, I develop a simple yet general Integrated Assessment Model – or climate-economy model – with countries’ heterogeneity. As in the standard DICE model of [Nordhaus \(2017\)](#), it builds on the Neoclassical Growth Model and includes a climate system, and

I extend it to multiple countries which interact through energy markets and are unequal along many dimensions of heterogeneity: (i) income, (ii) climate damages, and exposure to energy markets through (iii) differences in energy mix and (iv) fossil-fuel exports and energy rents. First, a representative firm in each country produces goods using capital and energy inputs. Differences in productivity reflect variations in GDP per capita and energy use. Second, a representative household makes consumption-saving choices leading to capital accumulation. Third, three energy firms in each country produce oil-gas, coal, and renewables, respectively, which are energy inputs used in good production. Countries differ in their energy mix due to differences in energy production costs. Moreover, differences in cost and in endowments in oil-gas – and dynamically depleting reserves – lead to countries being exporters or importers of fossil fuels. Finally, global fossil-fuel consumption – from oil, gas, and coal – emits carbon into the atmosphere, which then feeds back into the climate system. This affects temperatures across regions and results in heterogeneous damages for firms and households across countries. I calibrate the model to economic, climate, and energy data for a large sample of countries and match the dimensions of inequality at the heart of countries’ vulnerability to climate change and climate policies.

In this framework – along with a simplified static three-equation version of that model – I study the optimal carbon taxation. First, I characterize the Local Cost of Carbon LCC_i , which is the welfare climate cost for country i of one additional ton of carbon emitted. Moreover, given a utilitarian welfare metric with arbitrary Pareto weights over countries, we can also specify the Social Cost of Carbon (SCC), which is the global cost of climate change. I show that the welfare-relevant SCC is the sum of the Local Costs LCC_i weighted by the “*social welfare weights*”, i.e. the product of marginal utilities of consumption and Pareto weights. The planner places relatively more weight on poorer countries than on richer ones, affecting our measurement of the global cost of climate change. Additionally, relying on the continuous-time formulation of the model, I provide an analytical expression for the Costs of Carbon (LCC_i and SCC), which now depends on the climate system, climate damage, and scales with the path of output as in Golosov et al. (2014). We see that in this class of models, the Social Cost of Carbon with inequality is *lower* than the sum of the Local Cost of Carbon – what would hold in a “representative agent” version of the model. In our quantitative example, the SCC with inequality is 15% to 50% – depending on the different policies considered – than the sum of the local costs LCC_i .¹

Second, we solve the First-Best allocation, which can be decentralized using (i) carbon taxation and (ii) lump-sum transfers across countries. In that case, in the presence of redistributive motives, the planner equalizes marginal utilities across countries – and the social welfare weights are all equalized to one. Quantitatively, with uniform Pareto weights, the world planner would redistribute large amounts from the richest to the poorest countries: the US and Western Europe have to pay up to 50% of their GDP as transfers while consumption in South-Asia and Africa could double thanks to such kind of redistributive policy. With Negishi weights – which offset those pre-

¹In the Status-Quo equilibrium without climate policy, the SCC is \$100/ tCO_2 compared to a sum of LCC_i around \$200/ tCO_2 .

existing redistribution motives – the planner would use transfers to compensate the regions most affected by climate change, but also by the carbon tax itself, in particular large producers of fossil fuels. In that context, what is the optimal climate policy? We recover the Pigouvian Benchmark where the *carbon tax equals the Social Cost of Carbon*. Quantitatively, with uniform Pareto weights, the First-Best carbon tax – and the SCC – average $\$150/tCO_2$. Thanks to redistribution using other instruments, we also show that the Social Cost of Carbon is the sum of the Local Costs of Carbon for each country – a result that aligns with Representative-Agent models like Golosov et al. (2014), or models with perfect aggregation where redistribution motives are absent like Hillebrand and Hillebrand (2019); Kotlikoff, Kubler, Polbin and Scheidegger (2021b); Krusell and Smith (2022). This also recovers a standard “separation” result in public finance from Kopczuk (2003).

Third, I show that this benchmark for the optimal climate policy depends crucially on the availability of these redistributive transfer instruments across countries. Indeed, when cross-country transfers are constrained – e.g. for political reasons – I show that the optimal carbon tax needs to account for inequality and redistributive effects. In a Second-Best Ramsey policy, I consider a uniform carbon tax across countries – or a world carbon (ETS) price where the carbon revenue are rebated within each country. In that context, the optimal policy should account for three effects: (i) the climate externality, represented by the Social Cost of Carbon (SCC), (ii) a supply redistribution term summarizing the equilibrium effects on the price of fossil-fuel energy between exporters and importers, scaled by energy supply elasticities, and (iii) a demand distortion term that scales with energy input choices, cost shares, and demand elasticity. Moreover, (iv) these three aggregate terms are weighted by the “*social welfare weights*”, i.e., the product of marginal utilities of consumption and countries’ Pareto weights. Again, the planner places more weight on poorer countries than on richer ones, affecting the overall level of the climate policy. In that context, the world’s optimal carbon policy differs from the benchmark *Carbon tax = Social Cost of Carbon*, and should be adapted to the inequality and redistributive effects across country. Those redistributive effects – and the social welfare weights – change dynamically as climate becomes warmer, countries develop and become richer, and oil and gas become more scarce over time. Quantitatively, with uniform Pareto weights the optimal carbon is almost two times the level set in the First-Best Allocation, around $\$300/tCO_2$ instead of $\$150/tCO_2$, and more than three times the level of the Social Cost of Carbon – $\$300$ instead of $\$80/tCO_2$. This optimal climate policy is very distortive and more aggressive than in the Pigouvian benchmark for two reasons: higher carbon taxation allow to reach 2° , which benefits the warmer regions which tend to be poorer. Additionally, a higher global carbon tax is very distortive for energy choice and act as some form of “energy repression” which lower the world price of oil and gas in equilibrium. This allows developing countries to have access to cheaper energy thanks to indirect GE effects.

Fourth, I also study a Second-Best Ramsey policy without cross-country transfers but with country-specific carbon taxes. In that case, two motives remain – the Social Cost of Carbon and the supply redistribution of energy rent on the oil and gas market. The planner does not distort demand thanks to the ability to act optimally in each country. However, the carbon tax itself is

now inversely proportional to the social welfare weights: the planner strongly reduces the carbon tax required for relatively poorer countries or those with high Pareto weights. Quantitatively, this implies that if the “average” carbon tax is around $\$100/tCO_2$, the advanced economies should pay more than $\$1500/tCO_2$ while developing countries should only pay couple of dozen per tCO_2 . To sum up, most of those qualitative results would hold in a large class of climate-economy models.

Finally, this framework is general and I solve this dynamic Integrated Assessment Model with multiple countries and many dimensions of heterogeneity using global methods. Using the Pontryagin Maximum Principle, we simply have to follow the states – here capital/wealth and carbon concentration in the atmosphere – and the corresponding costates – the marginal value of wealth and of carbon. Inspired by the Mean-Field Games literature, this solution method is sequential, and we solve globally system of Forward-Backward ODEs in continuous time. As a result, it allows us to consider an arbitrary number of dimensions of ex-ante heterogeneity and a larger number of dimensions of time-varying states than what is typically covered in the literature using dynamic programming methods. This method is fast and efficient – solving the model in a couple of second. It is also particularly well adapted to optimal policy problems, where the resulting dynamic policies, here the carbon tax, tend to be expressed as function of costates, e.g. the ratio marginal value of carbon over marginal value of wealth for the Costs of Carbon (LCC_i and SCC).

Related literature

This paper stands at the intersection macroeconomics, climate economics, computational economics with the goal to optimal policy with heterogeneous agents.

First, I develop an Integrated Assessment model (IAM) with heterogeneous countries, and this naturally relates to the classical approach in Nordhaus’ DICE as in Nordhaus (1993, 2017); Barrage and Nordhaus (2024). Golosov, Hassler, Krusell and Tsyvinski (2014) and Barrage (2020) develop complete studies of the optimal taxation of fossil fuels in IAMs: in First-Best policy the optimal carbon tax is the Social Cost of Carbon (SCC), but it might differ in the presence of other distortive taxes. For survey of the recent literature see Hassler et al. (2016); Fernández-Villaverde et al. (2025); Bilal and Stock (2025). The current IAM also relates to a large literature merging advances in climate sciences into macroeconomics, such as in Dietz, van der Ploeg, Rezai and Venmans (2021); Dietz and Venmans (2019), Ricke and Caldeira (2014) or Folini et al. (2021). Following this literature, I consider a simple climate system and derive analytical expressions for the social cost of carbon and optimal carbon policies.

Extending such framework to handle country heterogeneity, I build on the RICE model – the multi-regions version of DICE – as in Nordhaus and Yang (1996); Nordhaus (2011). As studied in Hillebrand and Hillebrand (2019), the optimal carbon tax should be the Social Cost of Carbon, i.e. the sum of Local Damages, and the heterogeneity across regions determines the optimal transfer policies. When transfers are unrestricted, there is no need to adjust the carbon tax or the Social Cost of Carbon for inequality, as it is standard in the separation of instruments, see Kopczuk

(2003).²

More recently, frameworks with more realistic heterogeneity have been developed to study the impact of climate change such as Ricke, Drouet, Caldeira and Tavoni (2018); Krusell and Smith (2022); Hassler, Krusell, Olovsson and Reiter (2020); Kotlikoff, Kubler, Polbin and Scheidegger (2021b); Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021); Lang (2025). Rich frameworks also incorporate migration, agglomeration and congestion externality, and meaningful spatial heterogeneity, see Cruz and Rossi-Hansberg (2024a, 2022a); Bilal and Rossi-Hansberg (2023); Rudik et al. (2021); Desmet and Rossi-Hansberg (2024). The contribution of this article is to provide both a theoretical and quantitative analyses the optimal policy with heterogeneity, in the First-Best where we recover the Pigouvian benchmark, as well as in Second-Best with missing instruments where inequality matter for policy.

Moreover, a blooming literature has studied the redistributive effects of carbon taxation within countries and across households. For example, Belfiori, Carroll and Hur (2024) provides similar theoretical results for optimal policy in the First-Best and Second-Best without transfers. In similar heterogeneous agents frameworks, Le Grand, Oswald, Ragot and Aurélien (2023); Wöhrmüller (2024); van der Ploeg, Rezai and Tovar (2024); Fried, Novan and Peterman (2024); Benmir and Roman (2022); Kuhn and Schlattmann (2024); Schlattmann (2024); Douenne, Hummel and Pedroni (2023); Douenne, Dyrda, Hummel and Pedroni (2024) have made significant contributions to understand the redistributive effects – due to differences in goods, car and durables, housing, or adaptation mechanisms along the wealth distribution. The present paper study the case of inequality *across* countries where other redistributive instruments are not available.

This article also relate to a thriving literature that studies optimal policy design in Heterogeneous Agents (HA) models. Solving Ramsey policies, Le Grand et al. (2021), Bhandari et al. (2021a), Davila and Schaab (2023) or McKay and Wolf (2022) propose different approaches to conduct monetary and fiscal policy in HANK models. In my framework, I solve the Ramsey policy sequentially and solve climate externalities and Pigouvian taxation in the presence of heterogeneity rather than managing business cycle fluctuations.

Lastly, this work also relates to advances in global solution methods for HA models. Inspired by literature on Mean Field Games – which has leveraged dynamic programming approaches with PDE,³ see Cardaliaguet (2013/2018), or Achdou et al. (2022) – I study the case with a sequential approach using the Pontryagin Maximum Principle (PMP). If the PMP, using Hamiltonian, states and costates formulation is used for solving the neoclassical model, it has interestingly never been used with Heterogeneous Agents. The setting does extend to the stochastic case, (Yong and Zhou, 1999), or with a distribution of agents – Mean-Field / McKean Vlasov dynamics – as in Carmona et al. (2015), Carmona and Delarue (2018) or Carmona and Laurière (2022). Using this approach in the deterministic case, I solve the model globally in large dimensions, compute the

²Some of these principle have been known in the climate or public finance literatures, e.g. Schelling (1992); Tol (2011); Anthoff et al. (2009); Anthoff and Emmerling (2019); Belfiori (2018); van der Ploeg et al. (2024) or Diamond and Mirrlees (1971); Atkinson and Stiglitz (1976); Diamond (1973).

³Recently, the solution of the master equation has been very fruitful, as in Cardaliaguet et al. (2019); Bilal (2021); Bilal and Rossi-Hansberg (2023).

social cost of carbon analytically, and design optimal policy. I claim that this approach is fruitful for solving dynamic policies globally as the Primal Approach usually express policies as a function of multipliers or costates. In particular, for climate policies the carbon tax is simply the ratio of two costates for carbon and wealth.

In future work, see [Bourany \(2023\)](#), this can be extended to the case with climate risk, touching on a literature related to Stochastic DICE models, see [Cai and Lontzek \(2019\)](#) and [Lontzek, Cai, Judd and Lenton \(2015\)](#) or the general approach to study model uncertainty and ambiguity aversion applied to climate change in [Barnett, Brock and Hansen \(2020, 2022\)](#). If the inclusion of aggregate risk is preliminary in the present paper, I provide intuitions in the toy model. For the general dynamic and stochastic setting, this borrows intuitions from [Carmona et al. \(2016\)](#), [Bourany \(2019\)](#), and [Carmona and Delarue \(2018\)](#) to solve the Stochastic FBSDE systems.

The remainder of this paper is organized as follows. In Section 2, I study the optimal taxation carbon in a simple model to provide most of the intuitions. In Section 3, I lay out the Integrated Assessment Model that we study in the policy analysis. In Section 4, I present how I match the model to the data and solve the model. In Section 5 I derive the optimal carbon tax – First-Best and Second-Best Ramsey policy – in this context.

2 Toy model

In this section, I develop the simplest climate economy model to highlight the intuition behind the design of the optimal climate policy. The goal is to provide intuitions on the effects of heterogeneity across countries, the source of climate externality related to energy markets, and how it change the level of carbon taxation. In the next section, I develop a more general quantitative model that will be used for policy recommendation.

The model is static and all the decisions are taken in one period. Consider I countries $i \in \mathbb{I}$, heterogeneous in three dimensions that will be detailed below. A unique household in each country consumes the good c_i , produced by the representative firm with labor ℓ_i and energy e_i .⁴ In each of these countries, an energy producer extracts energy and sells this input at price q^e on international markets. It earns profits and is owned by the household. Moreover, the countries are subject to climate damage on production. I describe each agent's problem in turn. Finally, a government, whose objective is specified in the next section, imposes a tax on emission t_i^ε and distributes lump-sum transfers t_i^{ls} in each country.

First, a representative household consumes their labor income $w_i \bar{\ell}_i$, where the exogenous labor supply is normalized to $\bar{\ell}_i = 1$, the profit of the energy firm of its country π_i^e and the

⁴Generalization of this model, with differing population p_i , endowments of inputs in the production function (e.g. capital k_i), do not change the qualitative implication of this framework, as we will see in the quantitative model.

lump-sum transfers given by the government t_i^{ls} .

$$\begin{aligned}\mathcal{V}_i &= U(c_i) \\ c_i &= w_i \bar{\ell}_i + \pi_i^e + t_i^{ls} .\end{aligned}$$

Second, a representative firm produces a homogeneous good⁵ using energy e_i and household labor ℓ_i with a constant return to scale technology. Since the labor supply is normalized to 1, e_i represents the energy use per capita. The production function $\tilde{F}(\ell_i, e_i)$ is concave in (ℓ_i, e_i) , and $\tilde{F}_e(\ell, e) > 0$ and $\tilde{F}_{ee}(\ell, e) < 0$ and features Inada conditions. This firm maximizes profits:

$$\max_{\ell_i, e_i} \mathcal{D}_i(\mathcal{S}) z_i \tilde{F}(\ell_i, e_i) - (q^e + t_i^e) e_i - w_i \ell_i \quad , \quad (1)$$

where t_i^e is a carbon tax paid per unit of energy.

Both countries are subject to climate damages $\mathcal{D}_i(\mathcal{S})$ caused by climate externalities related to the world fossil-fuel energy consumption that release greenhouse gas emissions in the atmosphere:

$$\mathcal{S} = \mathcal{S}_0 + \overbrace{\sum_{i \in \mathbb{I}} e_i}^{\text{GHG emissions } \mathcal{E}} \quad ,$$

where energy use and emissions are measured in (metric) tons of Carbon or CO_2 . This depends on the mix between fossil fuels and renewable energies, taken as given in this static model. The quantitative model introduces this endogenous channel of energy substitution.

The global carbon emission stock is not internalized by households in their energy consumption decision, leading to damage $\mathcal{D}_i(\mathcal{S})$ that affects country i 's effective productivity, as in standard Integrated Assessment models, e.g. Nordhaus DICE models.

In each country, a competitive energy producer extracts energy e_i^x – for example oil, gas, or coal – maximizing its profit, subject to convex cost $c(e^x)$, i.e. $c'(e^x) > 0$ and $c''(e^x) > 0$ that is paid in the homogenous good.

$$\begin{aligned}\pi_i^e &= \max_{e_i^x} q^e e_i^x - c_i(e_i^x) \quad , \\ \Rightarrow \quad q^e &= c'_i(e_i^x) \quad \Rightarrow \quad \begin{cases} e_i^x &= \varepsilon_i(q_i^e) = c'^{-1}_i(q^e) \\ \pi_i^e &:= q^e \varepsilon_i(q_i^e) - c_i(\varepsilon_i(q_i^e)) \end{cases} \quad ,\end{aligned}$$

subject the energy price q^e . This corresponds to a decreasing-return-to-scale extraction technology, and implies positive profits $\pi_i^e > 0$. Since energy is traded without friction on international markets, this price is set to clear the supply and demand:

$$E = \sum_{i \in \mathbb{I}} e_i = \sum_{i \in \mathbb{I}} e_i^x .$$

Since the good firm's technology is constant return to scale (CRS), define $F(e_i) = \tilde{F}(1, e_i)$ to

⁵This good is traded costlessly across countries and its price is the numeraire, and hence normalized to 1.

aggregate firms and household budgets into a single constraint:

$$c_i + (q^e + t_i^\varepsilon)e_i = \mathcal{D}_i(\mathcal{S})z_iF(e_i) + q_i^e e_i^x - c_i(e_i^x) + t_i^{ls} \quad [\lambda_i] . \quad (2)$$

with λ_i the shadow value of that constraint, which plays an important role in redistribution motives.

Heterogeneity. The countries $i \in \mathbb{I}$ are symmetric in all regards, except for differences in three parameters. To fix ideas, consider two regions, *North* and *South*, to give qualitative predictions of the policy results. First, countries differ in terms of productivity z_i . Here, I consider a wide definition of z_i that accounts for technology, efficiency, market frictions, and institutions. This results in some countries – e.g. the North – producing more and being richer, leading to inequality in consumption.⁶ Second, energy reserves endowments are unequally distributed, which results in differences in costs of extraction. I assume that northern countries, e.g. US, Canada, Russia, Norway, etc. have lower costs of extraction $c'_N(e) < c'_S(e)$, implying larger production and energy rents $\pi_N^e > \pi_S^e$. Third, some countries, e.g., Southern Hemisphere, are more vulnerable to the damages of climate, $\mathcal{D}_S(\mathcal{S}) < \mathcal{D}_N(\mathcal{S})$ for all $\mathcal{S}$ the stock of carbon. In this sense, the damage parameter $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$ is higher in the South, $\gamma_S > \gamma_N$. All these differences yield heterogeneity in consumption in the competitive equilibrium and motives for redistribution, e.g. $c_N > c_S$.

Definition 2.1 (Competitive Equilibrium). *Given a carbon policy t^ε , a competitive equilibrium (CE) is an allocation $\{c_i, e_i, e_i^x\}_i$ and energy price q^e such that (i) the good firm chooses input e_i maximizing profit, and (ii) the energy firm chooses extraction e_i^x maximizing profit, and both goods and energy markets clear:*

$$\sum_{i \in \mathbb{I}} c_i + c_i(e_i^x) = \sum_{i \in \mathbb{I}} \mathcal{D}_i(\mathcal{S})z_iF(e_i) \quad \quad E = \sum_{i \in \mathbb{I}} e_i = \sum_{i \in \mathbb{I}} e_i^x .$$

This results in the following optimality conditions. First, for consumption, the multiplier λ_i represents the marginal value of wealth or the marginal utility of consumption.

$$\lambda_i = U'(c_i) \quad \text{with} \quad c_i = \mathcal{D}_i(\mathcal{S})z_iF(e_i) + q^e(e_i^x - e_i) - c_i(e_i^x) + t_i^{ls}$$

where consumption depends on production, energy cost, and net energy export.

The second and third optimality for energy use and energy extraction write as follow:

$$\begin{aligned} MPe_i &= q^e + t_i^\varepsilon & \text{with} & \quad MPe_i := \mathcal{D}_i(\mathcal{S})z_iF'(e_i) , \\ q^e &= c'(e_i^x) , \end{aligned}$$

and this corresponds to the standard condition Marginal Product = Marginal Cost for Energy.

⁶Indeed, assuming $F(e)$ is Cobb Douglas $F(e) = \bar{\ell}^{1-\alpha}e^\alpha$, with $\bar{\ell} = 1$, we obtain $\alpha\mathcal{D}_i(\mathcal{S})z_ie_i^{\alpha-1} = q^e$ leading to

$$e_i = (\alpha\mathcal{D}_i(\mathcal{S})z_i/q^e)^{1/(1-\alpha)} \quad y_i - q^e e_i = (\mathcal{D}_i(\mathcal{S})z_i)^{1/(1-\alpha)} (q^e)^{-\alpha/(1-\alpha)} [\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}]$$

which is increasing in z_i and $\mathcal{D}_i(\mathcal{S})$

This competitive equilibrium is inefficient: climate damages $\mathcal{D}_i(\mathcal{S})$ are not internalized, and energy consumption is too high in view of the economic costs of global warming. Moreover, economic inequality results from the heterogeneity in productivity, energy endowment, and climate damage. In our two regions example, $c_N > c_S$ results in $\lambda_S > \lambda_N$. Redistribution from the North to the South could be desirable from a utilitarian point of view. This inequality in consumption and damages arises despite trade openness.⁷ I explore how the social planner allocates consumption and energy in such an environment.

2.1 First-Best: Social planner allocation with full transfers

Consider a Social Planner choosing the agent's decisions, subject to the resource constraints in goods and energy as well as the climate externality.

$$\begin{aligned}
\mathcal{W} &= \max_{\{c_i, e_i, e_i^x\}_{i \in \mathbb{I}}} \sum_{i \in \mathbb{I}} \omega_i U(c_i) \\
\sum_{i \in \mathbb{I}} c_i + c_i(e_i^x) &= \sum_{i \in \mathbb{I}} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) & [\phi] \\
E &= \sum_{i \in \mathbb{I}} e_i = \sum_{i \in \mathbb{I}} e_i^x & [\mu^e] \\
\mathcal{S} &:= \mathcal{S}_0 + \sum_{i \in \mathbb{I}} e_i
\end{aligned} \tag{3}$$

where ϕ is the shadow value of the good market clearing and μ^e the one of the energy market clearing. The welfare function is the weighted sum of countries' utilities, with Pareto weights ω_i . In the following, I denote the social planner allocation $\{\hat{c}_i, \hat{e}_i\}_{i \in \mathbb{I}}$ to distinguish it from the competitive equilibrium.

Choosing the consumption on behalf of the agents yields a redistribution motive:

$$[c_i] \quad \phi = \omega_i U'(\hat{c}_i) \quad \Rightarrow \quad \omega_i U'(\hat{c}_i) = \omega_j U'(\hat{c}_j) \quad \forall i, j \in \mathbb{I}.$$

Depending on the Pareto weights, there is a motive for transferring consumption across countries.

Regarding the choice of energy inputs:

$$[\hat{e}_i] \text{ \& \; } [\hat{e}_i^x] \quad c'(\hat{e}_i^x) = \frac{\mu^e}{\phi} = \mathcal{D}_i(\mathcal{S}) z_i F'(\hat{e}_i) + \underbrace{\sum_{j \in \mathbb{I}} \mathcal{D}'_j(\mathcal{S}) z_j F(\hat{e}_j)}_{=: \overline{SCC}}$$

with an additional term that represents the cost of emitting one ton of carbon in terms of forgone production. This term is the *Social Cost of Carbon (SCC)* in the social planner allocation and represents the marginal global damage of climate change. With $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{SD}_i(\mathcal{S})}$, which is constant with Nordhaus' Damage function $\mathcal{D}_i(\mathcal{S}) = e^{-\frac{\gamma_i}{2}(\mathcal{S}-\mathcal{S}_0)^2}$, I redefine it as $\overline{SCC} = \sum_{j \in \mathbb{I}} \mathcal{S} \gamma_j y_j$.

I turn to the decentralization of such allocation. I consider a planner who has access to all instruments $\{t_i^e, t_i^{ls}\}_i$, and, in particular, lump-sum transfers t_i^{ls} across countries.

⁷We could consider trade and financial autarky preventing production from being exported to other countries. This would strengthen heterogeneity and redistributive motives.

Proposition 1 (First-Best Policy and Decentralization).

The optimal policy decentralizing the First-Best allocation is a uniform carbon tax, $t_i^\varepsilon = t^\varepsilon$, equal to the Social Cost of Carbon $t^\varepsilon = SCC$, and uses lump-sum transfers for redistribution. Indeed, optimality writes:

$$MPe_i := \mathcal{D}_i(\mathcal{S})z_i F'(\hat{e}_i) = q^e + t^\varepsilon \quad q^e = c'(\hat{e}_i^x)$$

$$\text{with} \quad t^\varepsilon = \overline{SCC} := - \sum_{j \in \mathbb{I}} \mathcal{D}'_j(\mathcal{S})z_j F'(\hat{e}_j) = \sum_{j \in \mathbb{I}} \mathcal{S}\gamma_j y_j = I\mathbb{E}_j[\mathcal{S}\gamma_j y_j]$$

and transfers $\{t_i^{ls}\}_i$ which are implicitly defined by

$$\omega_i U'(\hat{e}_i) = \omega_j U'(\hat{e}_j) ,$$

$$\hat{e}_i = \mathcal{D}_i(\mathcal{S})z_i F'(\hat{e}_i) + q^e(\hat{e}_i^x - \hat{e}_i) - c_i(\hat{e}_i^x) + (t_i^{ls} - t^\varepsilon \hat{e}_i) .$$

For arbitrary Pareto weights ω , lump-sum transfers are redistributive: $\exists i, j$ s.t. $t_i^{ls} > t^\varepsilon \hat{e}_i, t_j^{ls} < t^\varepsilon \hat{e}_j$.

To see this last point, summing the budget constraints yields:

$$\sum_{i \in \mathbb{I}} t_i^{ls} = t^\varepsilon \sum_{i \in \mathbb{I}} \hat{e}_i$$

implying there is lump-sum redistribution from richer countries to poorer ones in the presence of heterogeneity across countries.⁸ In our North-South example, with $z_S < z_N, c'_N < c'_S$ or $\mathcal{D}'_S > \mathcal{D}'_N$, and for arbitrary Pareto weights⁹, we obtain that $t_S^{ls} > t^\varepsilon \hat{e}_S$, and $t_N^{ls} < t^\varepsilon \hat{e}_N$. This implies that some funds are taxed from the North and redistributed lump-sum to the South. However, there exists a unique set of Pareto weights $\omega_i = 1/U'(\hat{e}_i)$ – the so-called Negishi weights – such that this motive disappears $t_S^{ls} = t^\varepsilon \hat{e}_S$ and $t_N^{ls} = t^\varepsilon \hat{e}_N$ and there are no transfers across countries.

In the following, I rule out this flexible lump-sum transfers assumption: if development aid exists, in practice, full redistribution with lump-sum taxes and transfers to cover the differences in technology, market frictions, and institutions is politically unfeasible.

2.2 Second Best: Ramsey problem, uniform carbon tax, and limited transfers

Consider now a social planner designing the optimal climate policy, taking into account the constraints preventing transfers across countries. Subject to the competitive equilibrium optimality conditions, the climate externality and the absence of financial instruments for full lump-sum redistribution, the planner takes the decisions of consumption and energy to maximize global welfare. I denote the Ramsey allocation $\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i$ to distinguish it from the competitive equilibrium $\{c_i, e_i, e_i^x\}$ and the First-best allocation $\{\hat{c}_i, \hat{e}_i, \hat{e}_i^x\}$.

$$\mathcal{W} = \max_{\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i, q^e, t^\varepsilon} \sum_{i \in \mathbb{I}} \omega_i U(\tilde{c}_i) \quad (4)$$

⁸Given that $t_i^{ls} = t^\varepsilon \hat{e}_i + U'^{-1}(\frac{\phi}{\omega_i}) - \mathcal{D}_i(\mathcal{S})z_i F'(\hat{e}_i) - q^e(\hat{e}_i^x - \hat{e}_i)$.

⁹In particular, this is the case if the Pareto weights are large enough, i.e. $\omega_S \geq \phi/U'(c_S)$ i.e. more than the weight imposed by the shadow value of good discounted by South' marginal utility

I consider a uniform carbon tax for all countries $t_i^\varepsilon = t^\varepsilon$, and this, for several reasons. First, the goal is to provide a direct comparison to the standard Pigouvian framework, where the natural outcome is a global uniform tax on fossil energy. Second, following the arguments of [Weitzman \(2015\)](#), the uniform carbon tax or price of carbon serves as a “focal point”, where the social-planner policy is a representation of the bargaining outcome in an agreement coming from all the countries in the world. Third, in the next section, I consider different tax rates for each country.

In both cases – uniform tax or country-specific taxes – I assume away cross-country transfers: as the revenue of the tax is redistributed lump-sum to the household $\tilde{t}_i^{ls} = t^\varepsilon \tilde{e}_i$. Moreover, in the Second-Best Ramsey policy, the planner internalizes the optimality conditions of the competitive equilibrium. Using the Primal Approach in public finance, the Ramsey problem accounts for the countries’ budgets and the firms’ optimality conditions and is written as:

$$\begin{aligned}
\mathcal{W} &= \max_{\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i, q^e, t^\varepsilon} \sum_{i \in \mathbb{I}} \omega_i U(c_i) \\
s.t. \quad & \tilde{c}_i + (q^e + t^\varepsilon) \tilde{e}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\tilde{e}_i) + q^e \tilde{e}_i^x - c_i(\tilde{e}_i^x) + t^\varepsilon \tilde{e}_i & [\phi_i] \\
& q^e = c'_i(\tilde{e}_i^x) & q^e + t^\varepsilon = MPe_i & [v_i] \\
& \mathcal{S} := \mathcal{S}_0 + \sum_i e_i & E = \sum_i e_i = \sum_i e_i^x & [\mu^e]
\end{aligned} \tag{5}$$

with the Lagrange Multipliers ϕ_i, v_i, μ^e , respectively for the budget constraint, the energy choice of the good firm, and the energy market clearing.

The Lagrangian of this problem, as well as the detailed optimality conditions, are derived in detail in Section B. They yields the following optimality conditions for consumption c_i , demand e_i and supply e_i^x for energy:

$$\begin{aligned}
\omega_i U'(\tilde{c}_i) &= \phi_i = \text{marginal value of income/wealth} \\
\phi_i t^\varepsilon &= \underbrace{v_i \mathcal{D}_i(\mathcal{S}) z_i F''(\tilde{e}_i)}_{=\text{demand distortion}} + \underbrace{\mu^e}_{=\text{supply redistribution}} \underbrace{- \sum_j \phi_j \mathcal{D}'_j(\mathcal{S}) z_j F'(\tilde{e}_j)}_{\propto \text{Social Cost of Carbon}}
\end{aligned}$$

The planner chooses a single carbon policy instrument and thus accounts for several redistribution channels across countries through (i) the marginal value of income ϕ_i , (ii) the distortion of energy demand symbolized by the shadow value of energy choice v_i , (iii) the redistributive effects on energy market with the market clearing multiplier μ^e , and (iv) the Social Cost of Carbon summarizing the marginal cost of climate change. There, before deriving the main formula for the optimal carbon tax t^ε and explaining the economic intuition behind it, let us introduce these key objects.

First, I define the “social welfare weight” $\hat{\phi}_i = \phi_i / \bar{\phi}$ that represents the relative weight that the planner uses for global policy. I define it as a ratio, rescaling the multiplier ϕ_i , of the shadow value of relaxing the budget constraint for country i .

$$\hat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{I} \sum_{i \in \mathbb{I}} \omega_i U'(c_i)} \leq 1$$

where $\bar{\phi} = \partial\mathcal{W}/\partial c = \frac{1}{I}(\sum_{i \in \mathbb{I}} \omega_i U'(c_i))$ is the average marginal utility. $\bar{\phi}$ is the “money $\leftrightarrow$ welfare” conversion factor for the social planner. When there is no full redistribution, this factor $\hat{\phi}_i$ is high for relatively poorer countries or countries with a high Pareto weight ω_i .

Second, climate change affects countries differently according to their marginal damages $\mathcal{D}'_j$. The social cost of carbon scales those damages by the social welfare weights/inequality factor $\hat{\phi}_i \propto \omega_i U'(c_i)$ given that the planner does not have access to full redistribution. Rescaled in monetary unit, with the conversion factor $\bar{\phi}_i$, the SCC writes:

$$SCC := -\frac{\partial\mathcal{W}/\partial\mathcal{S}}{\partial\mathcal{W}/\partial c} = -\frac{1}{\bar{\phi}} \sum_j \phi_j \mathcal{D}'_j(\mathcal{S}) z_j F(e_j) = -\sum_j \hat{\phi}_j \mathcal{D}'_j(\mathcal{S}) z_j F(e_j) = \sum_{j \in \mathbb{I}} \hat{\phi}_j \mathcal{S} \gamma_j y_j$$

with the definition $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{D}_i(\mathcal{S})\mathcal{S}}$, the slope of the climate damage function, as in Nordhaus’ DICE model. In particular, in this heterogeneous countries model with limited redistribution, the Social Cost of Carbon integrates the distribution of consumption/income under the factor $\hat{\phi}_i$:

$$\begin{aligned} SCC &:= \sum_{j \in \mathbb{I}} \hat{\phi}_j \mathcal{S} \gamma_j y_j = I \mathbb{E}_j \left(\hat{\phi}_j \mathcal{S} \gamma_j y_j \right) \\ SCC &= I \mathbb{E}_j [\mathcal{S} \gamma_j y_j] + I \text{Cov}_j \left(\hat{\phi}_j, \mathcal{S} \gamma_j y_j \right) \leq I \mathbb{E}_j [\mathcal{S} \gamma_j y_j] =: \overline{SCC} \end{aligned}$$

with the damage slope $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{D}_i(\mathcal{S})\mathcal{S}}$, and country i ’s output $y_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i)$. The $\overline{SCC} = I \mathbb{E}_j [-\mathcal{D}'_j(\mathcal{S}) z_j F(e_j)] = I \mathbb{E}_j [\mathcal{S} \gamma_j y_j]$ is the Social Cost of Carbon when full redistribution is available, or equivalently in representative agent models where redistributive concerns are absent. Since $\mathbb{E}_j(\cdot)$ is a mean¹⁰ over countries j , we need to multiply by the number of countries (I here) to obtain the sum of local damages.

Is the SCC higher in the model with inequality compared to the one-agent setting? Take the North-South economy as an example. First, low-income countries have a lower consumption and hence higher marginal utility of consumption, e.g. $c_S < c_N$ and $\hat{\phi}_S > \hat{\phi}_N$. Second, South is suffering from stronger damages $\mathcal{D}'_S > \mathcal{D}'_N$. However, third, productivity is higher in the North $z_N > z_S$ implying $F(e_N) > F(e_S)$ and income $\bar{y}_N := z_N F(e_N) > \bar{y}_S$. Therefore, the covariance between $\hat{\phi}_i$, $\bar{y}_i$ and $\gamma_i \propto \mathcal{D}'_i$ is ambiguous. Quantitatively, in a large class of Integrated Assessment models, the local cost of climate change $\mathcal{D}'_i(\mathcal{S}) y_i$ is strongly correlated with income y_i , as there larger production loss of climate change in richer countries. In such cases, the covariance $\text{Cov}_j(\hat{\phi}_j, \gamma_j y_j)$ is negative, and as a result:

$$SCC = I \mathbb{E}_j [\mathcal{S} \gamma_j y_j] + I \text{Cov}_j \left(\hat{\phi}_j, \mathcal{S} \gamma_j y_j \right) < I \mathbb{E}_j [\gamma_j y_j] = \overline{SCC} \quad \text{if} \quad \text{Cov}_j \left(\hat{\phi}_j, \gamma_j y_j \right) < 0$$

Third, I explore the redistributive effect of carbon taxation on the energy supply. Changing the price affects the market clearing, with shadow value μ^e . I formulate this supply side channel as a redistribution between energy importers and exporters, weighted by a factor representing the

¹⁰The formula of the expectation of a product writes $\mathbb{E}_i[x_i y_i] = \mathbb{E}_i[x_i] \mathbb{E}_i[y_i] + \text{Cov}_i[x_i y_i]$

curvature of aggregate energy supply:

$$\begin{aligned}
\text{Supply Redistribution} &= \frac{\mu^e}{\phi} = \mathcal{C}_{EE} \frac{1}{I} \sum_j \frac{\phi_j}{\phi} (e_j - e_j^x) & \text{with} & \quad \mathcal{C}_{EE} = \left(\sum_j c_j''(e_j^x)^{-1} \right)^{-1} \\
&= \mathcal{C}_{EE} \mathbb{E}_j \left(\hat{\phi}_j (e_j - e_j^x) \right) \\
&= \mathcal{C}_{EE} \text{Cov}_j \left(\hat{\phi}_j, e_j - e_j^x \right) \leq 0
\end{aligned}$$

What is the sign of this covariance? In our two regions example, we assumed that the North had a larger endowment in energy resources and hence higher net energy exports $e_N - e_N^x < e_S - e_S^x$. Therefore, since the net import of energy correlates with lower consumption, and hence a higher marginal value of consumption $U'(c_i)$, the covariance term is positive. Moreover, the magnitude of this terms-of-trade redistribution ultimately depends on the aggregate supply elasticity:

$$\mathcal{C}_{EE} = \left(\sum_j c_j''(e_j^x)^{-1} \right)^{-1} = q^e \frac{\bar{\nu}}{E} \quad \text{with} \quad \bar{\nu} = \left(\sum_j \lambda_j^x \nu_j^{-1} \right)^{-1}$$

with ν_j the inverse supply elasticity, constant in the iso-elastic case $q^e = c_i'(e) = \bar{\nu}_i e^{\nu_i}$ and the share of country i in energy production $\lambda_j^x = e_i^x/E$. As a result, this Social “Supply Redistribution” is positive. It is larger when the energy supply is inelastic – price and profits vary a lot for small changes in quantity produced. It is null when the energy production is Constant Return to Scale (CRS) when $\nu_j = 0$, and therefore, no energy rents are redistributed.

Third, carbon taxation distorts energy choice across users and changes the equilibrium energy price along the demand curve. We derive the Social “Demand Distortion” term as:

$$\begin{aligned}
\text{Demand Distortion} &= \frac{1}{I} \sum_j \frac{v_j}{\phi} \mathcal{D}_j(\mathcal{S}) z_j F''(e_j) = \mathbb{E}_j (\hat{v}_j \mathcal{D}_j(\mathcal{S}) z_j F''(e_j)) \\
&= \text{Cov}_j \left(\frac{v_j}{\phi}, \mathcal{D}_j(\mathcal{S}) z_j F''(e_j) \right) \leq 0
\end{aligned}$$

with v_j is the multiplier on the energy demand optimality condition: positive value implies that the planner would like to relax the constraint, increase the quantity e_i , lower the MPE_i , and conversely for negative values. I define $\hat{v}_j = v_j/\bar{\phi}$ as the rescaled shadow value of country j ’s energy demand.

There is no aggregate distortion, only redistributive distortions across countries. This comes from the optimality of the tax t^e , as $\mathbb{E}_j(\hat{v}_j) = 0$, and this yields the last line as a covariance. How to determine its sign? In our North-South example, lower-income economies have energy demand more sensitive to price distortions since $z_i F''(e_i)$ relates to the energy share and demand elasticity:

$$\mathcal{D}_i(\mathcal{S}) z_i F''(e_i) = -\frac{q^e}{e_i \sigma_i^e} (1 - s_i^e) \quad \Rightarrow \quad \mathcal{D}_S(\mathcal{S}) z_S F''(e_S) < \mathcal{D}_N(\mathcal{S}) z_N F''(e_N)$$

where $s_i^e = \frac{e_i q^e}{y_i} < 1$ is the energy share in production and σ_i^e is country i ’s energy demand elasticity. The North relies “more” on energy – since $z_N > z_S$ implies that $e_N > e_S$: more productive countries have higher energy demand ceteris paribus. It would also be the case if energy is more substitutable

in richer countries, i.e. large σ^e , and demand varies a lot with price. The covariance would then be negative: if the planner values more the production, high v_j , of the most inelastic countries, lower $F''(e_i)$ then the tax would be lower. Moreover, that term is null if the energy demand/production function is constant return in energy such that $s_i^e = 1$, or if energy is perfectly substitutable $\sigma^e \rightarrow \infty$, or if we are in a representative agent economy $\mathcal{D}_N(\mathcal{S})z_N F''(e_N) = \mathcal{D}_S(\mathcal{S})z_S F''(e_S)$ and there is no heterogeneity in demand across countries.

Proposition 2 (Second-Best Ramsey Policy with limited transfers).

The optimal Second-Best carbon tax accounts for three distributional motives when setting a single uniform level: (i) climate damage in the **Social Cost of Carbon (SCC)**, (ii) **Supply Redistribution** in energy markets through terms-of-trade and energy rents and (iii) **Demand Distortion** through distorted firms' energy choices. This includes redistribution motives due to the presence of inequality through the **social welfare weights** $\hat{\phi}_j = \phi_j / \bar{\phi} = \omega_j U'(c_j) / \frac{1}{I} \sum_i \omega_i U'(c_i)$. The optimal energy tax writes:

$$t^\varepsilon = SCC + \text{Supply Redistribution} + \text{Demand Distortion}$$

$$t^\varepsilon = - \sum_j \hat{\phi}_j \mathcal{D}'_j(\mathcal{S}) z_j F(e_j) + \mathcal{C}_{EE} \frac{1}{I} \sum_j \hat{\phi}_j (e_j - e_j^x) + \frac{1}{I} \sum_j \hat{v}_j \mathcal{D}_j(\mathcal{S}) z_j F''(e_j)$$

$$t^\varepsilon = I \mathbb{E}_j(\mathcal{S} \gamma_j y_j) + I \text{Cov}_j(\hat{\phi}_j, \mathcal{S} \gamma_j y_j) + q^e \frac{\bar{\nu}}{E} \text{Cov}_j(\hat{\phi}_j, e_j - e_j^x) - q^e \text{Cov}_j(\hat{v}_j, \frac{1-s_i^e}{\sigma_i^e e_i}),$$

for the carbon tax such that $MPe_i = c'(e^x) + t^\varepsilon$, where $\gamma_i = -\frac{\mathcal{D}'_i(\mathcal{S})}{\mathcal{D}_i(\mathcal{S})\mathcal{S}}$ is the marginal damage of climate change¹¹, $y_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i)$ is total production, $\bar{\nu} = (\sum_j \lambda_j^x \nu_j^{-1})^{-1}$ the average inverse energy supply elasticity, s_i^e the energy cost shares, and σ_i^e the energy demand elasticity. We see these three motives matter with a single tax and lump-sum rebate. Compared to the economy with full redistribution, the carbon tax is **smaller** if (i) the cost of climate $\gamma_j y_j$ is concentrated in richer countries, with low $\hat{\phi}_i$, (ii) the net energy imports are high, with higher $e_i - e_i^x$, in richer, low $\hat{\phi}_i$ countries, (iii) the energy is more essential, with low demand elasticity and high $\frac{1-s_i^e}{\sigma_i^e e_i}$, in poorer, high distortion $\hat{v}_i$, countries. For (ii) and (iii), carbon taxation is isomorphic to an energy terms-of-trade manipulation between the exporters and the importers in trade theory.

In the next corollary, we reexpress the carbon tax as a function of observable sufficient statistics. Indeed, given its dependence on the multipliers for individual demand v_i , the Demand Distortion term can be quite opaque.

Corollary 3 (Second-Best Ramsey Policy with limited transfers, sufficient statistics).

The optimal Second-Best uniform carbon tax can be rewritten as:

$$t^\varepsilon = \frac{1}{1 + \text{Cov}_j(\hat{\phi}_j, \hat{e}_j^{s,\sigma})} \left[SCC + \text{Supply Redistribution} \right] \quad \text{with} \quad \hat{e}_j^{s,\sigma} := \frac{\frac{\sigma_j^e e_j}{1-s_j^e}}{\sum_i \frac{\sigma_i^e e_i}{1-s_i^e}}$$

$$t^\varepsilon = \frac{1}{1 + \text{Cov}_j(\hat{\phi}_j, \hat{e}_j^{s,\sigma})} \left[I \mathbb{E}_j(\mathcal{S} \gamma_j y_j) + I \text{Cov}_j(\hat{\phi}_j, \mathcal{S} \gamma_j y_j) + q^e \frac{\bar{\nu}}{E} \text{Cov}_j(\hat{\phi}_j, e_j - e_j^x) \right]$$

for a uniform carbon tax t^ε , with marginal climate damage γ_i , output y_i , average inverse energy supply elasticity $\bar{\nu}$, energy cost shares s_i^e , energy demand elasticity σ_i^e . Note that $\hat{e}_i^{s,\sigma} = \hat{e}_i = e_i / E$ if the production function has a Cobb-Douglas form.¹² Demand distortion amplifies or dampens the

¹¹The parameter γ_i is constant in the damage function used in the DICE model $\mathcal{D}_i(\mathcal{S}) = e^{-\gamma_i(\mathcal{S}-\mathcal{S}_0)^2}$.

¹²If all countries i have the same Cobb-Douglas production of the form $F(\ell_i, e_i) = e_i^\alpha \ell_i^{1-\alpha}$, we get $\hat{e}_i^{s,\sigma} := \frac{\sigma_i^e e_i}{1-s_i^e} = \frac{1 \times e_i}{1-\alpha}$

carbon taxation motives, i.e. the Social Cost of Carbon (SCC) and Supply Redistribution. The carbon tax is **lower** if the largest energy consumers e_i , with high energy share s_i^e and demand elasticity σ_i^e have low consumption c_i and thus high social welfare weights $\hat{\phi}_j = \omega_j U'(c_j) / \frac{1}{I} \sum_i \omega_i U'(c_i)$.

Note, in representative agent models, as in Nordhaus (2017), or Golosov et al. (2014), there is no heterogeneity across countries, making all the covariances trivially null. Hence, we obtain the standard Pigouvian result $t^\varepsilon = SCC = I\mathbb{E}_j[\mathcal{S}\gamma_j y_j] = \overline{SCC}$. Moreover, models with unconstrained transfers, which can be aggregated, yield $\hat{\phi}_j = \hat{\phi}_i = 1$, also reducing these covariances to zero. Models with heterogeneity in income, climate damage, or country size, like Nordhaus and Yang (1996) or Krusell and Smith (2022), but no heterogeneity in energy demand nor energy rent redistribution also yield the Pigouvian result $t^\varepsilon = SCC$, where the Social Cost of Carbon is adjusted for inequality.

These Second-Best carbon tax formulas hold for a single uniform carbon tax. If the planner has access to a distribution of carbon tax rates (or carbon prices), the *distribution* of the tax changes with the presence of inequality, as we see in the next section.

2.3 Ramsey Problem with heterogeneous carbon tax & limited transfers

We consider a case where the Social Planner implements a distribution of *country-specific* carbon taxes t_i^ε . I again assume away cross-country transfers, and the revenue of the carbon tax is rebated lump-sum $t_i^{ls} = t_i^\varepsilon e_i$. The welfare objective and the constraints internalized by the planner are the same as before: (i) the budget $\tilde{c}_i = \mathcal{D}_i(\mathcal{S})z_i F(\tilde{e}_i) + q^e(\tilde{e}_i^x - \tilde{e}_i) - c_i(\tilde{e}_i^x)$ with multiplier ϕ_i , (ii) the energy firm optimality $q^e = c'(e_i^x)$, (iii) the energy demand optimality $q^e + t_i^\varepsilon = MP e_i$ with multiplier v_j , and (iv) the energy market clearing $E = \sum_i e_i = \sum_i e_i^x$. The only exception is that the carbon tax is country-specific t_i^ε . The planner optimality conditions become:

$$\omega_i U'(c_i) = \phi_i \quad \hat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{I} \sum_{i \in \mathbb{I}} \omega_i U'(c_i)} \leq 1$$

the same as before, as the planner keeps the same motive for redistribution. The social welfare weights, or inequality factor, come for heterogeneity in the marginal value of income ϕ_i . However, when the planner can choose one instrument per country, the distortion of demand is absent:

$$v_j = 0 \quad \Rightarrow \quad \text{Demand Distortion} = 0$$

and hence $\hat{e}_j^{s,\sigma} := e_j^{s,\sigma} / \sum_i e_i^{s,\sigma} = e_i / E$

Proposition 4 (Second-Best Ramsey Policy, heterogeneous taxes & limited transfers).

The optimal Second-Best energy taxation policy with heterogeneous taxes when transfers are absent becomes:

$$\begin{aligned} t_i^\varepsilon &= \frac{1}{\widehat{\phi}_i} \underbrace{\sum_j \widehat{\phi}_j (-\mathcal{D}'_j(\mathcal{S}) z_j F(\tilde{e}_j))}_{\propto \text{SCC}} + \frac{1}{\widehat{\phi}_i} \underbrace{\mathcal{C}_{EE} \frac{1}{I} \sum_j \widehat{\phi}_j (e_j - e_j^x)}_{=\text{Supply Redistribution}} \\ t_i^\varepsilon &= \frac{1}{\widehat{\phi}_i} \left[\text{SCC} + \text{Supply Redistribution} \right] \\ t_i^\varepsilon &= \frac{1}{\widehat{\phi}_i} \left[I\mathbb{E}_j(\mathcal{S}\gamma_j y_j) + ICov_j(\widehat{\phi}_j, \mathcal{S}\gamma_j y_j) + q^e \frac{\bar{\nu}}{E} \text{Cov}_j(\widehat{\phi}_j, e_i - e_i^x) \right] \end{aligned}$$

where γ_i is the marginal damage of climate change, y_i is total output and $\bar{\nu}_i$ the average inverse energy supply elasticity, and $\widehat{\phi}_j = \phi_j / \bar{\phi} \propto \omega_j U(c_j)$ are the social welfare weights

The planner accommodates country-specific levels of inequality for the distribution of carbon prices. Indeed, for a given – potentially arbitrary – distribution of Pareto weights ω_i , the optimal carbon tax is relatively lower for poorer countries. The two motives for carbon taxation, (i) the Pigouvian Social Cost of Carbon and (ii) the supply redistribution, changing terms-of-trade and energy rent in general equilibrium, both need to be *discounted* by the country level of inequality $\widehat{\phi}_i \propto \omega_i U'(c_i)$. The tax is reduced for countries with low consumption – due to inherently low income (due to TFP) or climate damage – or high Pareto weight in the global welfare. Lastly, the energy demand is not affected by this country-specific tax.

These main findings – that the *level* and the *distribution* of carbon taxes change with inequality – are general and hold in a dynamic quantitative model that I develop in the next sections.

2.4 From carbon taxation to carbon pricing in emissions markets

In this section, I investigate how to implement the optimal climate policy when the planner chooses to design a “cap-and-trade” emission market, such as the European Union’s “emission trading system (ETS)”. Emissions markets are a privileged policy solution as they simultaneously provide the strict regulation of a cap and the market efficiency of emission trading.

Consider a social planner designing the cap-and-trade system, choosing the number of allowances, or “quotas” or “permits”, $\bar{\mathcal{E}}$ to be auctioned in a world market. Moreover, it also chooses the number of permits that are given “for free” to each country $\bar{\varepsilon}_i$. As a result, it controls the total supply of permits $\bar{\mathcal{E}} + \sum_i \bar{\varepsilon}_i$, which are traded on a global market at price q^ε . Finally, the representative firm in each country chooses how many permits to purchase ε_i to cover its emissions from energy use. The country i ’s representative household/firm faces the following problem:

$$\max_{c_i, e_i, \varepsilon_i} U(c_i) \quad s.t \quad \begin{cases} c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e (e_i^x - e_i) - c_i(e_i^x) + q^\varepsilon (\bar{\varepsilon}_i - \varepsilon_i) + t_i^{ls} \\ e_i \leq \varepsilon_i \end{cases}$$

where the firm pay for ε_i carbon permits at price q^ε . This yields the optimality condition for

energy e_i and carbon permits ε_i :

$$MPe_i = q^e + q^\varepsilon$$

which implies that the implicit carbon tax is $MPe_i - q^e = \tilde{t}^\varepsilon = q^\varepsilon$. The complete treatment of this example is detailed in Section B.1. Below, I summarize several lessons on the optimal design of this type of policy.

First, the unconstrained distribution of “free carbon permits” $\bar{\varepsilon}_i$ acts as implicit money transfers across countries. Indeed, if the planner “offers” the ownership of carbon permits, countries/firms can sell them on the market to get $\bar{\varepsilon}_i q^\varepsilon$ and redistribute that money to households. Moreover, due to the government budget constraints, the revenue and cost of those permits are redistributed lump-sum to the household $\sum_i t_i^{ls} = q^\varepsilon \sum_i (\varepsilon_i - \bar{\varepsilon}_i)$. If these implicit transfers are allowed, the planner can achieve full redistribution $\omega_i U'(c_i) = \omega_j U'(c_j)$. As a result, we can recover the First-Best allocation with $q^\varepsilon = SCC$, exactly as in theorem 1 and Section 2.1.

Second, if we prevent both explicit and implicit transfers, i.e. $\bar{\varepsilon}_i = 0$, the planner again needs to adjust the carbon price for the presence of inequality, as represented by the social welfare weights $\hat{\phi}_i = \omega_j U(c_j) / \frac{1}{I} \sum_i \omega_i U(c_i)$. This implies that the total quantity $\bar{\mathcal{E}}$ to be auctioned on a global market should be chosen to target a carbon price of q^ε as in the Second-Best in Section 2.2.

Corollary 5 (Global carbon price, cap-and-trade system with limited transfers).

The carbon price target on a global cap-and-trade emission market needs to target the following level, accounting for redistribution motives, exactly as in theorem 2:

$$\begin{aligned} q^\varepsilon &= SCC + \text{Supply Redistribution} + \text{Demand Distortion} \\ q^\varepsilon &= - \sum_j \hat{\phi}_j \mathcal{D}'_j(\mathcal{S}) z_j F(e_j) + \mathcal{C}_{EE} \frac{1}{I} \sum_j \hat{\phi}_j (e_j - e_j^x) + \frac{1}{I} \sum_j \hat{v}_j \mathcal{D}_j(\mathcal{S}) z_j F''(e_j) \\ q^\varepsilon &= I \mathbb{E}_j \left(\mathcal{S} \gamma_j y_j \right) + I \text{Cov}_j \left(\hat{\phi}_j, \mathcal{S} \gamma_j y_j \right) + q^e \frac{\bar{v}}{E} \text{Cov}_j \left(\hat{\phi}_j, e_j - e_j^x \right) - q^e \text{Cov}_j \left(\hat{v}_j, \frac{1 - s_i^\varepsilon}{\sigma_i^\varepsilon e_i} \right). \end{aligned}$$

Third, if the planner wants to achieve additional redistribution, it needs to design *segmented markets* with $I = \#\mathbb{I}$ different prices q_i^ε . In each market, the planner needs to set the supply $\bar{\mathcal{E}}_i = e_i$ to achieve the target prices $q_i^\varepsilon \equiv t_i^\varepsilon = (1/\hat{\phi}_i)(SCC + \text{Supply redistribution})$, which are the same level as theorem 4 in Section 2.3. In that context, the carbon permits/allowances are not tradeable across countries unless against the exchange rate $q_i^\varepsilon/q_j^\varepsilon$ for one ton of CO_2 .

We see that the question of the number of instruments prevails over the *nature* of the instruments – price or quantity – as I explain in the next section.

2.5 Prices vs. Quantity

I now discuss the pros and cons of a global carbon tax or country-specific carbon taxes, or *price instruments*, in comparison to quantity targets in a cap-and-trade system, either with a global carbon budget or country-specific carbon targets. In that sense, I explore the arguments made in Weitzman (2003) and Weitzman (2015) on these two types of instruments.

Weitzman (2015) argues strongly against quantity targets and emissions cap-and-trade systems. He defends that a universal carbon tax ideally possesses (1) cost-effectiveness, (2) a natural one-dimensional *focal point* (as in Schelling), and (3) a built-in self-enforcement mechanism that internalizes the externality, balancing out countervailing forces and heterogeneous interests. According to him, n different quantity assignments, as in the top-down approach of the Kyoto Protocol, fail on the second and third points and are harder to bargain upon in international agreements. It spurs free-riding as each nation has a self-interest to negotiate a low cap on their own carbon emissions – lower than socially optimal and “disagreements over the subdivision of an aggregate world cap into n national quantity targets” prevent the achievement of an efficient solution.

I showed above that (i) the availability of transfers and (ii) the choice of instruments – whether a global policy or country-specific ones, are the most important determinants of the policy effectiveness, rather than the choice of quantity or price instruments. Indeed, as noted in Section 2.4, there is a direct mapping between a quantity of carbon permits $\bar{\mathcal{E}}$ along with a distribution of “free allowances” $\{\bar{\varepsilon}_i\}_i$ and a carbon tax t^ε along with cross-countries transfers t_i^{ls} , and both replicate the First-Best allocation. If, like Weitzman, we note that allocating carbon permits appears as “visible transfer payments across national borders”, we could instead advocate for $\bar{\varepsilon}_i = 0$ to refrain from transfers that are undermined by free-riding incentives.

In that context, there is also a direct link between choosing a total quantity of permits $\bar{\mathcal{E}}$ and a single carbon price $q^\varepsilon = t^\varepsilon$ that replicates Weitzman’s idea of an “internationally harmonized but nationally retained carbon tax” exactly as detailed in Section 2.2, theorem 2 and theorem 5. Regarding policy uncertainty and the volatility of carbon price in cap-and-trade systems for a given quantity target $\mathcal{E}$, one could note that policymakers could control the quantity of permits supplied and perform “open-market operations” to reach a stable carbon price target. Finally, country-specific quantity $\bar{\mathcal{E}}_i = e_i$ and country-specific carbon taxes $\{t_i^\varepsilon\}$ both suffer from the same pitfalls because of free-riding, bargaining frictions, and transaction costs in international negotiations. These considerations are examined in Bourany (2025).

2.6 Extensions and implication for carbon taxation

In the previous sections, I showed how the carbon tax should differ from the representative agent Pigouvian framework in the simplest model with heterogeneous countries. In Bourany (2025), I analyze the case with two important extensions. First, international trade has redistributive “carbon leakage” effects that dampen the effectiveness of carbon policy. For example, imposing a carbon tax on energy in country i reallocates production toward other countries j that now may export toward i . In Bourany (2025), I show that the presence of inequality and trade create a fourth motive that differentiates the optimal carbon tax from the social cost of carbon.

Second, the carbon policy is chosen by a social planner at the world level. However, free-riding – when individual countries deviate without implementing the socially optimal policies – create participation constraints that limit what can be achieved by the planner. In Bourany (2025), I study how to optimally design climate agreements accounting for such free-riding incentives.

2.7 Climate Risk and Social Cost of Carbon

Until now, the present analysis is deterministic: both the future gains and losses of climate policies and climate change are known to all agents. In [Bourany \(2023\)](#), I study how uncertainty about future climate damage and economic opportunities affect carbon policies. I show a summary of those results here.

Let us consider an extension of the previous where the damage caused by climate change and productivity is affected by a random variable $\epsilon \in \mathcal{E}$ following the probability distribution $\epsilon \sim \varphi(\epsilon)$, such that the productivity becomes stochastic: $\mathcal{D}_i(\mathcal{S}|\epsilon)$. This is a simple example of “climate risk”, where for a given amount of carbon emissions, the economic damage of climate change is unknown. The model is set up in two stages: (i) the agents and the planner take the energy – and thus emissions – decisions ex-ante, and (ii) the productivity/climate shock is realized and production and consumption $c_i(\epsilon)$ adjust ex-post to satisfy the budget and resources constraints.

In the competitive equilibrium, the representative agents maximize their expected utility:

$$\max_{e_i, e_i^x} \int_{\mathcal{E}} \max_{c_i(\epsilon)} U(c_i(\epsilon)) d\varphi(\epsilon)$$

In that case, the energy input decision is taken in expectation but does not imply fundamentally different choice. The optimality condition writes:

$$\int_{\mathcal{E}} MPe_i(\epsilon) d\varphi(\epsilon) = q^e$$

as before the households and firms do not internalize the climate externality caused by emissions.

I now consider the case of a social planner maximizing the world’s welfare. In the two-step decision process, the planner would choose the decisions in energy inputs, extraction, and a uniform carbon tax t^e for all countries:

$$\max_{\{e_j, e_j^x\}_j, q^e, t^e} \int_{\mathcal{E}} \max_{\{c_j(\epsilon)\}_j} \sum_{i \in \mathbb{I}} \omega_i U(c_i(\epsilon)) d\varphi(\epsilon)$$

As before, we consider the Social welfare weights, that are now adjusted for the ex-post allocation of risk:

$$\hat{\phi}_i(\epsilon) = \frac{\phi_i(\epsilon)}{\mathbb{E}_{i, \epsilon}[\phi_i(\epsilon)]} = \frac{\omega_i U'(c_i(\epsilon))}{\frac{1}{I} \sum_i \int_{\mathcal{E}} \omega_j U'(c_j(\epsilon)) d\varphi(\epsilon)}$$

The planner’s optimality condition for the energy choice gives us an analytical formulation for the carbon tax, which consist of four terms: the three terms we considered above: (i) the Social Cost of Carbon, (ii) the Supply redistribution, and (iii) the Demand Distortion, all of them adjusted for the presence of ex-post heterogeneity due to risk. There is also a new additional term: (iv) the effect of risk on the the energy choice. This last term correct for the redistributive motives caused by the presence of risk and is expressed as a covariance term: if the countries with the highest need for energy – i.e. with a high marginal product of energy – are also the poorest – with high social weights and marginal utility – this implies an energy subsidy ex-ante to provide

redistribution in the absence of insurance markets.

$$\mathbb{E}_\epsilon(MPe_i(\epsilon)) = q^e + \mathbb{E}_\epsilon[SCC(\epsilon) + \textit{Supply Redistribution}(\epsilon) + \textit{Demand Distortion}(\epsilon)] - \underbrace{\text{Cov}_\epsilon(\hat{\phi}_i(\epsilon), MPe_i(\epsilon))}_{\text{effect of agg. risk } \epsilon \text{ on energy choice}}$$

The three terms seen above are taken in expectation and the carbon taxation accounts for the presence of risk. Take the example of the Social Cost of Carbon: following the approach above to decompose how the heterogeneity ex-ante and ex-post affect the level of the Pigouvian motive. The Social Cost of Carbon, in expectation $\mathbb{E}_\epsilon[SCC(\epsilon)]$ is written:

$$\begin{aligned} \mathbb{E}_\epsilon[SCC(\epsilon)] &= \sum_{i \in \mathbb{I}} \int_{\mathcal{E}} \hat{\phi}_i(\epsilon) LCC_i(\epsilon) d\varphi(\epsilon) \\ &= \mathbb{E}_j \left[\underbrace{\text{Cov}_\epsilon(\hat{\phi}_i(\epsilon), LCC_j(\epsilon))}_{=\text{effect of aggregate risk } \epsilon} \right] + \underbrace{\text{Cov}_j \left[\mathbb{E}_\epsilon(\hat{\phi}_i(\epsilon)), \mathbb{E}_\epsilon(LCC_j(\epsilon)) \right]}_{=\text{effect of heterogeneity across } j} + \underbrace{\mathbb{E}_{j,\epsilon}[LCC_j(\epsilon)]}_{=\text{average exp. damage}} \end{aligned}$$

The second and third terms are the same as in the previous sections: the presence of inequality implies a covariance term for *ex-ante heterogeneity* that can increase or decrease the carbon tax. However, this time, it is taken in expectations, for the local cost of carbon $\mathbb{E}_\epsilon(LCC_j(\epsilon))$ and the social welfare weights $\mathbb{E}_\epsilon(\hat{\phi}_i(\epsilon))$. Similarly, the last terms is the average, ex-ante and ex-post, of the local cost of carbon.

The novelty is in the first term, representing the effect of risk, and hence *ex-post heterogeneity* on the cost of climate change. If the climate damages $LCC_i(\epsilon)$ are highest in the poorest countries, i.e. with the highest $\hat{\phi}_i(\epsilon)$, the covariance is positive and it can then increases the *SCC*. This create insurance motives for climate policy, as the Social Cost of Carbon is heightened due to climate risk when the poorest countries are the most affected by climate change.

We will see that these general lessons also hold in a richer dynamic setting.

3 Quantitative model

I develop a neoclassical framework with rich heterogeneity across regions. Time is continuous $t \in [t_0, \infty)$, and the countries are indexed by $i \in \mathbb{I}$. They can be heterogenous in an arbitrary number of dimensions as summarized in the state variables s_{it} ¹³.

In each country, we consider five representative agents: (i) a household making consumption and saving decisions, (ii) a firm using capital, labor, and different energy sources to produce the final good, and three energy firms that (iii) extract fossil fuels (oil and gas), (iv) produce coal, and (v) produce renewable/non-carbon energy. Finally, each country has a government that collects taxes and redistributes lump-sum rebates.

3.1 Household

In each country $i \in \mathbb{I}$, the representative household is of population size $\mathcal{P}_i$ – which we consider constant for exposition purpose.¹⁴

Households consume final goods c_{it} and are directly affected by their region’s temperature τ_{it} . They can save and invest in capital k_{it} to be rented to the good producer at rate r_t^k . Households also supply their inelastic labor $\bar{\ell}_i = 1$ to that final good firm, receiving the wage income $\bar{\ell}_i w_{it}$. Moreover, the households receive energy rent π_i^f generated by the fossil fuel producers, as will be detailed below. They maximize their present discounted utility with discount rate ρ , and solve the following intertemporal problem, with value function for states s_{it} :

$$\mathcal{V}_{it_0}(\{s_{it_0}\}_i) = \max_{\{c_{it}, k_{it}\}_t} \int_{t_0}^{\infty} e^{-\rho t} u_i(c_{it}, \tau_{it}) dt \quad (6)$$

The utility that households receive from consumption is also scaled by a damage function $\mathcal{D}_i^u(\tau)$, which represents the direct impact of temperature τ_{it} . I consider standard CRRA preference with the intertemporal elasticity of substitution $1/\eta$.

$$u_i(c_{it}, \tau_{it}) = u(\mathcal{D}_i^u(\tau_{it})c_{it}) \quad u(\mathcal{D}c) = \frac{(\mathcal{D}c)^{1-\eta} - 1}{1-\eta} . \quad (7)$$

As a result, the country i household budget constraint or wealth/capital dynamics evolves according to:

$$\dot{k}_{it} = r_{it}^k k_{it} + w_{it} \bar{\ell}_i + \pi_{it}^f - c_{it} + t_{it}^{ls} , \quad (8)$$

starting from initial condition k_0 . This setting features financial autharky as Finally, the household also receives lump-sum transfers t_{it}^{ls} from the government. Wealth k_{it} and temperature τ_{it}

¹³More precisely, state variables of heterogeneity can be split in two, $s_{it} = \{\underline{s}_i, \bar{s}_{it}\}$, where $\underline{s}_i$ represents ex-ante heterogeneity and states variables $\bar{s}_{it}$ represent ex-post heterogeneity that changes over time. In practice, the dimensionality of $\underline{s}$ can be arbitrarily large, as I explain in the computational section below.

¹⁴In the quantitative application, we consider the population dynamics following $\dot{\mathcal{P}}_{it} = n_{it} \mathcal{P}_{it}$ with exogenous growth rate n_{it} which needs to converges in finite time $t \leq T$ to a constant growth $\bar{n}$. Similarlry, we consider TFP growth of productivity $\dot{z}_{it} = g_{it} z_{it}$ with $g_{it} \rightarrow \bar{g}$ for $t \leq T$. Both assumption allow for flexible dynamics while ensuring a stable BGP after terminal date T . In such case, we could normalize all the economic variables of the model per “effective capita”, renormalizing the discount rate to $\rho_{it} = \rho - n_{it} - (1-\eta)g_{it}$.

are the two dimensions of ex-post heterogeneity that change with the transition. We consider a deterministic environment with financial autarky, which make countries unable to borrow/save on international markets.

3.2 Final good firm

In each country $i \in \mathbb{I}$, a representative firm produces a homogeneous final good, used for consumption and investment, with constant-return-to-scale (CRS) technology $F(\cdot)$ and different inputs: labor $\bar{\ell}_i$ at wage w_{it} , capital k_{it} at the rental rate r_{it}^k , and energy services e_{it} at price q_{it}^e as detailed below.

The firm maximizing profit, i.e. output per capita $y = \mathcal{D}^y(\tau)zF(\cdot)$ net of input costs:

$$\max_{k_{it}, e_{it}} \mathcal{D}_i^y(\tau_{it}) z_i F_i(k_{it}, \bar{\ell}_i, e_{it}) - w_{it} \bar{\ell}_i - q_{it}^e e_{it} - r_{it}^k k_{it}. \quad (9)$$

First, firm's productivity levels differ across countries due to institutional and efficiency time-invariant factors represented by the factor z_i . Second, temperatures τ_{it} affect output directly through climate damages $\mathcal{D}_i^y(\tau_{it})$, the source of climate externality which we detail below. For simplicity of exposition, the production function is Cobb-Douglas with capital-labor bundle $k_i^\alpha \ell_i^{1-\alpha}$ and CES in energy e_i with energy share ε_i with elasticity σ_y

$$F_i(k_{it}, \ell_i, e_{it}) = \left[(1 - \varepsilon_i)^{\frac{1}{\sigma}} (k_{it}^\alpha \ell_i^{1-\alpha})^{\frac{\sigma-1}{\sigma}} + \varepsilon_i^{\frac{1}{\sigma}} e_{it}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

The firm optimal inputs decision for capital demand k_{it} , labor ℓ_i , and energy e_{it} is such that the marginal product of the inputs equals its price. With the marginal product of capital, labor and energy are defined as: $MPx_{it} = \frac{d}{dx} [\mathcal{D}_i^y(\tau_{it}) z_i F_i(k_{it}, \ell_i, e_{it})]$ for $x \in \{k, \ell, e\}$, we obtain the first-order conditions:

$$r_{it}^k = MPk_{it} - \delta \quad w_{it} = MP\ell_{it} \quad q_{it} = MPe_{it} \quad (10)$$

Energy demand

Given the demand for energy inputs e_t , the firm in country i has the choice among three sources of energy: fossil fuels (or oil and gas) e_{it}^f , coal e_{it}^c and low-carbon/renewables e_{it}^r . These three sources are substitutable, and total energy e_{it} has constant elasticity of substitution $\sigma_e \in (0, \infty)$.

$$e_{it} = \left(\omega_f^{\frac{1}{\sigma_e}} (e_{it}^f)^{\frac{\sigma_e-1}{\sigma_e}} + \omega_c^{\frac{1}{\sigma_e}} (e_{it}^c)^{\frac{\sigma_e-1}{\sigma_e}} + \omega_r^{\frac{1}{\sigma_e}} (e_{it}^r)^{\frac{\sigma_e-1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e-1}}$$

subject to the budget for energy expenditures, where q_t^f is the international price of oil and gas, q_{it}^c the local price of coal energy, and q_{it}^r the local price of low-carbon energy.

Additionally, energy from oil-gas, e_i^f , and coal, e_i^c emit greenhouse gases affecting climate, with respective carbon concentration ξ^f and ξ^c , as we will see in Section 3.4. As a result, there is a motive for taxing oil, gas, and coal energy with the carbon tax τ_{it}^ε , which is a tax per ton of CO_2 .

As a result, this pins down the price of the energy bundle q_{it} :

$$q_{it}^e e_{it} = e_{it}^f (q_t^f + \xi^f t_{it}^\varepsilon) + e_{it}^c (q_{it}^c + \xi^c t_{it}^\varepsilon) + e_{it}^r q_{it}^r$$

$$q_{it}^e = \left(\omega_f (q_t^f + \xi^f t_{it}^\varepsilon)^{1-\sigma_e} + \omega_c (q_{it}^c + \xi^c t_{it}^\varepsilon)^{1-\sigma_e} + \omega_r (q_{it}^r)^{1-\sigma_e} \right)^{\frac{1}{1-\sigma_e}},$$

and, the representative final good firm choose the energy inputs according to the first-order condition:

$$q_t^f + \xi^f t_{it}^\varepsilon = M P e_{it}^f = q_{it}^e \omega_f^{\frac{1}{\sigma_e}} \left(\frac{e_{it}^f}{e_{it}} \right)^{-\frac{1}{\sigma_e}} \quad (11)$$

and similarly for the other energy inputs $q_{it}^c + \xi^c t_{it}^\varepsilon = M P e_{it}^c$ and $q_{it}^r = M P e_{it}^r$.

3.3 Energy producers

We consider three kinds of energy sources – oil and gas, coal, or renewable (non-carbon) – supplied by three representative energy firms in each country $i \in \mathbb{I}$.

3.3.1 Fossil firm

A competitive energy producer extracts fossil fuels oil and gas e_i^x from its pool of resources $\mathcal{R}_{it}$. The energy is extracted with convex production cost $\nu_i^f(e_{it}^x, \mathcal{R}_{it})$, where these costs are paid in units of final good.

The fossil-fuel reserves $\mathcal{R}_{it}$ are depleted with extraction e_{it}^x and discovered at exogenous rate d_{it}^x – all denoted in per-capita terms – such that $\dot{\mathcal{R}}_{it} = d_{it}^x - e_{it}^x$. We assume that neither the fossil firms nor the social planners internalize the scarcity of these resources. Internalizing the resource depletion would imply a more involved dynamic Hotelling problem – with stock effects – with Hotelling rents rising over time, dampening the extraction rate e_i^x . I suggest an extension in appendix XX to see how these motives would change the taxation of fossil fuel and carbon theoretically. For simplicity, I refrain from considering this extension in the quantitative analysis. Richer models developed in [Bornstein, Krusell and Rebelo \(2023\)](#), [Heal and Schlenker \(2019\)](#), and [Asker, Collard-Wexler, De Canniere, De Loecker and Knittel \(2024\)](#) study the dynamic aspects of the oil market and the considerations for carbon emissions.

As a result, the static maximization problem of the fossil firm is given by :

$$\pi_{it}^f = \max_{e_{it}^x} q_{it}^f e_{it}^x - \nu_i^f(e_{it}^x, \mathcal{R}_{it}),$$

$$\dot{\mathcal{R}}_{it} = d_{it}^x - e_{it}^x \quad (12)$$

with initial condition $\mathcal{R}_{i0}$ for reserves and $\nu_i^f(e_{it}^x, \mathcal{R}_{it})$ the extraction cost per capita. Since extraction costs are convex, energy production has decreasing return to scale.¹⁵ As a result, positive

¹⁵We can also define a fossil production function with inputs x_i^f such that $e^x = g(x_i^f)$ and profit $\pi = q^f g(x) - x$ instead of $\pi = q^f e^x - \nu(e^x)$, in which case $g(x) = \nu^{-1}(x)$ and profit are positive when productive has DRS, i.e. $g(\cdot)$ is concave.

energy rents π_{it}^f exist, even when the competitive firms take fossil price q_t^f as given. Moreover, I abstract from market power in the oil market, for example with the OPEC as a cartel – even though this framework could easily allow for such an extension. Any sources of misallocation or other frictions can be accounted for in the calibration of the cost function $\nu_i^f(\cdot)$ as we will see in the quantification Section 4. I consider a functional form for cost that yields isoelastic supply curves for fossil energy extraction.

$$\nu_i^f(e_{it}^x, \mathcal{R}_{it}) = \frac{\bar{\nu}_i}{1 + \nu_i} \left(\frac{e_{it}^x}{\mathcal{R}_{it}} \right)^{1 + \nu_i} \mathcal{R}_{it} \quad (13)$$

which is homogeneous of degree one in $(e_i^x, \mathcal{R}_i)$ and where the elasticity $\nu_i = \frac{\nu_i^{f''}(e^x, \mathcal{R})}{\nu_i^{f'}(e^x, \mathcal{R})e^x}$ is constant.

Optimal oil and gas extraction follows from the optimality condition:

$$q_t^f = \bar{\nu}_i \left(\frac{e_{it}^x}{\mathcal{R}_{it}} \right)^{\nu_i} \quad e_{it}^x = \mathcal{R}_{it} (q_t^f / \bar{\nu}_i)^{1/\nu_i} \quad (14)$$

. Finally, energy rent comes from fossil firms' profits $\pi^f(q^f, \mathcal{R}_{it})$ which writes:

$$\pi_i^f(q_t^f, \mathcal{R}_{it}) = q_t^f e_{it}^x - \nu_i^f(e_{it}^x, \mathcal{R}_{it}) = \frac{\nu_i}{1 + \nu_i} q_t^f e_{it}^x = \frac{\nu_i \bar{\nu}_i^{-1/\nu_i}}{1 + \nu_i} \mathcal{R}_{it} (q_t^f)^{1 + \frac{1}{\nu_i}}. \quad (15)$$

Energy profit $\pi_i^f(q^f, \mathcal{R})$ and its share in income $\eta_{it}^f = \frac{\pi_{it}^f}{y_i + \pi_{it}^f}$ are key to determine the exposure of a country to carbon taxation. Indeed, reducing carbon emissions by phasing out of fossil fuels reduces energy demand and its price q^f and hence hurt energy rent π_i^f and the welfare of oil and gas exporters.

Oil and gas are traded frictionlessly in international markets.¹⁶ The market clears such that

$$E_{it}^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e_{it}^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e_{it}^x. \quad (16)$$

Countries have different exposure to this fossil energy market, summarized by its net exports of oil and gas $e_i^x - e_i^f \leq 0$.

3.3.2 Coal firm

In each country i , a representative firm produces coal consumed by final good firms. We differentiate coal from other fossil fuels like oil and gas because (i) empirically, coal production does not generate large energy rents as a share of GDP,¹⁷ and we thus assume that extraction technology is constant return to scale (CRS), implying no profit in equilibrium. Then, (ii) large coal consumers also produce the major part of that coal locally, as trade costs for coal transportation are larger. Hence, I make the empirically grounded assumption that coal is not traded internationally. Moreover, (iii) coal production is not subject to reserves dynamics, as indeed coal is not scarce

¹⁶We consider here the simplifying assumption that fossil fuels produced in different countries are not distinguishable – crude oil or natural gas from Nigeria, Saudi Arabia, or Russia are not differentiated varieties.

¹⁷Motivated by evidence that even the largest coal producers do not have coal rents above 1% of GDP.

with a reserve/production ratio above a hundred for most large producers.

The production $\bar{e}_{it}^c$ has constant returns to scale and uses final good inputs, which yield the profit maximization problem:

$$\pi_{it}^c = \max_{\bar{e}_{it}^c} q_i^c \bar{e}_{it}^c - \kappa_i^c \bar{e}_{it}^c = 0 ,$$

where the marginal cost κ_i^c is constant, which implies that the price for coal and the market clearing condition are given by:

$$q_i^c = \kappa_i^c , \quad \bar{e}_i^c = e_i^c . \quad (17)$$

This implies a perfectly elastic supply curve for coal energy, something we observe in practice as coal production is easily scalable in response to oil and gas price fluctuations. We also use the free cost parameter κ_i^c to calibrate the share of coal

3.3.3 Low-carbon, renewable, firm

Renewable and other non-carbon energy sources, such as solar, wind, hydro, or nuclear electricity (thereafter generalizing under the name “renewables”) are a third source of energy and provide a way of substituting away from fossil fuels in the production function $F(\cdot)$.

A representative firm produces renewables or non-carbon energy, and this supply, $\bar{e}_{it}^r$, is not traded. Similarly, this assumption follows the fact that electricity is rarely traded across countries – and when it is, it is only the result of temporary differences in electricity production due to intermittency rather than large structural imbalances. The production $\bar{e}_{it}^r$ also has constant returns to scale, and this input is paid in units of the final good. Hence, the renewable firm maximization problem is:

$$\pi_{it}^r = \max_{\bar{e}_{it}^r} q_{it}^r \bar{e}_{it}^r - \kappa_{it}^r \bar{e}_{it}^r ,$$

where κ_{it}^r is the marginal cost of producing renewables, resulting in zero profits $\pi_i^r = 0$ and a perfectly elastic supply curve. The price of renewable and the market clearing are given by:

$$q_{it}^r = \kappa_{it}^r = \bar{\kappa}_i e^{-g_r t} , \quad e_{it}^r = \bar{e}_{it}^r \quad (18)$$

Later, I assume that marginal costs κ_{it} decrease exogenously at rate g_r . Such technological progress can result from learning-by-doing positive externalities – where κ_{it} decreases with installed capacity – potentially yielding a decreasing supply curve as in [Arkolakis and Walsh \(2023\)](#). We consider such extension in the future, while we refrain from adding additional market failures and Schumpeterian externalities to focus on optimal policies against climate externalities.

3.4 The climate system

Economic activity and fossil fuel consumption in country i emit greenhouse gases – called carbon or CO_2 here for simplicity – which feed back in the climate system, raising temperatures and causing heterogeneous damages over regions j . Such externalities create Pigouvian motives for carbon taxation summarized by the Social Cost of Carbon (SCC).

3.4.1 Emissions

Oil and gas e_{it}^f and coal e_{it}^c consumptions in each country $i \in \mathbb{I}$ release total CO_2 emissions:

$$\mathcal{E}_t = \bar{\xi}_t \sum_{i \in \mathbb{I}} \mathcal{P}_i \epsilon_{it} = \bar{\xi}_t \sum_{i \in \mathbb{I}} \mathcal{P}_i \left(\xi^f e_{it}^f + \xi^c e_{it}^c \right) .$$

where ξ^f and ξ^c denote the carbon content of respectively oil-gas and coal energy. We consider emissions plateauing in finite time $t \leq T$ and I follow [Krusell and Smith \(2022\)](#) by assuming that part of emissions $\mathcal{E}_t$ is abated via carbon capture and storage (CCS) modeled by the exogenous parameter $\bar{\xi}_t$, with $\bar{\xi}_{t_0} = 1$. The share of emissions abated grows in the long-run, stabilizing cumulative carbon emissions and temperatures.

3.4.2 Climate system and temperature

Moreover, these emissions are released in the atmosphere, adding up to the cumulative stock of greenhouse gas $\mathcal{S}_t$ – or atmospheric carbon concentration:

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t . \quad (19)$$

A part of these emissions exit the atmosphere or is stored in oceans or the biosphere, discounting the current stock by an amount δ_s . Moreover, these cumulative emissions push the global atmospheric temperature $\mathcal{T}_t$ upward linearly with climate sensitivity χ .

$$\mathcal{T}_t = \mathcal{T}_{t_0} + \chi \mathcal{S}_t . \quad (20)$$

We adopt this simple climate system for tractability, and the methodology developed below can be extended to large-scale climate models.¹⁸

3.5 Damage and externality

Climate damages relate to local temperatures: warmer regions are more affected and vulnerable to extreme events such as heatwaves and hurricanes. Local temperatures in country i is affected by atmospheric warming $\mathcal{T}_t$ with linear pattern scaling Δ_i following:

$$\begin{aligned} \dot{\tau}_{it} &= \Delta_i \dot{\mathcal{T}}_t \\ \tau_{it} &= \tau_{i0} + \Delta_i (\mathcal{T}_t - \mathcal{T}_{t_0}) \end{aligned} \quad (21)$$

¹⁸Climate models with more complex climate block add 3 to 4 additional state variables $\mathbf{J}$, or carbon concentration and temperatures in “boxes” or layers of the atmosphere and sinks such as the oceans:

$$\begin{aligned} \dot{\mathbf{J}}_t &= \Phi^J \mathbf{J}_t + \rho^S \sum_{i \in \mathbb{I}} \epsilon_{it} \\ F_t &= \mathcal{F}(\rho^F \mathbf{J}_t) \quad \dot{\mathcal{T}}_t = \Phi^T \mathcal{T}_t + \rho^F F_t \end{aligned}$$

with F_t carbon forcing with $\mathcal{F}(\cdot) \sim \log(\cdot)$, ρ^S , ρ^F , and ρ^T are vectors are Φ^J and Φ^T Markovian transition matrices.

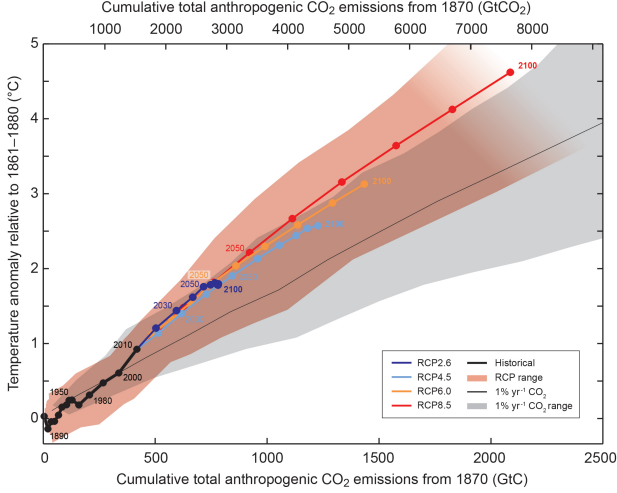


Figure 1: Linear temperature model - IPCC report

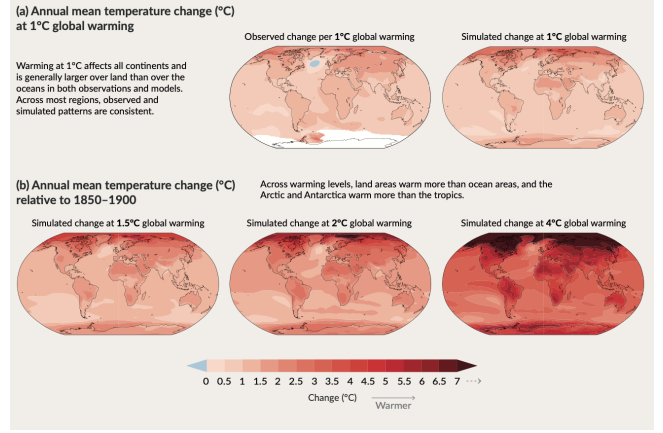


Figure 2: Local pattern scaling

As shown in Section 4, I estimate this pattern scaling by regressing local temperatures on global temperature.

Finally, I consider a period damage function $\mathcal{D}_i^y(\tau_{it}) := \mathcal{D}^y(\tau_{it} - \tau_i^*)$ for productivity and $\mathcal{D}^u(\tau_{it} - \tau_i^*)$ for damage to utility. The target τ_i^* is the “optimal” temperature for country i . The function $\mathcal{D}^y(\hat{\tau})$ is a reduced-form representation of the economic damage. In the baseline quantification, I assume damages are quadratic, as in standard Integrated Assessment Models such as the DICE framework. This methodology follows [Krusell and Smith \(2022\)](#), [Kotlikoff, Kubler, Polbin, Sachs and Scheidegger \(2021\)](#) and [Burke et al. \(2015\)](#). Moreover, those damages are such that productivity decays to zero when temperatures are extremely cold or hot.

$$\mathcal{D}_i^y(\tau) = \mathcal{D}^y(\tau - \tau_i^*) = \exp \left(-\gamma^y \mathbb{1}_{\{\tau > \tau_i^*\}} (\tau - \tau_i^*)^2 - \alpha^\gamma \gamma^y \mathbb{1}_{\{\tau < \tau_i^*\}} (\tau - \tau_i^*)^2 \right), \quad (22)$$

where γ_y represents the damage parameter on output for warm temperatures, with an asymmetric impact $\alpha^\gamma < 1$ for cold temperatures following the quantification in [Rudik et al. \(2021\)](#), who show that productivity impact is much weaker for cold than for hot temperatures. The damage function for utility $\mathcal{D}_i^u(\tau)$ has the same functional form with damage γ^u .

This creates winners and losers: countries warmer than their target temperature τ_i^* are extremely affected by global warming. In contrast, regions with negative $\tau_{it} - \tau_i^*$ benefit – at least in the short-run – from a warmer climate. I deviate from the above articles by assuming that the target temperature τ_i^* differs across countries: an already warm regions have different adaptation costs compared to a country which is historically cold. The target temperature $\tau_i^* = \alpha^\tau \tau^* + (1 - \alpha^\tau) \bar{\tau}_{it0}$ is more or less tilted toward historical baseline. I discuss this quantification in Section 4.

3.6 Competitive Equilibrium

Final goods are freely traded, and with output $y_i = \mathcal{D}_i(\tau_{it}) z_{it} F_i(\cdot)$, world market clears:

$$\sum_{i \in \mathbb{I}} \mathcal{P}_i \left[c_{it} + \dot{k}_{it} + \delta k_{it} + \nu_i^f(e_{it}^x, \mathcal{R}_{it}) + \kappa_i^c e_{it}^c + \kappa_{it}^r e_{it}^r \right] = \sum_{i \in \mathbb{I}} \mathcal{P}_i \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, \ell_i, e_{it}). \quad (23)$$

Definition. Competitive equilibrium (C.E.): For a set of policies $\{t_{it}^\varepsilon, t_{it}^{ls}\}_{it}$ across countries, a C.E. is a set $\{c_{it}, k_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x, \bar{e}_{it}^c, \bar{e}_{it}^r\}_{it}$ of decisions, and prices $\{r_{it}^k, q_t^f, w_{it}, q_{it}^c, q_{it}^r\}_{it}$, and states $\{\mathcal{S}_t, \tau_{it}, \mathcal{R}_{it}\}_{it}$ such that:

- (i) Households choose consumption and saving $\{c_{it}, k_{it}\}_{it}$ to maximize utility in equation (6) subject to budget constraint equation (8).
- (ii) Final good firms choose inputs $\{k_{it}, \ell_i, e_{it}, e_{it}^f, e_{it}^c, e_{it}^r\}_{it}$ to maximize profits, resulting in equation (11) and equation (10).
- (iii) Fossil energy firms maximize profits in equation (12) and extract $\{e_{it}^x\}_{it}$ as in equation (14)
- (iv) Renewable and coal energy firms supply $\{\bar{e}_i^c, \bar{e}_i^r\}$ given respectively by equation (17) and ??
- (v) Energy markets clears for fossils in equation (16) and coal and renewables in equation (17) and equation (18)
- (vi) Emissions $\mathcal{E}_t$ affect climate $\{\mathcal{S}_t, \mathcal{T}_t, \tau_{it}\}_{it}$, following equation (19), equations (20) and (21).
- (vii) Good markets clear for final good as in equation (23).

Heterogeneity. This model features many dimensions of heterogeneity, that can be summarized by the state variable $s_{it} = \{\underline{s}_i, \bar{s}_{it}\}$, describing the ex-ante dimensions of heterogeneity $\underline{s}_i$ – constant differences across countries – and ex-post heterogeneity $\bar{s}_{it}$ changing endogeneously over time. The states are: $\underline{s}_i = \{p_i, z_i, \bar{\nu}_i, \nu_i, \kappa_i^c, \kappa_i^r, \Delta_i, \tau_i^*\}$ and $\bar{s}_{it} = \{k_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}$, yielding agent value $\mathcal{V}_{it}(\{s_{it}\}_{it})$. For solving the model, we keep track of the dynamical system with heterogeneity, as we see next.

3.7 Sequential formulation and Solution method

First, since households own the four firms – final good, fossil, coal, and renewable energy – we can aggregate profits and household budget constraint:

$$\dot{k}_{it} = \mathcal{D}_i^y(\tau_{it}) z_{it} F(k_{it}, \ell_i, e_{it}^f, e_{it}^c, e_{it}^r) - \delta k_{it} - (q_t^f + \xi^f t_{it}^\varepsilon) e_{it}^f - (q_{it}^c + \xi^c t_{it}^\varepsilon) e_{it}^c - q_{it}^r e_{it}^r + \pi_i^f(q_t^f, \mathcal{R}_{it}) - c_{it} + t_{it}^{ls}, \quad (24)$$

and yields a single optimal control problem.

To solve for the competitive equilibrium and the optimal decision of the Household, we solve this class of Integrated Assessment Model with the sequential formulation and rely on the Pontryagin Maximum Principle. The household in each country has individual states $\mathbf{s} = \{\underline{s}_i, \bar{s}_{it}\}_{it} = \{\underline{s}_i, k_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$, individual controls, $\mathbf{c} = \{c_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x\}_{it}$, take prices as given $\mathbf{q} = \{r_t^k, q_t^f, w_{it}, q_{it}^c, q_{it}^r\}_{it}$, and has costates or Lagrange multipliers, $\boldsymbol{\lambda} = \{\lambda_{it}^k, \lambda_{it}^S\}$, each of which represents shadow value of the respective states dynamics.¹⁹ The Hamiltonian for country i is written as follow:

$$\mathcal{H}(\mathbf{s}_i, \mathbf{c}_i, \mathbf{q}, \boldsymbol{\lambda}_i) = u(c_i, \tau_i) + \lambda_i^k \dot{k}_i + \lambda_i^S \dot{\mathcal{S}}$$

for dynamics $\dot{k}_{it}$ in equation (24), $\dot{\mathcal{S}}_t$ in equation (19).

¹⁹The costates for τ_{it} or for $\mathcal{S}_t$ can be used interchangeably due to linear pattern scaling.

As a result, equilibrium relations for household problem boil down to the standard neo-classical model dynamics, yielding a system of coupled ODEs for each country $i \in \mathbb{I}$, with state equation (24) and costate $\lambda_{it}^k = \frac{\partial \mathcal{V}_{it}}{\partial k_{it}}$ or “marginal value” of capital k_{it} , equal to marginal utility $u_c(c_{it}, \tau_{it})$:

$$\dot{\lambda}_{it}^k = \lambda_{it}^k(\rho - r_{it}^k) \quad \lambda_{it}^k = u_c(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it})u'(\mathcal{D}^u(\tau_{it})c_{it}) \quad (25)$$

We obtain the Euler equation, including local temperature dynamics:

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{1}{\eta} (r_{it}^k - \rho) + \gamma^c(\tau_{it} - \tau_i^*)\dot{\tau}_{it} \quad (26)$$

Indeed, because marginal utility of consumption is affected directly by climate damages γ^c , future global warming $\dot{\tau}_{it} > 0$ triggers intertemporal substitution through saving.

Note that this states-costates formulation is *very general* and hold in a large class of heterogeneous agents models. To close the control problem, note that household income is determined by the firms energy decisions which here simply result from static optimization between the price, marginal return in production.

3.8 Social and Local Cost of Carbon

Climate change affect damages and agents’ valuation, which can be represented by “Social Cost of Carbon” (SCC), a marginal welfare measure in monetary terms. This cost of carbon is an equilibrium concept: it depends on the path of temperatures τ_{it} but also on economic variables and policies. In the competitive equilibrium – or laissez faire, or status-quo – climate externalities are not internalized and households and firms do not take climate damages into account for choosing consumption, production, and energy decisions. Still, forward-looking agents can anticipate perfectly the evolution of climate and its welfare impact.

The Local Cost of Carbon (LCC) represents such a welfare valuation, for the cost incurred by country i of one additional ton of CO_2 released in the atmosphere $\mathcal{S}_t$. In continuous time, and using our the Pontryagin Maximum Principle approach, the Local Cost of Carbon (LCC) is expressed as the ratio of the two costates:

$$LCC_{it} := -\frac{\frac{\partial \mathcal{V}_{it}}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{V}_{it}}{\partial k_{it}}} = -\frac{\lambda_{it}^S}{\lambda_{it}^k}. \quad (27)$$

The welfare cost of carbon $\lambda_{it}^S = \frac{\partial}{\partial \mathcal{S}_t} \mathcal{V}_i$ represents the marginal welfare change from an additional ton of carbon $\mathcal{S}_t$, normalized in monetary units with the marginal value of wealth (or capital here) $\frac{\partial \mathcal{V}_{it}}{\partial k_{it}} = \lambda_{it}^k = u_c(c_{it}, \tau_{it}) \neq u_c(c_{jt}, \tau_{jt}) = \lambda_{jt}^k$, which differs across regions i due to inequality in consumption. This notion is exactly analogous to the Local Cost of Carbon concept developed in [Cruz and Rossi-Hansberg \(2022a\)](#), among many others.

As a result, following the LCC dynamics amount to solving for both costates dynamics λ_{it}^k and λ_{it}^S . Using the Pontryagin principle, we can pin down the dynamics for λ_{it}^S , the shadow value of adding carbon in the atmosphere, through both the productivity effect $y = \mathcal{D}^y(\tau)zF(k, e)$ and

the utility effect $\lambda^w = \mathcal{D}^u(\tau) u'(\mathcal{D}^u(\tau)c)$.

$$\dot{\lambda}_{it}^S = \lambda_{it}^S(\rho + \delta_s) + \chi \Delta_i (\tau_{it} - \tau_i^*) (\gamma^y y_{it} + \gamma^u c_{it}) \lambda_{it}^w . \quad (28)$$

which pins down the welfare cost of an extra ton of carbon.²⁰ Using the Euler equation, or costate dynamics of equation (25), we get $\lambda_{it}^k = \lambda_{is}^w e^{-\int_t^s (\rho - r_{iu}^k) du}$ for $s > t$, which gives the Local Cost of Carbon:

$$\begin{aligned} LCC_{it} &= \int_t^\infty e^{-\delta_s(s-t) - \int_t^s r_{iu}^k du} \chi \Delta_i (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] ds \\ &\xrightarrow{t \rightarrow \infty} \frac{\chi \Delta_i}{\rho + \delta_s} (\tau_{i\infty} - \tau_i^*) [\gamma^y y_{i\infty} + \gamma^u c_{i\infty}] . \end{aligned} \quad (29)$$

We obtain a result that is conventional in Integrated Assessment Models (like DICE) with multiplicative damages $y = \mathcal{D}^y(\tau) z F(\cdot)$: Local Cost of Carbon LCC_{it} scales with output y_{it} as in Golosov, Hassler, Krusell and Tsyvinski (2014) as well as climate parameters – sensitivity χ , carbon life-cycle δ_s , and pattern scaling Δ_i . Moreover, they are discounted with preference ρ and, thanks to intertemporal substitution, it depends on the path of interest rate $\{r_{is}^k\}_{t \geq s}$. Additionally, with quadratic damage in temperature, the LCC also increases with the path of temperatures τ_{it} . When the temperature stabilizes to $\tau_{it} \rightarrow \tau_{i\infty}$, so do damages and output/consumption in the long-run, yielding the stationary formula.²¹ From those regional Local Costs of Carbon, we can aggregate the global effects of climate change into one measure of welfare, the Social Cost of Carbon, which we introduce next.

4 Quantification and calibration

The theoretical results are general and hold in a large class of integrated assessment models. We present throughout the next sections a quantitative application to provide realistic predictions on the impact of optimal carbon policy. The model is calibrated to a sample of 32 regions (25 largest economies and 7 “rest of continents”). I first describe the data used. I then provide details on the quantification and how the parameters are calibrated to match the data. I summarize in Table 1 the dimensions of heterogeneity of the model. Table 2 contains the summary table for the calibration described in this section.

²⁰We can express it in level as

$$\lambda_{it}^S = - \int_t^\infty e^{-(\rho + \delta_s)(s-t)} \chi \Delta_i (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \lambda_{is}^k ds ,$$

²¹This formula can be naturally extended to the case with BGP with TFP growth and population growth, where all the variables are expressed per “efficient capita” as:

$$LCC_{it} \xrightarrow{t \rightarrow \infty} \frac{\chi \Delta_i}{\rho + \eta g_i + \delta_s} (\tau_{i\infty} - \tau_i^*) [\gamma^y y_{i\infty} + \gamma^u c_{i\infty}]$$

4.1 Data

The model is calibrated on data for years 2018-2023, taking the average over that period to smooth out the effect of the COVID-19 recession on energy and macroeconomic data. Data for GDP per capita, in Purchasing Power Parity (PPP, in 2016 USD) come from the World Bank, as collected and processed by the Maddison Project, [Bolt and van Zanden \(2023\)](#). For the energy variables, I use the comprehensive data collected and processed in the Statistical Review of Energy [Energy Institute \(2024\)](#), that includes the production and consumption of various energy sources, including Oil, Gas, Coal and Renewable. It also includes proven reserves of those fossil fuels. For energy rent, I use the World Development Indicators that use national accounts to measure the share of GDP coming from energy (oil, gas and coal) and natural resource rents. Finally, for temperature, I use the same time series as [Burke et al. \(2015\)](#), which use the temperature at country level, averaged over the year and weighted by population across locations.

4.2 Welfare and Pareto weights

The welfare function that the social planner maximize, in Section [D](#) is the weighted sum of individual utilities in all countries, with p_i the population size and ω_i the Pareto weights.

I consider two sets of Pareto weights. First, I consider the utilitarian benchmark, where the planner weight every individual in the world equally: $\omega_i = 1$. In case, a large weight is set on developing countries.

Second, following the discussion in [Anthoff et al. \(2009\)](#), [Nordhaus \(2011\)](#) and [Nordhaus and Yang \(1996\)](#), one would like to choose Pareto weights that eliminate redistributive effects that are orthogonal to climate change and carbon policy. To that purpose, the “Negishi” Pareto weights make the preexisting competitive equilibrium efficient under that welfare metric. This implies that:

$$\omega_i = \frac{1}{u_c(\bar{c}_{it_0}, \tau_{it_0})} \quad \Leftrightarrow \quad C.E.(\bar{c}_i) \in \operatorname{argmax}_{\bar{c}_i} \sum_i p_i \omega_i u(\bar{c}_{it_0}, \tau_{it_0})$$

$$\omega_i u_c(\bar{c}_{it_0}, \tau_{it_0}) = \omega_j u_c(\bar{c}_{jt_0}, \tau_{jt_0}) \quad \forall i, j \in \mathbb{I}$$

where $\bar{c}_i$ is the consumption level in the present competitive equilibrium steady state – the period 2018-2023 – absent future climate damage. This implies that the carbon policy do not have redistributive motives through energy general equilibrium effects. However, global warming, and carbon taxation itself have redistributive effects, as they change the distribution of c_{it} . These effects would thus be taken into account in the choice of policies as shown in theorem [9](#). In Figure [19](#), we show the distribution of those weights for our quantification. Due to the large consumption differences, rich European and American countries carry 50 to 100 times more weights than the poorest ones in South Asia or Africa, which essentially shuts down redistribution motives.

4.3 Macroeconomy, trade and production

For the macroeconomic part of the framework, I consider standard utility and production functions. For utility in equation [\(7\)](#), we take the CRRA/IES parameter $\eta = 1.5$ from [Barrage](#)

and Nordhaus (2024). For production, I use a nested CES framework. The firm combines a Cobb-Douglas bundle of capital k_i and labor ℓ_i ²² with a composite of energy e_i , with elasticity $\sigma^y = 1$, the Cobb-Douglas case. Second, energy e_i aggregates oil and gas e_i^f , coal e_i^c , and renewable/non-carbon e_i^r , with elasticity σ^e . For the energy, I set $\varepsilon = 0.10$ to match the world average energy cost share $\frac{q_i^e e_i}{y_i} = 6\%$, as measured in Kotlikoff, Kubler, Polbin and Scheidegger (2021b) and used in Krusell and Smith (2022), and the energy mix for oil-gas, $\omega^f = 0.56$, coal $\omega^c = 0.27$, and non-carbon $\omega^r = 0.17$, to match the aggregate shares of each energy sources in the data. Below, we match the individual countries' energy mix using energy prices/costs. Finally, for the elasticity between energy inputs, I use the value $\sigma_e = 2$, following the rest of the literature, i.e. Papageorgiou et al. (2017), Kotlikoff, Kubler, Polbin and Scheidegger (2021b), and Hillebrand and Hillebrand (2019), among others.

I calibrate the productivity z_i of the production function $y_{it_0} = \mathcal{D}_i^y(\tau_{it_0}) z_i \bar{y}_{it_0}$ to match exactly GDP data y_{it_0} across countries. This parameter z_i , represents productivity residuals as well as institutional/efficiency differences across countries. In Figure 16, I show the GDP levels, as they replicated with this model.

4.4 Energy markets

For the energy markets, I match exposure of different countries. First, on the supply side, I calibrate the three parameters $\mathcal{R}_i$, ν_i and $\bar{\nu}_i$ to match the three country-level variables $\mathcal{R}_i$, e_i^x and π_i^f . The reserves $\mathcal{R}_i$ are taken directly from the data on oil and gas reserves documented by Energy Institute (2024). I calibrate the slope of this cost function $\bar{\nu}_i$ to match exactly the production of oil and gas e_i^x from the same data, as displayed in Figure 17. I calibrate the cost curvature to match the share $\eta_i^\pi = \frac{\pi_i^f}{y_i p_i + \pi_i^f}$ of fossil energy profit as share of GDP within a 10 percentage point range for all countries. Differences in oil and gas energy rent across countries are not only determined by differences in cost and technology, but also in differences in trade costs and market power²³, which explains why it is difficult to match exactly the value η_i^π . However, for simplicity and tractability, I refrain from adding an additional market failure.

Second, I match the energy mix of the different countries by relying on the two assumptions made in the model: (i) coal and renewable are only traded at the country level: $\bar{e}_i^c = e_i^c$ and $\bar{e}_i^r = e_i^r$ and (ii) their cost functions are linear in goods (CRS). By calibrating energy costs parameters κ_i^c and κ_i^r , one can match the data on coal share $\frac{e_i^c}{e_i^f + e_i^c + e_i^r}$ and non-carbon energy share $\frac{e_i^r}{e_i^f + e_i^c + e_i^r}$, successfully identifying countries that are more reliant on coal vs. oil and gas vs. non-carbon/renewable: for example, China and India are highly coal-dependent, and Russia, Middle-East and United-States/Canada are the biggest consumers of oil and gas.

²²Labor is inelastically supplied $\ell_i = \bar{\ell}_i$ in each country and normalized to 1 – since the country size p_i is already taken into account. As a result, all the variables can be seen as input per capita.

²³For example, OPEC controls more than 28% of oil supply and around 15% of natural gas supply, and has substantive market power

4.5 Climate system

Finally, I calibrate the climate model described in Section 3.4, following Dietz et al. (2021), to set the climate sensitivity to $\chi = 0.23$ and the carbon decay $\delta = 0.0004$ to match two moments of pulse experiments usually done in IAMs.

Moreover, this climate system is inherently unstable for emissions, $\mathcal{E}_t$ trends up with population growth, n , and long-term TFP growth $\bar{g}$, where $n = 0.0035$ and $\bar{g} = 0.01$, the long-term growth rates according to forecast by the UN and World-Bank. To counteract the non-stationarity of the climate system, I follow Krusell and Smith (2022) and assume that part of the emissions $\mathcal{E}_t$ are captured and stored, under the variable $\bar{\xi}_t$. I assume a logistic form such that the temperature are stable in the last period which correspond to a BGP. More optimistic/pessimistic scenarios for Carbon Capture and Storage (CCS) could be imagined without affecting the main result since most damages are discounted heavily after 2100.

Additionally, I calibrate initial temperatures τ_{it_0} with data from Burke et al. (2015), and I display selected countries in Figure 18. Furthermore, identify the linear pattern parameter $\tau_{it} = \Delta_i \mathcal{T}_t$ in reduced-form by estimating this linear regression over the period $t = 1950\text{--}2015$ for each country i .²⁴ This procedure does not require extensive granular data about geography, albedo, etc.

Finally, to calibrate the DICE-type quadratic damage function in equation (22), I consider two calibration: (i) $\gamma = 0.00340$ close to Krusell and Smith (2022) that matches Nordhaus' DICE calibration of 6.6% global GDP loss for $\mathcal{T}_t = 5$, and the updated calibration in Barrage and Nordhaus (2024) which calibrate it at $\gamma = 0.00347$. For cold temperature values, I consider $\gamma^- = 0.3\gamma$, following the quantification in Rudik et al. (2021), who show that the negative productivity impact of cold temperatures is much weaker than for hot temperatures. Alternatively, (ii) I consider the one $\gamma = 0.012$ estimated in Bourany and Rosenthal-Kay (2026) which implies a Social Cost of Carbon significantly higher.

Finally, to pin down τ_i^* , I use also an intermediary assumption between the following two polar cases: (i) representative agent economies, like Barrage and Nordhaus (2024), would assume $\tau_i^* = \tau_{it_0}$, and differences in damages only comes from increases in global temperature $\mathcal{T}_t$. This echoes Bilal and Känzig (2024) analysis showing that climate damages come in large part from the increase in global temperature causing extreme events. In contrast, (ii) a different view in heterogeneous economies would set an “ideal” temperature $\tau_i^* = \tau^*$ for all regions, as in Krusell and Smith (2022) and Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021), highlighting that climate damages come essentially from differences in initial temperatures. I take the intermediary route with $\tau_i^* = \alpha^T \tau^* + (1 - \alpha^T) \tau_{it_0}$, where $\alpha^T = 0.5$ and $\tau^* = 14.5$ is the average spring temperature of developed economies – and around the yearly average of places like California or Spain.

²⁴To control for the fact that country j has an influence on world temperature $\mathcal{T}_t = \sum_i g_i \tau_{it}$, I estimate the jackknife linear equation with $\mathcal{T}_{t,\neq j} = \sum_{i \neq j} g_i \tau_{it}$ for each j , i.e. $\tau_{jt} = \Delta_j \mathcal{T}_{t,\neq j}$.

4.6 Heterogeneity

In this section, I summarize the different dimensions of heterogeneity included in the model and aggregate the parameters of the calibration in Table 2.

Table 1: Heterogeneity across countries

Dimension of heterogeneity	Model parameter	Matched variable from the data	Source of the data
Population	Country size $\mathcal{P}_i$	Population $\mathcal{P}_i$	UN Population Prospect
TFP/technology/institutions	Firm productivity z_i	GDP per capita (2016-PPP) y_i	World Bank/Maddison project
Productivity in energy	Energy-augmenting productivity z_i^e	Energy cost share s_i^e	SRE Energy Institute (2024)
Cost of coal energy	Cost of coal production κ_i^c	Energy mix/coal share e_i^c/e_i	SRE Energy Institute (2024)
Cost of non-carbon energy	Cost of non-carbon production κ_i^r	Energy mix/coal share e_i^r/e_i	SRE Energy Institute (2024)
Local temperature	Initial temperature τ_{it0}	Pop-weighted yearly temperature	Burke et al. (2015)
Pattern scaling	Pattern scaling Δ_i	Sensitivity of τ_{it} to world $\mathcal{T}_t$	Burke et al. (2015)
Oil-gas reserves	Reserves $\mathcal{R}_i$	Proved Oil-gas reserves $\mathcal{R}_i$	SRE Energy Institute (2024)
Cost of oil-gas extraction	Slope of extraction cost $\bar{\nu}_i$	Oil-gas extracted/produced e_i^x	SRE Energy Institute (2024)
Cost of oil-gas extraction	Curvature of extraction cost ν_i	Profit π_i^f / energy rent	World Bank / WDI

4.7 Solution method

The global sequential method developed here for heterogeneous regions Integrated Assessment Models is a generic methodology adapted from the Pontryagin Maximum Principle. Agents' states can be either ex-ante heterogeneous $\underline{s}_i$ or states that change over time $\bar{s}_{it}$, with marginal value/costates λ_{it} . This gives rises to a system of coupled ODEs. The benefit is that the dimensionality of $\underline{s}_i$ can be potentially very large – we use 10 dimensions – and $\bar{s}_{it}$ can be increased compared to standard Dynamic Programming methods – we use 3 dimensions $\{k_{it}, \tau_{it}, \mathcal{R}_{it}\}$.

- (i) Choose a sample drawn from a distribution on $\underline{s}_i$: we have 32 countries here but can extend it to several hundreds. One should use Monte-Carlo draws over $\underline{s}_i$ for more complex models.
- (ii) Gake a guess for the paths of aggregate variables: in our case $\{\mathcal{S}_t, q_t^f\}_t$.
- (iii) given those aggregate states, simulate countries sequentially, states forward $\{k_{it}, \tau_{it}, \mathcal{R}_{it}\}$ and costates backward $\{\lambda_{it}^k, \lambda_{it}^S\}_{it}$. Individual controls – $\{c_{it}, e_{it}^f, e_{it}^x, \dots\}$ – can be solved jointly with states-costates using non-linear solvers. Countries can be solved in parallel.
- (v) Reiterate or use Newton methods for the aggregate trajectories given implicit equations, like market clearing: in our case consistency between climate system $\mathcal{S}_t$ and the sum of individual emissions $\mathcal{E}_t$, and oil-gas market clearing $\sum_i \mathcal{P}_i(e_i^f - e_i^x) = 0$.

The equilibria under optimal policy (see below) would include states, controls and multipliers resulting in large dynamical systems – all while solving in seconds on standard computer.²⁵ This can be applied in general Heterogeneous Agents settings – with discrete agents in our case, or atomistic agents/Mean-Field Games, as in [Carmona and Delarue \(2018\)](#).

²⁵The Competitive equilibrium and First-Best solve in 3 seconds for a standard MacBook (2023 M3 Pro 36 GB Ram), and the Second Best Optimal Policy (with many more costates and multipliers) solves in 18 seconds.

4.8 Dynamics of the Competitive Equilibrium

We can now analyze the transition path of the model exposed above in the “laissez-faire” equilibrium without climate policy, in Figure 3. Thanks to TFP growth, countries that the least affected by climate change are growing up to 1% per year, with a dampening effect from climate damage as we get closer to 2100, where world temperature go beyond $4^\circ C$ above pre-industrial levels. This is translated in a sharp increase of the Local Cost of Carbon. However, some countries are hurt much more, like India, Nigeria, or Saudi-Arabia, plotted here, and more generally South and South-East Asia, Middle-East, and Africa, with negative effect on production and consumption, where λ_{it}^k would increase faster than the rise in λ_{it}^S , inducing a stabilization of the LCC_{it} for those countries. Additionally, on the energy market, declining reserve increase the cost of extraction, reducing oil-gas supply e_{it}^x , and slowing down the total energy consumed e_{it} while increasing the price of fossil fuel q_t^f over time.

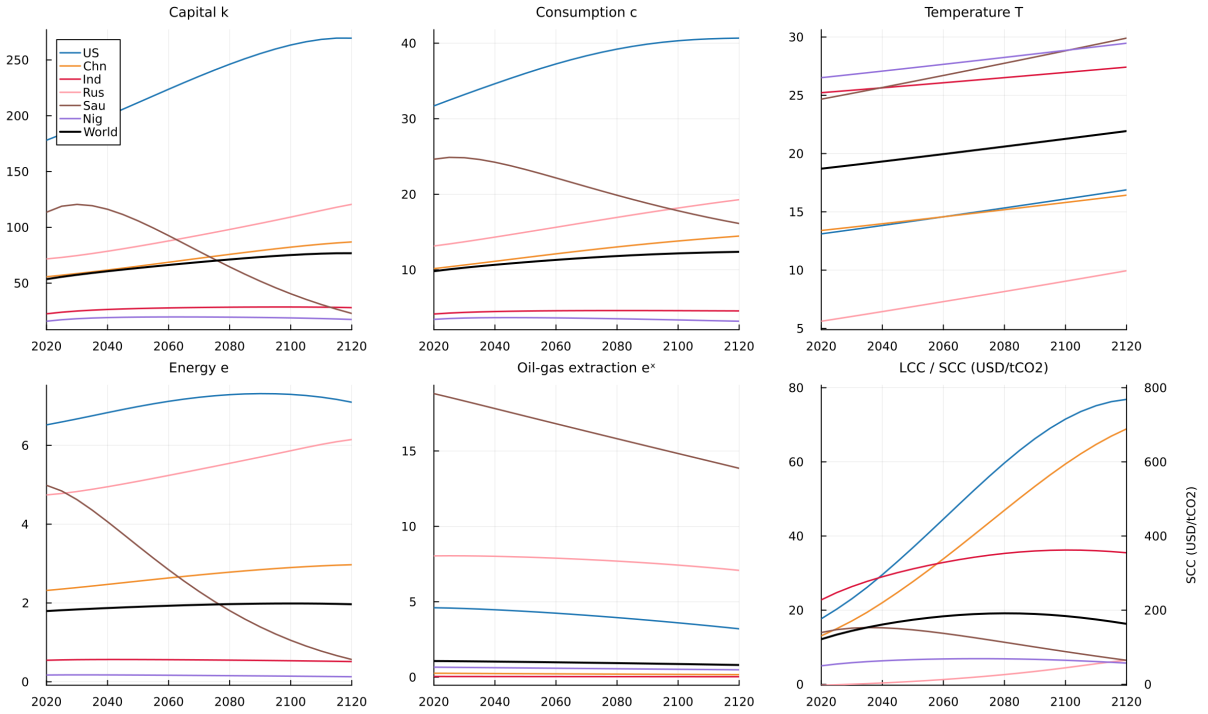


Figure 3: Competitive equilibrium – No Climate Policy

Additionally, we can plot the Local Cost of Carbon across countries – $p_i LCC_{it_0}$ – which are present discounted values of future climate damages $t \in [t_0, \bar{t}]$. As mentioned above, this costs scale (i) with output y_i and consumption c_i , which explain the high values of large economies like the US, (ii) with temperature τ_i , an implication of quadratic/convex damage functions, and (iii) with population size p_i by construction, such that large economies like China and India have country-level LCC_i among the highest. As a point of comparison, we add a naive measure of the Social Cost of Carbon – where $\overline{SCC}_{t_0} = \sum_i \omega_i LCC_{it_0}$ where $\omega_i = 1$ are the uniform weights, leading to a value $\approx 200/tCO_2$ in usual range for SCC in the literature.

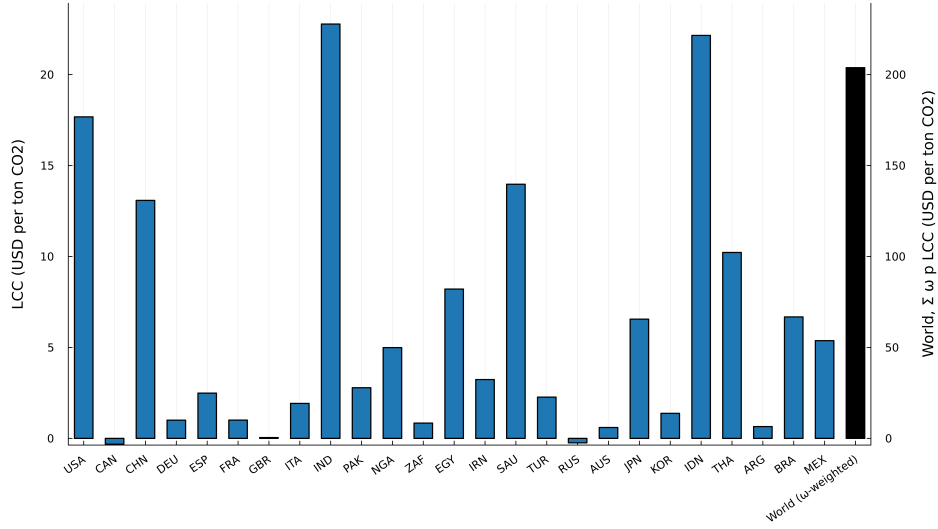


Figure 4: Competitive equilibrium – Local Costs of Carbon

5 Optimal Policy with Climate Change and Inequality

We now consider a world social planner that design the optimal climate policy to maximize global welfare. We follow three benchmarks: (i) the First-Best allocation, where the social planner has access to all the instruments, including cross-countries lump-sum transfers, (ii) the Second-Best policy where the social planner only chooses a single uniform carbon tax τ_t^ε , and (iii) the Second-Best allocation, choosing country-specific carbon taxes τ_{it}^ε . Despite the model being richer than Section 2's model, the theoretical results are analogous: I show how inequality and the lack of redistributive instruments change the path of the carbon tax.

5.1 First-Best Allocation

When a social planner can choose the allocation decision in each country, it maximizes global welfare – which is the weighted sum of household utilities, with Pareto weights²⁶ ω_i – subject to the economy resource constraints.

$$\mathcal{W}_{t_0} = \max_{\{c, k, e^f, e^c, e^r, e^x, \bar{e}^c, \bar{e}^r\}} \sum_{\mathbb{I}} \mathcal{P}_i \omega_i \int_{t_0}^{\infty} e^{-\rho t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) dt \quad (30)$$

subject to the good and energy resource constraints and the climate system. We associate – for *global* welfare – the Lagrange multipliers ϕ_t^k for the market clearing for final good, $\mu_t^f, \mu_{it}^c, \mu_{it}^r$ fossil (oil-gas), coal and renewable (low-carbon) resources respectively, and ϕ_t^S for the marginal value of

²⁶Pareto weights should average to one, s.t. $\sum_{\mathbb{I}} \frac{\mathcal{P}_i}{\mathcal{P}} \omega_i = 1$

emissions:

$$\begin{aligned}
\sum_{i \in \mathbb{I}} \mathcal{P}_i \left[c_{it} + \dot{k}_{it} + \delta k_{it} + \nu_i^f(e_{it}^x, \mathcal{R}_{it}) + \kappa_i^c e_{it}^c + \kappa_{it}^r e_{it}^r \right] &= \sum_{i \in \mathbb{I}} \mathcal{P}_i \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, \ell_i, e_{it}) . & [\phi_t^k] \\
E_{it}^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e_{it}^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e_{it}^x & \quad [\mu_t^f] & \quad \bar{e}_i^c = e_i^c & \quad [\mu_{it}^c] & \quad \bar{e}_i^r = e_i^r & \quad [\mu_{it}^r] \\
\dot{\mathcal{S}}_t = \sum_{\mathbb{I}} \mathcal{P}_i (\xi^f e_{it}^f + \xi^c e_{it}^c) - \delta_s \mathcal{S}_t & \quad [\phi_t^S] & \quad \tau_{it} = \tau_{it_0} + \Delta_i \chi \mathcal{S}_t
\end{aligned}$$

The following result extends the theorem 1, and the complete description is available in Section D.

5.1.1 Allocation and Redistribution

The world planner's optimality condition for consumption c_{it} shows a redistribution motive: it equalizes marginal utility subject to Pareto weights:

$$\omega_i u_c(c_{it}, \tau_{it}) = \phi_t^k = \omega_j u_c(c_{jt}, \tau_{jt})$$

with marginal utility $u_c(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it}) u'(\mathcal{D}^u(\tau_{it}) c_{it}) = \mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{-\eta}$, with the CRRA functional form. Despite the possibility, in the competitive equilibrium, to trade in goods and energy, strong inequality exists due to differences in productivity, energy rents or climate damage. As a consequence, in the First-Best, the social planner redistributes consumption across regions. For arbitrary weights $\{\omega_i\}_i$, this implies that in decentralized equilibria, they would use cross-countries lump-sum transfers to compensate the regions with lowest consumption, or warmer climate, ceteris paribus.²⁷ Similarly, optimality for capital yields the dynamics for the aggregate value of wealth ϕ_t^k which evolves following the Euler equation:

$$\dot{\phi}_t^k = (\rho - r_{it}^k) \phi_t^k = (\rho - r_{jt}^k) \phi_t^k \quad \forall j \in \mathbb{I}$$

which also equalizes marginal product of capital $r_{it}^k = r_{jt}^k =: r_t^{fb}, \forall t > t_0$, allocating capital k_{it} to its most productive use across countries and over time.

5.1.2 Social Cost of Carbon

Given the welfare function in equation (30), the marginal cost of adding one unit of carbon in the atmosphere $\mathcal{S}_t$ is summarized by the Social Cost of Carbon, defined as:

$$SCC_t^{fb} := - \frac{\frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{W}_t}{\partial c_{it}}} = - \frac{\phi_t^S}{\phi_t^k} \quad (31)$$

where the aggregate welfare change ϕ_t^S is normalized in monetary units with the aggregate marginal value of wealth $\partial \mathcal{W}_t / \partial c_t = \phi_t^k$. This second equation results from the equalization of marginal

²⁷Naturally, there exists one set of Pareto weights $\omega_i = \phi_t^k / u_c(c_{it}, \tau_{it})$ such that the transfers are null at some points in time t , but the planner would still use intertemporal transfers along state transitions.

utilities such that $\frac{\partial \mathcal{W}_t}{\partial c_{it}} = \phi_t^k, \forall i \in \mathbb{I}$.

As a result, the welfare cost of carbon, as costate of carbon dynamics, aggregate every regions climate damages, following:

$$\dot{\phi}_t^S = \phi_t^S(\rho + \delta_s) + \chi \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \Delta_i(\tau_{it} - \tau_i^*)(\gamma^y y_{it} + \gamma^u c_{it}) \phi_t^k .$$

which yields, thanks to equalization of interest rate $r_{it}^k = r_t^{fb}$ and the dynamics of marginal value of weath ϕ_t^k over time:

$$SCC_t^{fb} = \int_t^\infty e^{-\delta_s(s-t) - \int_s^t r_u^{fb} du} \chi \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \Delta_i(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] dt , \quad (32)$$

and this implies the following proposition.

Proposition 6 (Social Cost of Carbon – First-Best Allocation).

In the First-Best allocation, marginal utilities are equalized $\phi_i^k = \omega_i u_c(c_{it}, \tau_{it}), \forall i \in \mathbb{I}$, and as a result, we obtain that the Social Cost of Carbon is the sum of Local Costs of Carbon of the different locations $i \in \mathbb{I}$.

$$SCC_t^{fb} = \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i LCC_{it}$$

where the Local Cost of Carbon LCC_{it} is defined exactly as in equation (29), *i.e.*

$$LCC_{it} = \int_t^\infty e^{-\delta_s(s-t) - \int_t^s r_{iu}^k du} \chi \Delta_i(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] ds .$$

Social Cost of Carbon is the sum, weighted by population $\mathcal{P}_i$ and Pareto weights ω_i , of the Local Cost of Carbon LCC_{it} . Despite the model being substantially richer, the main result of the Section 2 holds.²⁸ This also aligns with models without heterogeneity in marginal value of wealth, or no heterogeneity in productivity, or where climate damages do not have redistributive effects on production, consumption and welfare, which can be aggregated. Such models include Nordhaus and Yang (1996), and Krusell and Smith (2022), where in the later the free mobility of capital and the absence of heterogeneity in TFP or production allows to aggregate the economy into an aggregate production function, which would yield an analogous result.

5.1.3 Dynamics of the First-Best Allocation

In Figure 5, we observe a clear change in consumption levels: the planner equalize countries' marginal value of wealth, redistributing across countries and over time. As a consequence, all the countries obtain consumption in the same range – depending on their temperature as a compensat-

²⁸The proof – whose sketch follows from the main text – relies on the optimality w.r.t. consumption and capital subject to a single resource constraint (over time and across countries), which implies equalization of marginal utility, and marginal product of capital (and thus labor). Technically, it also relies on Fubini's theorem to swap the sum and integral – trivial here with a finite number of countries and when integrals converge under transversality conditions.

ing differential – and growing at the same rate of a global interest rate $r_t^{fb} \approx \rho + \eta g$. Additionally, the capital is allocated such that marginal product are equalized after $t > t_0$, which implies the discontinuity after the first period, typically away from warm country.

The carbon tax is set to $t_t^\varepsilon = SCC_t = \sum_i p_i \omega_i LCC_{it}$ which helps lower emissions and curb the rise in temperature, where global temperature stay under $2.5^\circ C$ compared to preindustrial temperature. Since the collapse of several developing economies is avoided, this decreases the denominator $\phi_t \propto u_c(c_{it}, \tau_{it})$, along with the lower welfare cost of climate change, which results in a similar SCC_t as in the laissez-faire economy, between $\$100$ and $\$200/tCO_2$. This also explain why India's LCC_{it} rises to $\$25$ (compared to $\$20$ in the competitive equilibrium) despite having a better climate $25.5^\circ C$ (instead of $27^\circ C$ under laissez-faire).

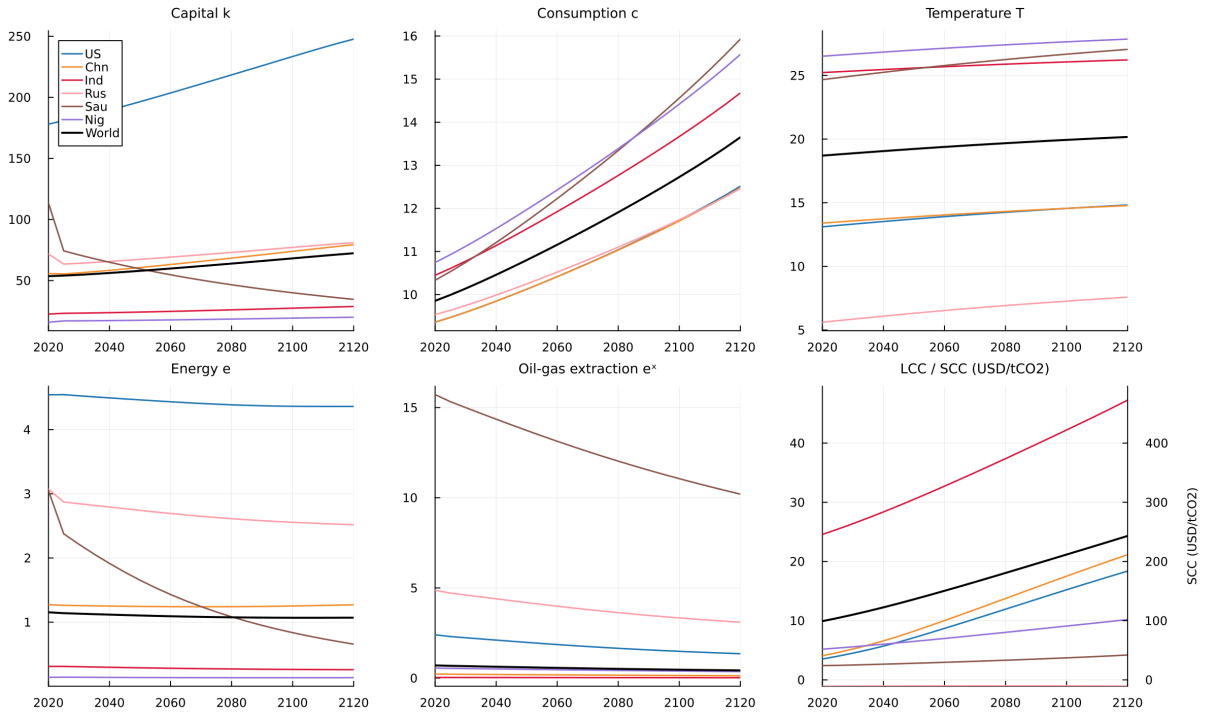


Figure 5: First-Best Allocation – Climate Policy $t_t^\varepsilon = SCC_t$

5.1.4 Decentralization of the First-Best: transfers and carbon tax

I show how this allocation can be decentralized with carbon taxation in the competitive equilibrium. The detailed treatment of this result is in Section B.

Proposition 7 (First-Best Allocation – Carbon taxation and transfers).

To decentralize the First-Best allocation, we need two set of instruments: (i) a carbon tax, (ii) cross-country lump-sum transfers. First, the optimal First-Best carbon tax equals the Social Cost of Carbon, as defined in theorem 6:

$$MPE_{it}^f = q_{it}^f + \xi^f t_t^\varepsilon \quad t_t^\varepsilon = SCC_t^{fb}$$

A similar optimality condition holds for coal $MPE_{it}^c = q_{it}^r + \xi^c t_t^\varepsilon$. In particular, the optimal carbon

tax is equal across countries. This results from the equalization marginal value of wealth. To achieve such equalization, the planner uses lump-sum transfers:

$$\omega_i u_c(c_{it}, \tau_{it}) = \phi_t^k = \omega_j u_c(c_{jt}, \tau_{jt}) \quad \Rightarrow \quad c_{it}^* = u_c^{-1}\left(\frac{\phi_t^k}{\omega_i} \middle| \tau_{it}\right)$$

which pins down the lump-sum transfers needed given the budget constraint:

$$t_{it}^{ls} = c_{it}^* - y_{it} - \pi_{it}^f + \delta k_{it} + \dot{k}_{it} + (q_t^f + \xi^f t_t^\varepsilon) e_{it}^f + (q_{it}^c + \xi^c t_t^\varepsilon) e_{it}^c + q_{it}^r e_{it}^r,$$

Additionally, such transfers reallocate capital across space and marginal product are equalized $r_t^{fb} = r_{it}^k \forall i \in \mathbb{I}$. Also, ceteris paribus, country with lower production (e.g. higher climate damages) or higher exposure to the carbon taxation itself t_t^ε could potentially receive high transfers.

A uniform carbon tax result aligns with the standard policy recommendations in representative agent models – which encompass most Integrated Assessment Models, like DICE, Nordhaus (2017), Barrage and Nordhaus (2024), FUND, PAGE, MERGE, and others – or the optimal carbon taxation result of Golosov, Hassler, Krusell and Tsyvinski (2014). It also reminiscent of models with unrestricted redistribution such as Hillebrand and Hillebrand (2019).

Lump-sum transfers redistribute across countries and across time:

$$\int_{t_0}^{\infty} e^{-\int_s^t r_u^{fb} du} \sum_{\mathbb{I}} p_i t_{it}^{ls} dt = 0$$

Situations where income y_{it} , energy rents π_{it}^f , climate damage and temperature τ_{it} , or Pareto weights ω_i are very heterogeneous such that consumption differentials in the equilibrium without policy intervention are large, the First-Best implies that some countries receive positive lump-sum transfers $\exists j, s.t. t_j^{ls} > 0$ and others pay lump-sum taxes $\exists j', s.t. t_{j'}^{ls} < 0$. Therefore, such decentralized allocation features direct lump-sum transfers across countries.

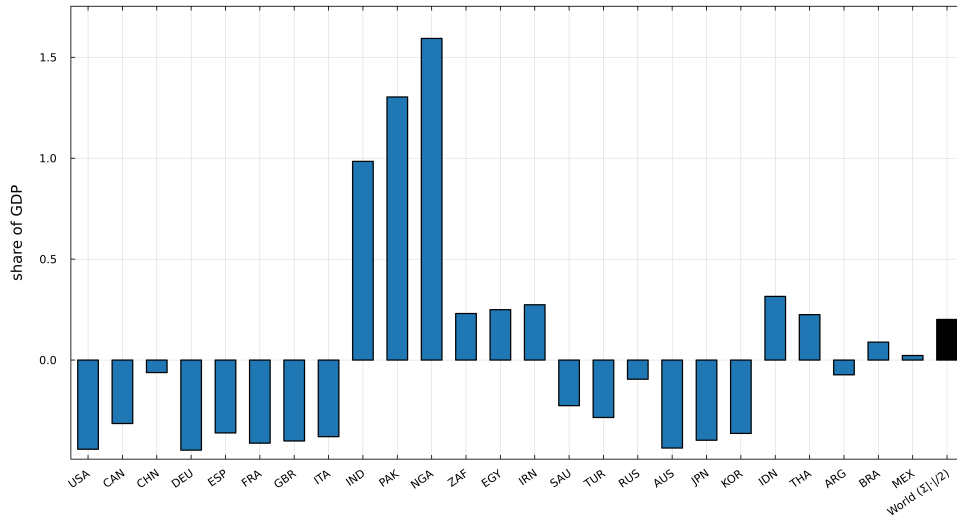


Figure 6: First Best Allocation, decentralized – Cross-countries Transfers

Note: Transfers / GDP (1 $\equiv$ 100%), present discounted value over 2020–2120, with $r = \rho + \eta g$.

We plot those transfers in Figure 6 where the transfers received t_i^{ls} are measured as a ratio of GDP of the receiving country, taken with an average over time at world interest r_t^{fb} . The question is whether such lump-sum transfers are politically feasible. Can a world central planner impose lump-sum transfers to solve world inequality, for example taxing North America and Europe and rebating it lump-sum to Africa or South Asia? In the next sections, I analyze a family of policies where transfers are not allowed.

5.2 Second-Best Ramsey problem and Uniform Carbon Taxation

I consider optimal carbon taxation in this framework, where the planner is prevented from achieving perfect redistribution. Lump-sum transfers are not available instruments, for political, governance, or economic reasons. This implies to solve a Ramsey taxation problem with limited instruments, where the planner internalize the constraints that are impose by the Competitive Equilibrium.

In particular, it maximizes global welfare, choosing a uniform carbon tax for the world $\{t_t^\varepsilon\}_t$ and rebates the revenue of that tax to the household of the country that pays it $t^{ls} = t_t^\varepsilon \varepsilon_{it}$.

$$\mathcal{W}_{t_0}(\{s_{it_0}\}_i) = \max_{\substack{\{c, k, e^f, e^c, e^r, e^x, \bar{e}^c, \bar{e}^r\} \\ \{t^\varepsilon, q^f, q^c, q^r\}}} \sum_{\mathbb{I}} \mathcal{P}_i \omega_i \int_{t_0}^{\infty} e^{-\rho t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) dt. \quad (33)$$

The planner takes the agent decisions, $\{c_{it}, k_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x, \bar{e}_{it}^c, \bar{e}_{it}^r\}_{it}$, prices $\{q_t^f, r_{it}^k, w_{it}, q_{it}^c, q_{it}^r\}_{it}$, and states $\{k_{it}, \tau_{it}, \mathcal{S}_t\}_{it}$ subject to the constraints of the Competitive equilibrium, which are (i) the budget constraint equation (24), with multiplier ψ_{it}^k , (ii) the climate dynamics for carbon $\mathcal{S}_t$, equation (19), with multiplier ψ_t^S , (iii) the firms optimality conditions for energy $e_{it}^f, e_{it}^c, e_{it}^r$, from equation (11), respectively with multipliers $v_{it}^f, v_{it}^c, v_{it}^r$, (iv) the energy firms optimality for extraction and production, from equations (14), (17) and (18), (v) the dynamic consumption/saving Euler equation from equation (26) with multiplier ψ_{it}^c , and finally (vi) the market clearing for goods equation (23), fossils, coal and renewable energy, from equations (16) to (18).

We apply the Pontryagin Maximum Principle and the details of the whole system are found in Section E. Note that the social planner has Full Commitment: all the decision paths are taken at the initial period t_0 and bind $\forall t \in (t_0, \infty)$. We provide intuitions of the most important results.

First, to study inequality and redistribution, we define the normalized social welfare weights using the marginal value of wealth ψ_{it}^k for an agent i for the planner with welfare $\mathcal{W}_t$:

$$\frac{\partial \mathcal{W}_t}{\partial k_{it}} = \omega_i \mathcal{P}_i \frac{\partial \mathcal{V}_{it}}{\partial k_{it}} = \omega_i \mathcal{P}_i \psi_{it}^k \propto \omega_i \mathcal{P}_i u_c(c_{it}, \tau_{it}) \quad \Rightarrow \quad \omega_i u_c(c_{it}, \tau_{it}) \neq \omega_j u_c(c_{jt}, \tau_{jt}) \quad (34)$$

$$\hat{\psi}_{it}^k := \frac{\omega_i \mathcal{P}_i \psi_{it}^k}{\bar{\psi}_t^k} \quad \text{with} \quad \bar{\psi}_t^k = \frac{1}{\bar{\mathcal{P}}} \sum_{\mathbb{I}} \omega_i \mathcal{P}_i \psi_{it}^k,$$

with $\bar{\psi}_t^k$ the marginal value of wealth of the “average” agent. The ratio $\hat{\psi}_{it}^k$ represents a relative “inequality index”: if it is higher than 1, the planner value capital, temperature, and consumption of country i at time t with a higher weight than the average household.

5.2.1 Second-Best, Social Cost of Carbon

Again, as in the previous section, the Social Cost of Carbon is measured as the ratio of the marginal value of carbon and the marginal value of wealth:

$$SCC_t^{sb} := -\frac{\frac{\partial \mathcal{W}_t}{\partial S_t}}{\frac{\partial \mathcal{W}_t}{\partial c_t}} = -\frac{\psi_t^S}{\bar{\psi}_t^w} \quad (35)$$

it differs slightly from the First-Best definition in equation (31), because now the planner consider the “average agent” marginal utility $\bar{\psi}_t^w$ to convert the Social Cost of Carbon in monetary unit.

To obtain the Social Cost of Carbon, we again solve the differential equation for the marginal value of carbon ψ_t^S . We summarize by m_{it} the marginal instantaneous damage for country i , which was $m_{it} = (\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}]$ in the Competitive Equilibrium and the First-Best. Now, in the Second-Best, the planner internalize production distortion caused by climate damage itself, which is:

$$m_{it} = (\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}] + \gamma^u (\tau_{it} - \tau_i^*) c_i \frac{\eta-1}{\eta} \left(1 - \frac{\lambda_{it}^k}{\psi_{it}^k}\right) - \gamma^y (\tau_{it} - \tau_i^*) \sum_{a \in \{e^f, e^c, e^r, k\}} MPa_{it} \tilde{v}_{it}^a$$

with MPa_{it} the marginal product of input a and $\tilde{v}_{it}^a := v_{it}^a / \bar{\psi}_{it}^w$ the reweighted shadow value for input a optimality. Two additional terms appears. The first one relates to agents’ saving decision, weighted by EIS $1/\eta$: it represents an additional “damage” if the planner marginal value of capital ψ_{it}^k is higher relative to the one of the agent λ_{it}^k . Additionally, when the choice of input a is too low, and their marginal product “too high”, additional damage lower TFP which help – hence the negative sign – to reduce distortions, weighted. Given the dynamics of ψ_t^S , the Cost of Carbon becomes:

$$SCC_t^{sb} = \int_t^\infty e^{-\delta_s(s-t)} \sum_{i \in \mathbb{I}} e^{-\int_t^s r_{iu}^k du} \hat{\psi}_{is}^k \chi \Delta_i m_{is} ds$$

This now differs from the First-Best because the marginal damages of temperatures m_{is} are weighted by relative welfare $\hat{\psi}_{it}^k$ of region i . We thus obtain an important result in Second-Best economies:

Proposition 8 (Social Cost of Carbon – Second-Best Allocation).

In the Second-Best allocation, inequalities across countries persist as measured by the social welfare weights $\hat{\psi}_{it}^k \neq 1, \forall i \in \mathbb{I}$. When redistribution is constrained, the Social Cost of Carbon is the weighted sum of Local Costs of Carbon of the different locations $i \in \mathbb{I}$, weighted by the social welfare weights $\hat{\psi}_{it}^k$ as given in equation (34):

$$SCC_t^{sb} = \sum_{i \in \mathbb{I}} \hat{\psi}_{it}^w LCC_{it} \propto \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i u_c(c_{it}, \tau_{it}) LCC_{it}$$

where the Local Cost of Carbon LCC_{it} is defined as in equation (29), i.e.

$$LCC_{it} = \int_t^\infty e^{-\delta_s(s-t) - \int_t^s r_{iu}^k du} \chi \Delta_i m_{is} ds .$$

As a result, we can express the Social Cost of Carbon as:

$$\begin{aligned}
SCC_t^{sb} &= \sum_{\mathbb{I}} \hat{\psi}_{it}^w LCC_{it} \\
&= \mathcal{P} \mathbb{E}^{\mathbb{I}}[\omega_i LCC_{it}] + \mathcal{P} \text{Cov}^{\mathbb{I}}(\hat{\psi}_{it}^w, LCC_{it}) \quad \leq \quad \mathcal{P} \mathbb{E}^{\mathbb{I}}[\omega_i LCC_{it}] =: SCC_t^{fb}
\end{aligned}$$

for the mean $\mathbb{E}^{\mathbb{I}}[\cdot]$ and covariance $\text{Cov}^{\mathbb{I}}(\cdot)$ over locations.²⁹ Since the Social Cost of Carbon is a sum – and not a mean – one needs to rescale by world population $\mathcal{P}$.

To summarize, the presence of heterogeneity and the correlation between local damage and income change the Social Cost of Carbon from the Social Planner’s perspective. In our quantitative experiment, plotted in Figure 7 with uniform Pareto weights, we see that poor countries tend to have higher weights than the richer and colder older, increasing their weights in the sum of the world Social Cost of Carbon. This implies a lower SCC from 75 to \$55/ tCO_2 in this Second-Best allocation.³⁰

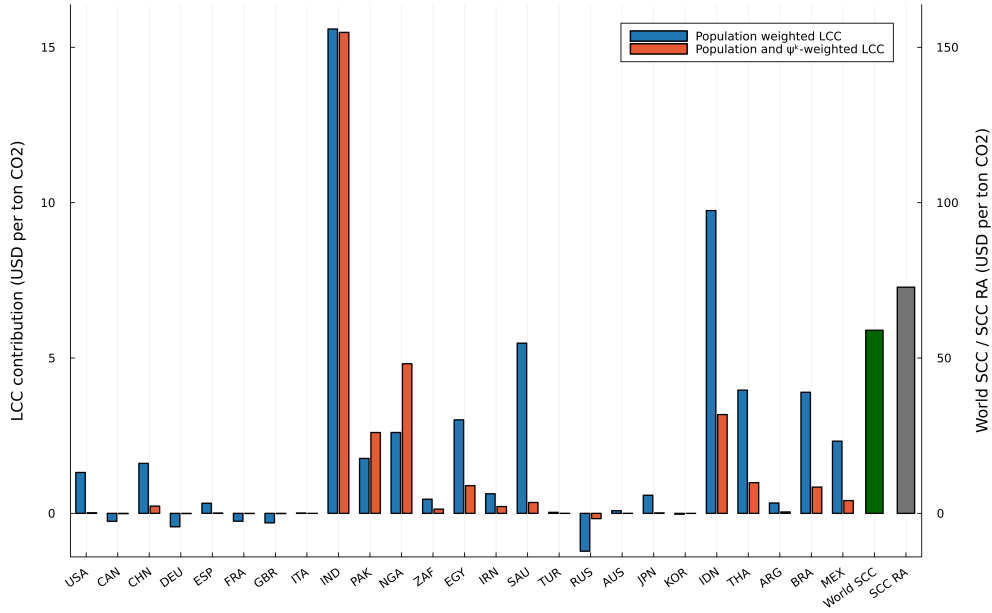


Figure 7: Second Best – Social Cost of Carbon and weighting Local Costs of Carbon

Note: LCC_{it_0} & weights: left bars (blues) $\mathcal{P}_i \omega_i LCC_{it}$, and right bars (orange) $\hat{\psi}_{it}^k LCC_{it} \propto \mathcal{P}_i \omega_i u_c(c_{it}, \tau_{it}) LCC_{it}$. World $SCC_{t_0} = \sum_i \hat{\psi}_{it_0}^k LCC_{it}$. vs. SCC_t “RA” = $\sum_i \mathcal{P}_i \omega_i LCC_{it}$, w/ uniform Pareto weights $\omega_i=1$.

²⁹We define them as $\mathbb{E}^{\mathbb{I}}[x_{it}] = \frac{1}{\mathcal{P}} \sum_{i \in \mathbb{I}} \mathcal{P}_i x_{it}$ and $\text{Cov}^{\mathbb{I}}(x_{it}, y_{it}) = \frac{1}{\mathcal{P}} \sum_{i \in \mathbb{I}} \mathcal{P}_i (x_{it} - \mathbb{E}^{\mathbb{I}}[x_{it}]) (y_{it} - \mathbb{E}^{\mathbb{I}}[y_{it}])$.

³⁰Note that in the Status Quo equilibrium without climate policy, this difference is even larger, from \$300 to \$150/ tCO_2 due to larger climate damages that affected the poorest and warmest countries.

5.2.2 Second-Best, Uniform Carbon Taxation

As shown in model of Section 2, taxation of carbon and fossil fuels have strong redistributive general equilibrium effects through energy markets, which we characterize now.

Fossil energy supply redistribution. First, the implementation of carbon taxation reduces demand for fossil fuels, which has redistributive effects on the energy rent and along the supply curve. Recall that v_{it}^f is the Lagrange multiplier for the optimality condition on fossil fuel demand, and $\hat{v}_{it}^f = v_{it}^f / \bar{\psi}_{it}^k$ its rescaled value by the marginal value of wealth. Moreover, a change in equilibrium quantity of oil-gas relaxes the market clearing, leading to the following mechanism:

$$\text{Supply Redistribution}_t^f = \left(\sum_i (\nu_{ie^x}^f)^{-1} \right)^{-1} \left(\sum_i \hat{\psi}_{it}^w (e_{it}^f - e_{it}^x) - \sum_i \hat{v}_{it}^f \right),$$

where $\left(\sum_i \nu_{ie^x}^f (e_{it}^x, \mathcal{R}_{it})^{-1} \right)^{-1}$ is the aggregate supply elasticity for oil and gas, which depends on the equilibrium extraction e_{it}^x and reserves $\mathcal{R}_{it}$ of each country. Moreover, as we saw above, carbon taxation lowers the energy price q_t^f , leading to a term-of-trade effect. This redistributes energy rents from exporters to importers, and hence scale with $e_{it}^f - e_{it}^x$. This change is weighted by the marginal valuation of wealth $\hat{\psi}_{it}^w$, and it is higher when the fossil-fuel producers are also relatively poorer or weighted more by the planner. This effect rewrites $\sum_i \hat{\psi}_{it}^w [e_{it}^x - e_{it}^f] = \text{Cov}^{\mathbb{I}}(\hat{\psi}_{it}^w (e_{it}^x - e_{it}^f))$, and we can measure this covariance empirically as a sufficient statistics. Moreover, the change in energy price also affects the oil and gas price, affecting the demand, as denoted by the term $\sum_i \hat{v}_{it}^f$. This term comes from the fact that carbon polity is affecting differentially oil-gas and coal. Recall that, if $\hat{v}_{it}^f > 0$, the planner would increase energy prices, moving up along the supply curve, which implies a lower the optimal carbon tax t_t^ε .

Fossil energy demand distortion. Second, carbon taxation is a distortionary tax that create a wedge for the demand of energy. This demand distortion differs across countries due to differences in energy mix, productivity and the endowment and cost of energy sources. From the planner's optimality for energy, we can define the following term:

$$\begin{aligned} \text{Demand Distortion}_{it}^f &:= \hat{v}_{it}^f = [\hat{v}_{it}^f \partial_{ef} MPe_{it}^f + \hat{v}_{it}^c \partial_{ef} MPe_{it}^c + v_{it}^r \partial_{ef} MPe_{it}^r + \hat{v}_{it}^k \partial_{ef} MPk_{it}] \\ &= z_i \mathcal{D}_i(\tau_{it}) \left[\hat{v}_{it}^f F_{efef} + \hat{v}_{it}^c F_{efec} + \hat{v}_{it}^r F_{efer} + \hat{v}_{it}^k F_{efek} \right] \\ &= \frac{1}{e_{it}^f} \left[-\hat{v}_{it}^f (q_{it}^f + \xi^f t_t^\varepsilon) \left(\frac{1-s_{it}^f}{\sigma^e} + s_{it}^f \frac{1-s_{it}^e}{\sigma^y} \right) + \hat{v}_{it}^c (q_{it}^c + \xi^c t_t^\varepsilon) s_{it}^f \left(\frac{1}{\sigma^e} - \frac{1-s_{it}^e}{\sigma^y} \right) \right. \\ &\quad \left. + \hat{v}_{it}^r q_{it}^r s_{it}^f \left(\frac{1}{\sigma^e} - \frac{1-s_{it}^e}{\sigma^y} \right) + \hat{v}_{it}^k (r_t^* + \delta) \frac{s_{it}^f s_{it}^e}{\sigma^y} \right], \end{aligned}$$

with energy share in production, $s_{it}^e = e_{it} q_{it}^e / y_{it}$, fossil share in energy mix $s_{it}^f = e_{it}^f q_{it}^f / e_{it} q_{it}^e$ and similarly s_{it}^c and s_{it}^r . Moreover, σ^e is the elasticity of substitution between energy sources, σ^y the one between energy and the capital/labor bundle, and $\hat{v}_{it}^f := \omega_i \mathcal{P}_i v_{it}^f / \bar{\psi}_{it}^w$, and $\hat{v}_{it}^c$, $\hat{v}_{it}^r$ and $\hat{v}_{it}^k$ are rescaled Lagrange multipliers for fossil energy, coal, renewables, and capital respectively. Note that

we can find a similar expression for coal energy, $Demand\ Distortion_{it}^c$, as function of $F_{e^c e^f}, F_{e^c e^c}$, etc.

To understand the intuition behind this term, take the first term, $-\frac{1-s_{it}^f}{\sigma^e} - s_{it}^f \frac{1-s_{it}^e}{\sigma^y}$, as an example. We see that this demand channel of taxation has two effects: the first lowers fossil consumption due to direct substitution between the three energy inputs, lowering marginal product MP_{it}^f with elasticity σ^e . The second effect is indirect through total energy use e_{it} , proportionally to fossil share s_{it}^f . Weighting these different distortions with the shadow values v_{it}^x for input x and scaling it for the input prices q_t^f, q_{it}^c , etc. we obtain the total distortion caused by taxation of oil and gas. This term is more involved than in the simpler model of Section 2 and theorem 2, because of the general substitution patterns across energy and heterogeneity across countries. Moreover, the aggregate level of the carbon tax balances out all these distortions across countries and energies, and the optimality condition for t_t^ε gives:

$$\sum_{i \in \mathbb{I}} (\xi^f \hat{v}_{it}^f + \xi^c \hat{v}_{it}^c) = 0$$

This minimization of total distortion relates to the standard principles of Ramsey taxation, as in [Diamond and Mirrlees \(1971\)](#); [Diamond \(1973\)](#); [Atkinson and Stiglitz \(1976\)](#).

Optimal carbon tax. We now present our main result for the optimal Second-Best uniform carbon tax. Details on how this formula is derived from the optimality condition for energy choices can be found in Section E. The optimal level of carbon taxation integrates the different redistribution motives that we detailed above. As a result, the Ramsey planner accounts for these general equilibrium effects as symbolized by the curvature of demand and supply of energy.

Proposition 9 (Uniform Carbon Taxation – Second-Best Allocation).

*The optimal Second-Best carbon tax accounts for three distributional motives when setting a single uniform level t_t^ε , in the absence of cross-countries transfers, when revenues of the carbon are rebated locally $t_{it}^{ls} = \epsilon_{it} t_t^\varepsilon$: (i) climate damage in the **Social Cost of Carbon (SCC)**, (ii) **Supply Redistribution** in energy markets through terms-of-trade and energy rents and (iii) **Demand Distortion** through distorted firms' energy choices. This includes redistribution motives due to the presence of inequality through the **social welfare weights** $\hat{\psi}_{it}^w = \omega_i \mathcal{P}_i \psi_{it}^w / \bar{\psi}_{it}^w \propto \omega_i \mathcal{P}_i u_c(c_{it}, \tau_{it})$. The optimal carbon tax writes:*

$$\xi^f t_t^\varepsilon = \xi^f SCC_t^{sb} + Supply\ Redistribution_t^f + \sum_{i \in \mathbb{I}} Demand\ Distortion_{it}^f$$

$$SCC_t = \mathcal{P} \mathbb{E}^\mathbb{I}[\omega_i LCC_{it}] + \mathcal{P} \text{Cov}^\mathbb{I}(\hat{\psi}_{it}^w, LCC_{it}) \tag{36}$$

$$Supply\ Redistribution_t^f = \left(\sum_i (\nu_{ie^x e^x}^f)^{-1} \right)^{-1} \left[\mathcal{P} \text{Cov}^\mathbb{I}(\hat{\psi}_{it}^w, e_{it}^x - e_{it}^f) - \mathcal{P} \mathbb{E}^\mathbb{I}[\hat{v}_{it}^f] \right]$$

$$Demand\ Distortion_{it}^f = \sum_{x \in \{e^f, e^c, e^r, k\}} \hat{v}_{it}^x \partial_{e^f} MP_{it}^x$$

where $\hat{v}_{it}^x$ are the rescaled multipliers for the optimality condition $MPx_{it} = q_{it}^x$ for the choice of input x , i.e. $\hat{v}_{it}^f := \omega_i \mathcal{P}_i v_{it}^x / \bar{\psi}_{it}^w$, for x being fossil (oil-gas) e^f , coal e^c , renewable (low-carbon) e^r and capital k . Moreover, the supply redistribution depends on the curvature of the oil-gas supply with $\nu_{ie^x}^f$ the supply elasticity, and the demand distortion depends on $\partial_{ef} MPx_{it}$ the curvature of the production function, i.e. the own-elasticity in oil-gas F_{efef} and cross-elasticity, e.g. F_{efer} .

In addition, even without climate externality $SCC_t = 0$, the carbon tax t_t^ε could be positive, accounting for energy terms-of-trade manipulations, for example with wealthy exporters and relatively poorer importers, or when richer countries are consuming more fossil-fuels with higher elasticity, following the intuitions of Section 2.

Such a result holds as long as different agents – countries, firms, or households – have different marginal utilities of consumption, i.e. different $\hat{\psi}_{it}^w$. However, these motives would be absent in models like Golosov et al. (2014) for two reasons: First if the supply curve for energy is perfectly elastic, because of constant return to scale, which yields $\nu_{e^x e^x} = 0$. Second, when the agent/firm is “representative” or multiple agents can be aggregated – at least in the inputs demand decisions – and a single energy tax instrument in the First-Best, the social planner is not “distorting” the energy demand across agents: the planner and the agents would achieve the same optimality condition for fossil fuel demand.

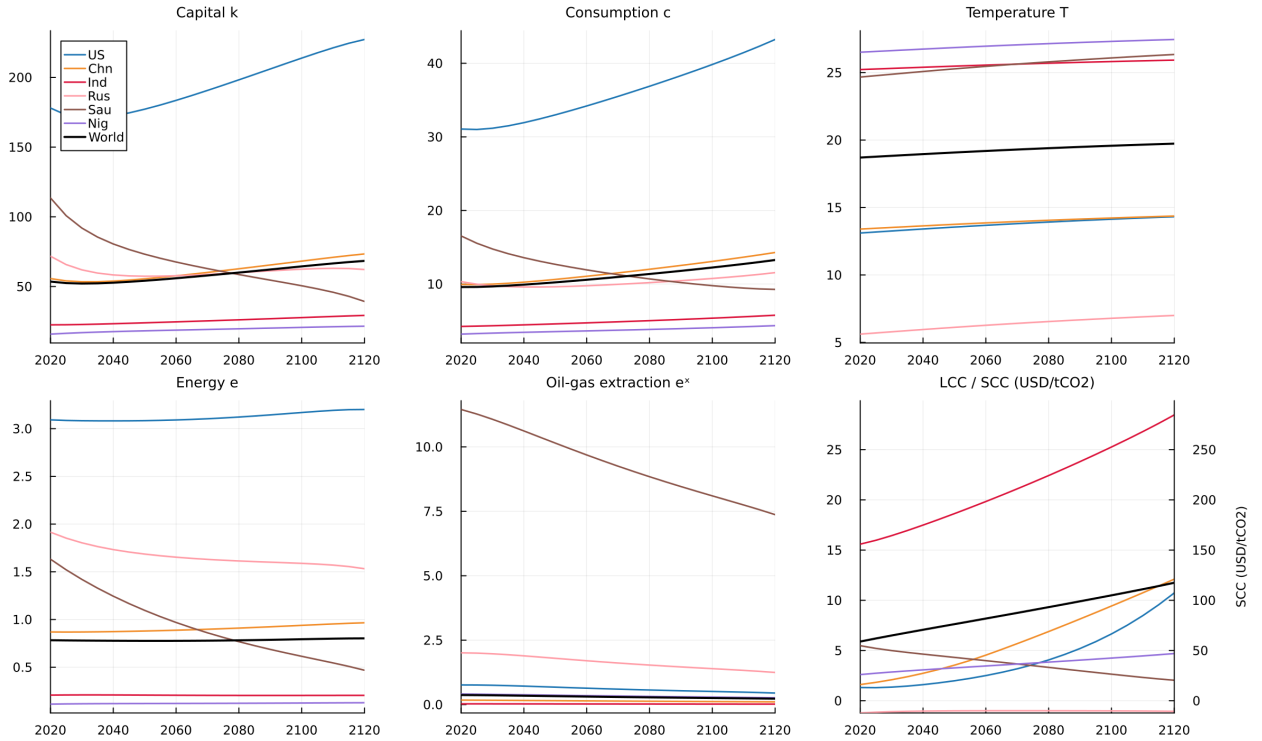


Figure 8: Second-Best Climate Policy without transfers

Note: Second-Best Carbon tax: $t_t^\varepsilon = SCC_t + \text{Supply Redistribution}_t^\circ + \text{Demand Distort}_t^\circ$ as in theorem 9

5.3 Dynamics and carbon taxation with inequality

In Figure 8 we plot the path of the equilibrium where the carbon policy accounts for the redistributive and distortive effects through energy markets. In that second-best allocation, the planner does not close the gap between the different countries consumption. However, thanks to high carbon taxation around $\$250/tCO_2$ – which we will see is higher than the Social Cost of Carbon – the planner is effective as reducing emissions, as a cost of reducing energy use twofold, from 2 to 0.8MTOE per capita yearly. As a result, the equilibrium price of oil-gas q_t^f falls by a factor 2 compared to the First-Best, and by a factor 4 compared to the Competitive Equilibrium, along with supply and extraction e_{it}^x . Thanks to this ambitious policy, temperature stays around $2^\circ C$ in 2100, which lower the cost of Climate Change to $SCC_t \approx \$50\text{--}100/tCO_2$.

In Figure 9, we compare the Social Costs of Carbon and Carbon taxes in our different policy experiments, averaging all the transition paths over time with effective interest rate $\bar{r} = \rho + \eta g$.³¹ We see that the SCC declines from $\approx \$180/tCO_2$ in the equilibrium without policy to $\$140/tCO_2$ in the First Best with cross country transfers.

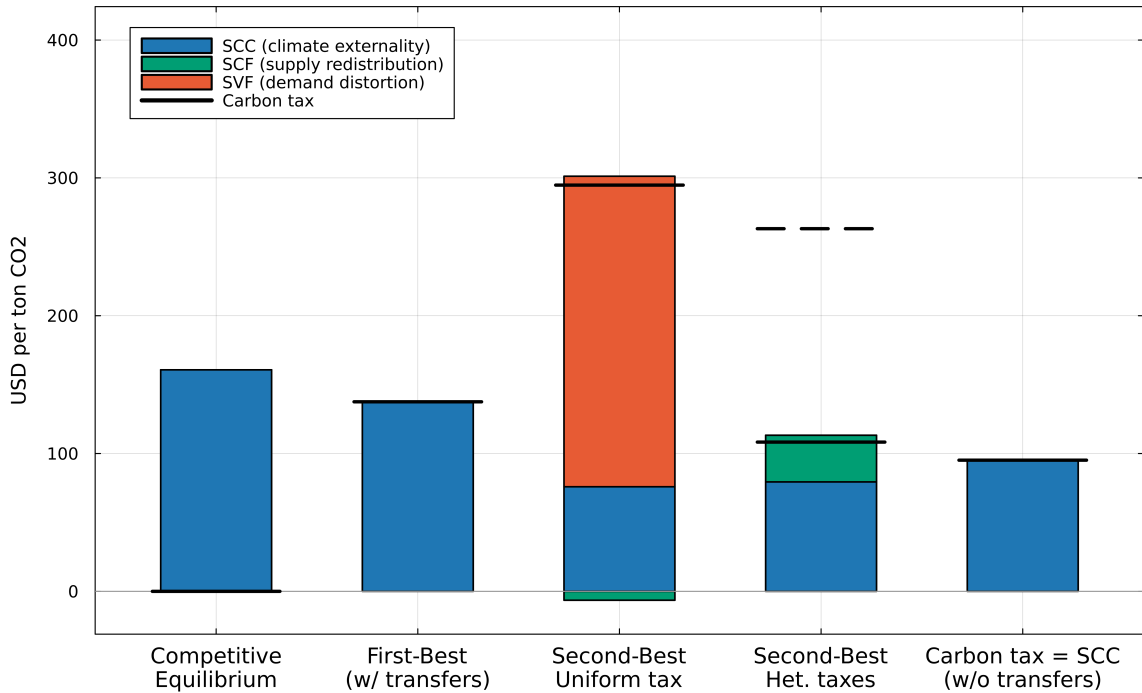


Figure 9: Climate policy t_t^ε in different allocations

Note: Carbon tax t_t^ε and SCC_t , and decomposition for oil-gas, $t_t^\varepsilon = SCC_t + \text{Supply Redistribution}_t^\circ + \text{Demand Distort}_t^\circ$. Present discounted value over 2020–2120, with $r = \rho + \eta g$.

However, in the Second-Best allocation, the world uniform carbon tax is set at around $\$300/tCO_2$, from $\$250$ to close to $\$400$ at the horizon 2100. This level is much higher and can be

³¹Social Cost of Carbon is a already a Present Discounted Value of future damage. Here we are interested in the future path of carbon taxes, which typically increase over time.

explain for the largest by a large aggregate distortion caused by the carbon tax on the demand side of the fossil energy markets. The planner set the tax over three times the level of the SCC to distort the equilibrium of energy q_t^f : the only global lever it can use to redistribute from rich to poor. It does so by making the effective price $q_t^f + \xi^f t_t^\varepsilon$ higher for the whole world, which distort the energy input choice of final good firms. That has no direct redistributive effect since the carbon tax revenues $t_t^\varepsilon e_{it}^f$ are rebated lump-sum as transfers to the households. However, it can have large *indirect* redistributive terms of trade effect on the fossil fuel market. This reduces heavily global demand for oil and gas, curbing down q_t^f which lower the cost for emerging and developing economies. Additionally, this distortive effect is causing a faster transition away from fossil-fuels, which provide larger welfare climate gains to the warmer countries.

5.4 Country-specific carbon taxation

We now consider an experiment where the social planner design country-specific taxes that would allow to correct some of these redistributive concerns. In that case, not only the *level* but also the *distribution* of the fossil fuel/carbon tax is affected by redistribution motives. The planner maximizes global welfare as in equation (33), choosing a countries-specific carbon taxes $\{t_{it}^\varepsilon\}_t$ over time and rebates the carbon tax revenue to the household of the country that pays it $t^{ls} = t_t^\varepsilon \varepsilon_{it}$. We solve the planner's problem choosing the agent decisions, prices, and states, subject to the constraints of the Competitive equilibrium.

Proposition 10 (Country-specific Carbon Taxation – Second-Best Allocation).

The optimal Second-Best country-specific carbon taxation, when transfers are absent and revenues rebated locally, can be written as:

$$\begin{aligned} t_{it}^\varepsilon &= \frac{1}{\hat{\psi}_{it}^w} [SCC_t + \text{Supply Redistribution}_t^f + \text{Demand Distortion}_{it}^f] \\ SCC_t &= \mathcal{P} \mathbb{E}^\Pi[\omega_i LCC_{it}] + \mathcal{P} \text{Cov}^\Pi(\hat{\psi}_{it}^w, LCC_{it}) \\ \text{Supply Redistribution}_t^f &= \left(\sum_i (\nu_{i e^x e^x}^f)^{-1} \right)^{-1} [\mathcal{P} \text{Cov}^\Pi(\hat{\psi}_{it}^w, e_{it}^x - e_{it}^f) - \mathcal{P} \mathbb{E}^\Pi[\hat{v}_{it}^f]] \\ \text{Demand Distortion}_{it}^f &= \sum_{x \in \{e^f, e^c, e^r, k\}} \hat{v}_{it}^x \partial_{e^f} MPx_{it} \end{aligned}$$

*The demand distortion is only local since the planner can choose country-specific taxes. However, this distortion is not null, since the planner does not have **energy-sources**-specific taxes. In particular, the carbon tax affects both oil-gas and coal,*

$$\xi^f \hat{v}_{it}^f + \xi^c \hat{v}_{it}^c = 0$$

These local distortive effects remain if energy demands e_{it}^f, e_{it}^c , energy shares, and elasticities differ.

Following the logic in Section 2, the optimal tax still depend on both the Social Cost of Carbon SCC_t and $\text{Supply Redistribution}_t^f$ as developed above. These two energy taxation motives

– climate externality correction and energy rents redistribution – remains the same. However, we observe that the demand distortion is reduced considerably, since the planner has access to I instruments. At the difference of Section 2, the local distortions of input choices – represented by the Lagrange multipliers $\hat{v}_{it}^f, \hat{v}_{it}^c, \hat{v}_{it}^r, \hat{v}_{it}^k$ – are no longer null in equilibrium as in the theorem 2. Despite the planner choosing a country-specific tax level, distortion and a reallocation between oil-gas, coal, and other inputs remains under the policy. The optimality for t_i^ε gives $\xi^f \hat{v}_{it}^f + \xi^c \hat{v}_{it}^c = 0$, which does not imply that $\hat{v}_{it}^c = 0$ and nor that $Demand\ Distortion_{it}^f = 0$.

Nevertheless, the tax is country i specific and depends on redistribution motives. Indeed the ratio $1/\hat{\psi}_{it}^w$ is the inverse of the social welfare weight – the inequality index developed earlier in this section in equation (34). It implies that richer/colder countries, which have higher consumption and lower marginal utilities are charged higher carbon taxes, and conversely poorer countries should be charged a lower tax:

$$\text{low } c_{it} \text{ high } \tau_{it} \quad \Rightarrow \quad \text{high } \hat{\psi}_{it}^w \propto u_c(c_{it}, \tau_{it}) \quad \Rightarrow \quad \text{low } t_{it}^f$$

everything else being constant, in particular SCC_t , $Supply\ Redistribution_t^f$ and $Demand\ Distortion_{it}^f$.

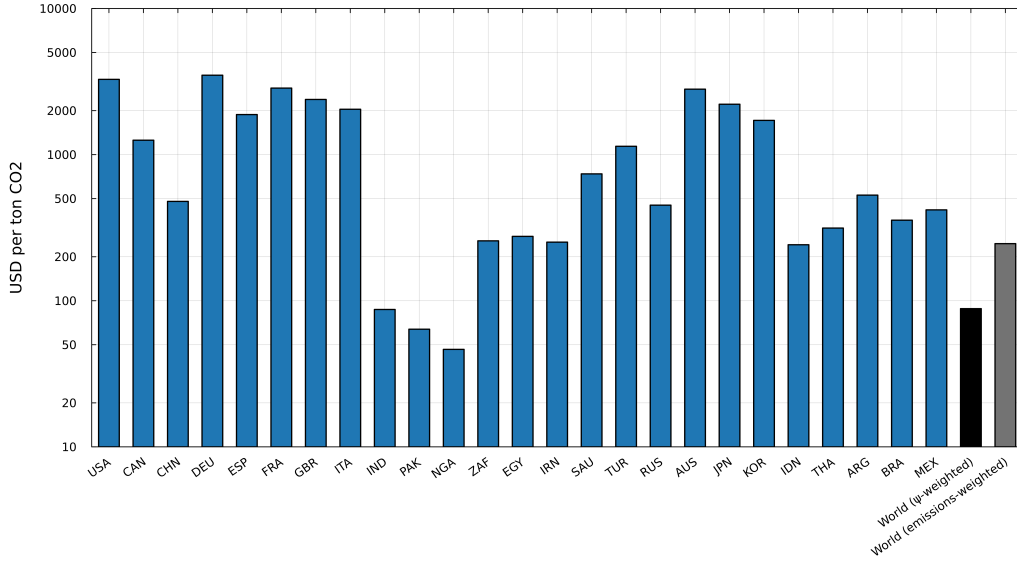


Figure 10: Second-Best: Country-Specific carbon taxes

Note: Carbon taxes: $t_{it}^\varepsilon \approx \frac{1}{\hat{\psi}_{it}^k} [SCC_t + Supply\ Redistribution_t^o]$, for $t = t_0 = 2020$. Y-axis is log scale.

This result provides a justification for a “tiered” carbon taxation – where carbon tax would be lower for developing economies and higher for advanced economies. This proposal, that tend to be promoted for moral reasons, results here from the efficiency–redistribution tradeoff, and welfare maximization. In our quantitative exercise, in Figure 10, we plot the t_{it}^ε per country. Due to high curvature of the utility, we obtain high carbon taxes for rich countries, up to thousands dollar / tCO_2 , and low ones for developing economies, below . This is again a consequence of using utilitarian weights $\omega_i = 1$ and the lack of redistribution instruments which imply that the carbon policies instruments t_{it}^ε inherit the goal to reduce inequality.

5.5 Comparison between the different policies

In Figure 11, we compare the dynamics in our different policy experiments. In the laissez-faire economy, world temperature rises up to 4° by 2100 – due to increasing emissions $\mathcal{E}_t$. Carbon policy and transfers in the First-Best allow to reduce emissions substantially while redistributing and increase every country's consumption over time. Carbon tax averages $\$160/tCO_2$ and temperature stay below $2.5^\circ C$ before 2100. However, in the second-best allocations, such redistribution is limited, with a single uniform carbon tax, or even with a menu of them $\{t_i^\varepsilon\}_i$. The carbon tax reaches $\$300$ as the planner manipulate terms-of-trade on the only international trade flow – oil-gas – and this overshoot of the carbon tax reduces equilibrium price q_t^f . Similarly, with heterogeneous taxes, t_{it}^ε , the optimal policy reins in emissions of rich countries, lowering fossil fuels demand and prices down to 80%, which is beneficial for emerging countries. This also reduces temperatures to $2^\circ C$ which helps the warmest countries. Alternatively, as in Figure 9 we can also consider a sub-optimal policy as a naive benchmark where the planner would set a common tax to $t_t^\varepsilon = SCC$, in that case, emissions do not fall as much, warming world temperature above $2.7^\circ C$ at the beginning of the 21st century.

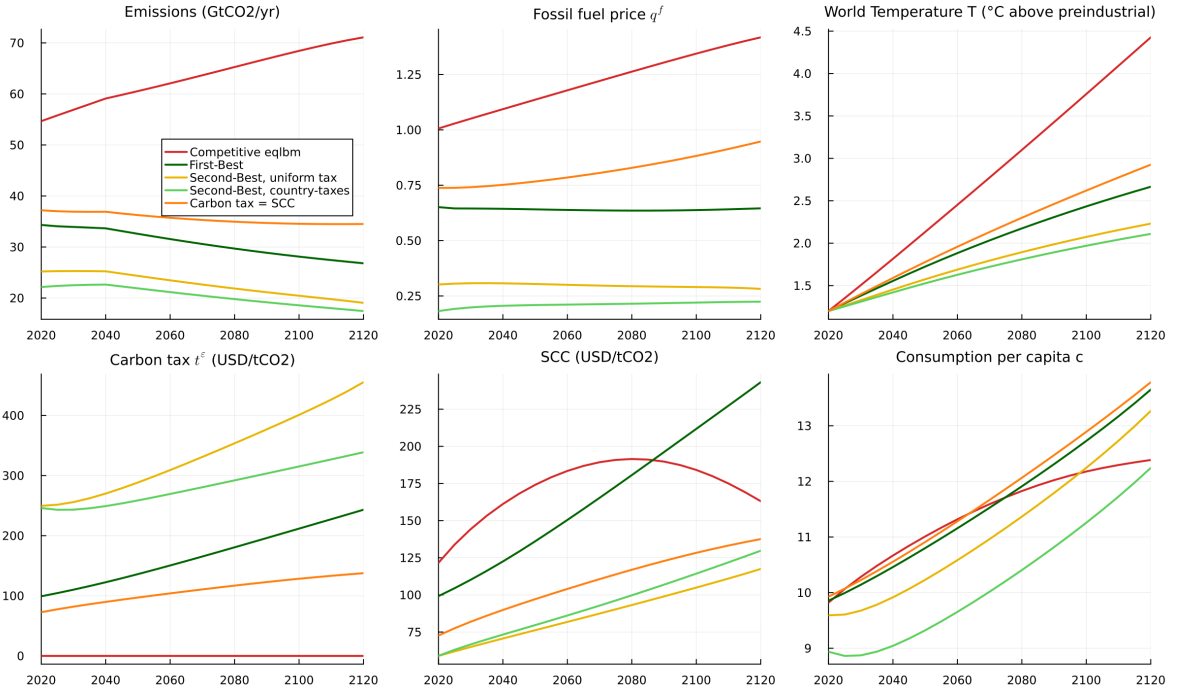


Figure 11: Dynamics and Climate policy t_t^ε in different allocations

5.6 The importance of Pareto weights

In the different experiments shown earlier, we considered the case of optimal policy with uniform weights per capita $\omega_i = 1$. However, the main results might change if we reorder the weights. As mentioned in Section 4.2 and shown in Figure 19 we consider a second polar case with Negishi weights, such that $\omega_i = 1/u_c(\bar{c}_{it_0}, \bar{\tau}_{it_0})$, which makes the initial equilibrium without climate change optimum under that metric.

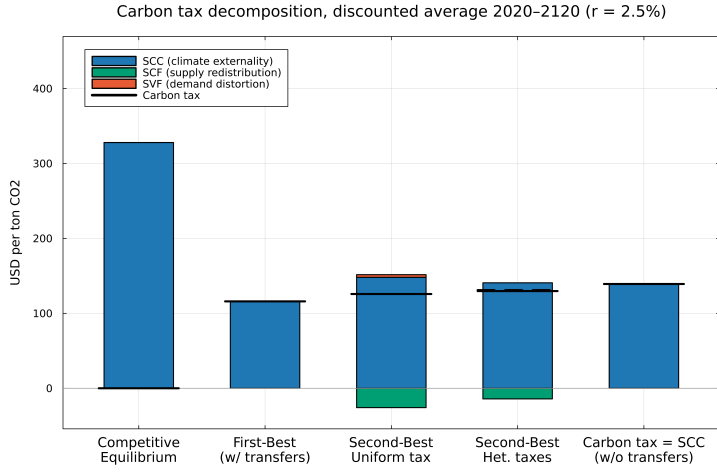


Figure 12: Climate policies – Comparison

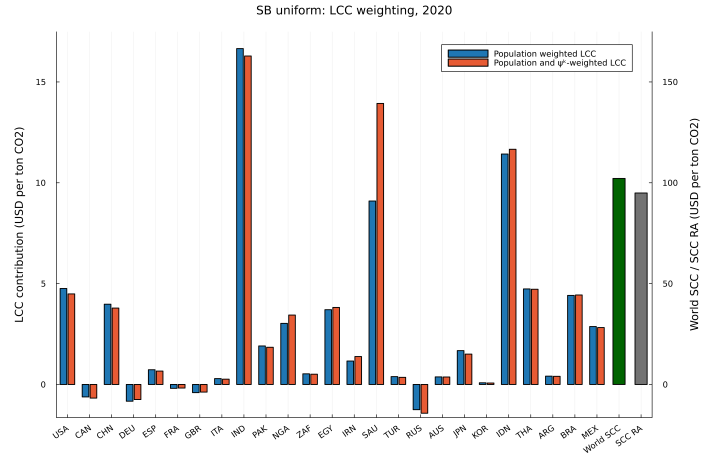


Figure 13: Local & Global Social Cost of Carbon

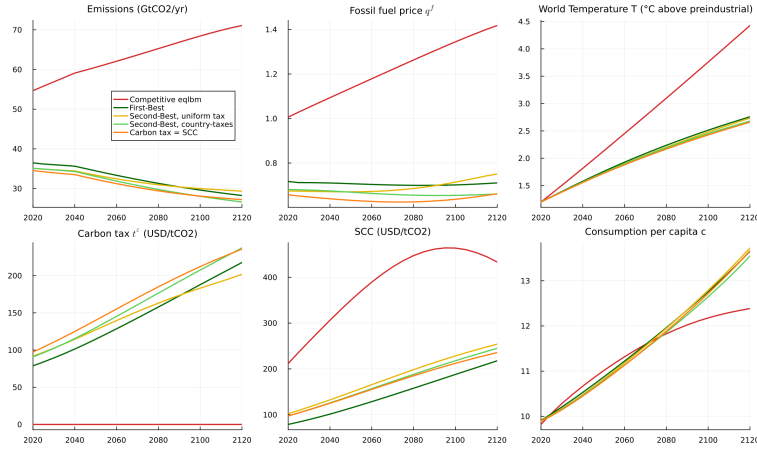


Figure 14: Aggregate Dynamics and Climate policies

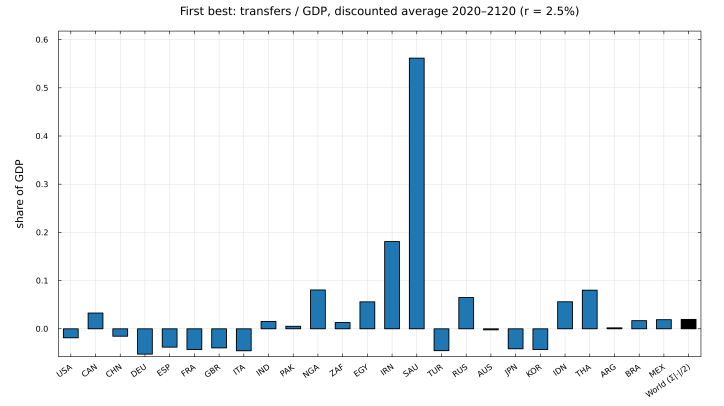


Figure 15: First Best - Transfers

In Figure 12, we plot the different taxes and Social Costs of Carbon in different experiments: in the status-quo policy, the SCC averages $\$320/tCO_2$ over time, compared $\$150$ for the uniform weights. We can see in Figure 13 that the reweighting with $\omega_i \psi_t^k \approx \omega_i u_c(c_t, \tau_t)$ is limited in the SCC, since $\omega_i \propto 1/u_c(c_{t_0}, \tau_{t_0})$ already eliminate most of the countries differences.

In the First-Best, the planner would like to compensate the losers – here the fossil-fuel producers using transfers, as shown in Figure 15. In a Second-Best world, transfers are unavailable and one needs to lower the optimal carbon tax from $\$150$ (the SCC) to $\$120/tCO_2$ to accomodate oil-gas exporters – as represented by the negative Supply Redistribution. However, Figure 12 shows that the optimal carbon tax barely changes across experiments, and the energy demand distortion is limited. As a result, in Figure 15, all our policy experiments manage to reduce emissions twofold – which compare to a decline by a factor 3 in our second-best policy with uniform weights – and limit global temperatures to 2.5° by 2100.

6 Conclusion

In this paper, I examine how to design the optimal carbon policy in a world marked by multiple layers of inequality. Through both theoretical and quantitative analysis, I demonstrate that the traditional approach of setting a global carbon taxation purely equal to the Social Cost of Carbon needs to be reconsidered when accounting for global inequalities in emissions, income, climate vulnerability, and energy policy impacts.

The key theoretical insights emerge from both a simplified model and a richer dynamic framework. In the First-Best scenario with available redistributive instruments, the optimal policy follows the Pigouvian principle, where the carbon tax equals the Social Cost of Carbon. However, in the more realistic Second-Best scenario without cross-country transfers, the optimal policy must balance climate externalities with redistributive considerations. This leads to two main findings: First, the uniform global carbon tax needs to be adjusted for both supply redistribution and demand distortion effects. Second, when country-specific carbon taxes are possible, they should be inversely proportional to social welfare weights – the product of the planners’ Pareto weights and the marginal utility of consumption – resulting in lower taxes for developing economies, or the ones directly affected by the energy policy.

The quantitative analysis reveals that accounting for inequality reduces the Social Cost of Carbon between 10% and 50% depending on the climate policy considered, reflecting higher marginal value of wealth for poorer countries. This framework also captures competing effects on energy markets for the choice of the optimal carbon tax: downward pressure to protect fossil-fuel exporters versus upward pressure to create energy distortions – and manipulation of the equilibrium price of fossil fuels – that favor the poorest countries. With uniform Pareto weights, the second redistributive aspect is more important and the optimal carbon tax is doubled – slightly above \$300/ tCO_2 .

These findings have important implications for international climate policy. They suggest that a one-size-fits-all approach to carbon taxation could be suboptimal when considering global inequalities. Instead, policymakers should consider differentiated carbon pricing schemes that account for countries’ development levels, climate damages, and energy market exposure. This research also highlights the importance of developing complementary redistributive mechanisms in international climate policy to achieve both environmental and equity objectives.

Future research would extend this analysis by integrating climate risk and tipping point uncertainty for optimal climate policies as in [Bourany \(2023\)](#). Additionally, investigating the political economy constraints and the free-riding challenges for the implementation of carbon policies matters considerably for practical policy design, as analyzed in [Bourany \(2025\)](#).

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A Quantification

A.1 Calibration

Table 2: Baseline calibration

<i>Technology & Energy markets</i>			
α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ^y	1	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ^e	2.0	Elasticity fossil-coal-non-carbon	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	0.01*	Long run TFP growth	Conservative estimate for growth
<i>Preferences & Time horizon</i>			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	IES / Risk aversion	Standard calibration
n	0.0035	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
ω_i	$1/u'(c_i)$	Pareto weights	Negishi / Status-quo Social Planner
T	400	Time horizon	Time for climate system to stabilize
<i>Climate parameters</i>			
ξ^f	2.761	Emission factor – Oil & natural gas	Conversion 1 $MTOE \Rightarrow 1 MT CO_2$
ξ^c	3.961	Emission factor – Coal	Conversion 1 $MTOE \Rightarrow 1 MT CO_2$
χ	$2.3/1e6$	Climate sensitivity	Pulse experiment: 100 $GtC \equiv 0.23^\circ C$ medium-term warming
δ_s	0.0004	Carbon exit from atmosphere	Pulse experiment: 100 $GtC \equiv 0.15^\circ C$ long-term warming
ζ	0.027	Growth rate, Carbon Capture and Storage	Starting after 2100, Follows Krusell Smith (2022)
$\gamma^\oplus$	0.003406	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.3 \times \gamma^\oplus$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
T^*	14.5	Optimal yearly temperature	Average spring temperature / Developed economies

A.2 Matching world heterogeneity

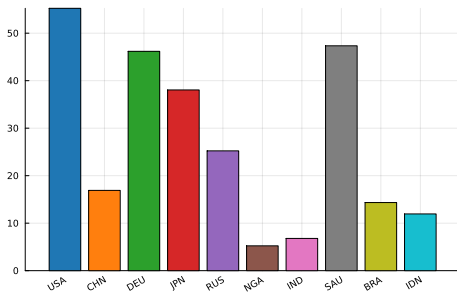


Figure 16: GDP per capita
Thsds 2011-USD PPP, avg. 2018-2023

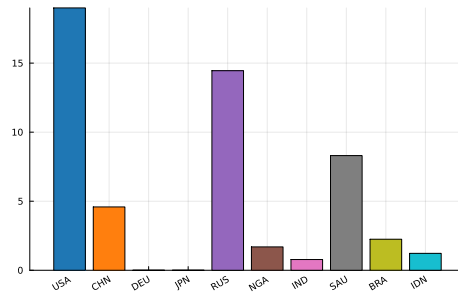


Figure 17: Oil and gas production
GTOE (gigatons oil equiv.), avg. 2018-2023

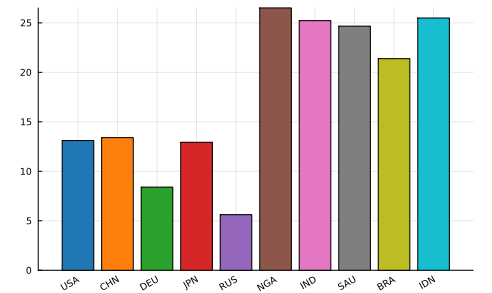


Figure 18: Temperatures
Avg., population-weighted, 2015

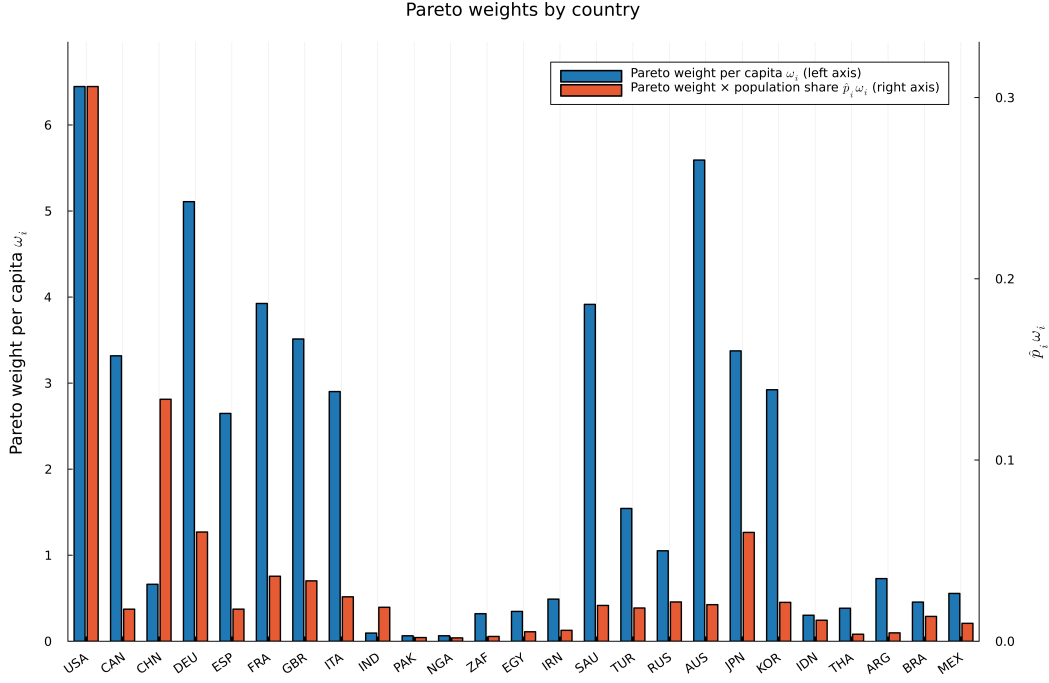


Figure 19: Negishi weights – ω_i and $\mathcal{P}_i\omega_i$

A.3 Pareto weights

In this graph, we plot the Negishi weights in our quantification.

B Toy model – Theoretical results

For a similar model – with multi-country trade – the proofs of the main theorems are analogous to the ones provided in the appendix of [Bourany \(2025\)](#).

B.1 Cap and Trade

Competitive equilibrium

$$\max_{c_i, e_i, \varepsilon_i} U(c_i) \quad s.t \quad \begin{cases} c_i = \mathcal{D}_i(\mathcal{S})z_i F(e_i) + q^e(e_i^x - e_i) - c_i(e_i^x) + q^e(\bar{\varepsilon}_i - \varepsilon_i) & [\lambda_i] \\ \xi e_i \leq \varepsilon_i & [\eta_i q^e] \end{cases}$$

where η_i is the Lagrange multiplier/shadow for each dollar of carbon permits – hence the normalization by q^e . The carbon intensity of energy is ξ : it requires ξ allowance to get enough carbon permit ε_i for one unit (e.g. ton of oil equivalent) of energy e_i . This yields the optimality condition of the firm for energy e_i and carbon permits ε_i :

$$[e_i] \quad MPe_i = q^e + \xi\eta_i q^e \quad [\varepsilon_i] \quad \eta_i q^e = q^e$$

which implies that the implicit carbon tax is $MPe_i - q^e = \xi \tilde{t}^e = \xi q^e$. Multiplier η_i for inequality constraint $e_i \geq \varepsilon_i$. Optimality of ε_i : $\eta_i = 1$.

The Lagrangian for the Second Best allocation in this context:

$$\begin{aligned} \mathcal{L}(c_i, e_i, \varepsilon_i, \bar{\varepsilon}_i, \bar{\mathcal{E}}, \boldsymbol{\lambda}) = & \sum_i \mathcal{P}_i \omega_i U(c_i) + \sum_i \omega_i \mathcal{P}_i \phi_i \left(\mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e (e_i^x - e_i) - c_i (\mathcal{P}_i e_i^x) / \mathcal{P}_i + q^e (\bar{\varepsilon}_i - \varepsilon_i) + t_i^{ls} - c_i \right) \\ & + \mu^e q^e \left(\sum_j \mathcal{P}_j (e_j^x - e_j) \right) + \mu^e q^e \left(\bar{\mathcal{E}} + \sum_j \mathcal{P}_j (\bar{\varepsilon}_j - \varepsilon_j) \right) + \mu^g \left(\sum_i q^e \mathcal{P}_i (\varepsilon_i - \bar{\varepsilon}_i) - \mathcal{P}_i t_i^{ls} \right) \\ & + \sum_i \mathcal{P}_i \omega_i \theta_i (q^e - c'_i(\mathcal{P}_i e_i^x)) + \sum_i \mathcal{P}_i \omega_i v_i (MPe_i - q^e - \xi q^e) \end{aligned}$$

Two options for rebate:

- (i) Global rebate (government budget): $\sum_i t_i^{ls} = \sum_i q^e (\varepsilon_i - \bar{\varepsilon}_i)$ (multiplier μ^g), or:
- (ii) Local lump-sum rebate $t_i^{ls} = q^e (\varepsilon_i - \bar{\varepsilon}_i)$ (in that case $\mu^g = 0$, we replace t_i^{ls} in budget constraint)

Planner's optimality conditions

- $[c_i]$

$$\phi_i = U'(c_i)$$

- $[e_i]$

$$\begin{aligned} \mathcal{P}_i \omega_i \phi_i [MPe_i - q^e] + \xi \sum_j \mathcal{P}_j \omega_j \phi_j \mathcal{D}'_i(\mathcal{S}) z_j F(e_j) - \mu^e + \mathcal{P}_i \omega_i v_i \mathcal{D}_i(\mathcal{S}) z_i F''(e_i) &= 0 \\ \mathcal{P}_i \omega_i \phi_i \xi q^e &= \xi \bar{\phi} SCC + \mu^e - \mathcal{P}_i \omega_i v_i \mathcal{D}_i(\mathcal{S}) z_i F''(e_i) \end{aligned}$$

- $[e_i^x]$

$$\begin{aligned} \underbrace{\mathcal{P}_i \omega_i \phi_i [q^e - c'_i(\mathcal{P}_i e_i^x)]}_{=0} + \mu^e \mathcal{P}_i - \mathcal{P}_i \omega_i \theta_i \mathcal{P}_i c''_i(\mathcal{P}_i e_i^x) \\ \mathcal{P}_i \omega_i \theta_i = \mu^e / (c''_i(\mathcal{P}_i e_i^x)) \end{aligned}$$

- $[q^e]$

$$\sum_i \mathcal{P}_i \omega_i \phi_i (e_i^x - e_i) + \sum_i \mathcal{P}_i \omega_i \theta_i - \sum_i \mathcal{P}_i \omega_i v_i = 0$$

- $[\bar{\mathcal{E}}]$

$$\mu^e q^e = 0$$

Given that the planner controls directly the supply of carbon allowances, the market clearing is not a binding constraint for the optimal policy.

- $[\varepsilon_i]$ & $[\bar{\varepsilon}_i]$ & $[t_i^{ls}]$ in the case of global rebate:

$$\begin{aligned} [\varepsilon_i] \quad & -\omega_i \mathcal{P}_i \phi_i q^e - q^e \mu^e \mathcal{P}_i + \mu^g q^e \mathcal{P}_i = 0 \\ & \mu^e - \mu^g = -\omega_i \phi_i \\ [\bar{\varepsilon}_i] \quad & \omega_i \mathcal{P}_i \phi_i q^e + q^e \mu^e - \mu^g q^e \mathcal{P}_i = 0 \\ & \mu^e - \mu^g = -\omega_i \phi_i \\ [t_i^{ls}] \quad & \sum_i \mathcal{P}_i \omega_i \phi_i = \mu^g \sum_i \mathcal{P}_i \end{aligned}$$

This implies that the planner implements full redistribution using the distribution of “free” carbon permits $\bar{\varepsilon}_i$ and lump-sum transfers t_i^{ls} . The exact mix $\{t_i^{ls}, \bar{\varepsilon}_i\}$ is undetermined as long as the following condition holds:

$$\omega_i \phi_i = \omega_i U'(c_i) = \mu^g = \omega_j U'(c_j)$$

- $[\varepsilon_i]$ & $[\bar{\varepsilon}_i]$ Local rebate: no impact in the budget constraint

$$[\varepsilon_i] - q^\varepsilon \mu^\varepsilon \mathcal{P}_i = 0$$

$$[\bar{\varepsilon}_i] q^\varepsilon \mu^\varepsilon \mathcal{P}_i = 0$$

Given that the purchase of carbon permits, and hence the revenue of “free” carbon permits, are redistributed/taxed lump-sum, we obtain that $\bar{\varepsilon}_i$ is a redundant policy instrument. Moreover, the planner chooses the same policy as the agent, such that $\varepsilon_i = \xi e_i$.

- $[q^\varepsilon]$

$$\sum_i \mathcal{P}_i \omega_i v_i = 0$$

Again, as in the carbon taxation case, this implies no "aggregate distortion" at the global level.

C Quantitative model - Competitive equilibrium

Dynamics of the individual state variables $\underline{s}_{it} = (w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t)$:

$$\dot{w}_{it} = (r_t^* - n_i - \bar{g}_i)w_{it} + \mathcal{D}(\tau_{it})F(k_{it}, e_{it}) - (r_t^* + \delta)k_{it} + \pi_i^f(q_t^f, \mathcal{R}_{it}) - q_{it}^e e_{it} - c_{it}$$

$$\mathcal{E}_t = \bar{\xi}_t \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi_{it}^c)$$

$$\dot{\tau}_{it} = \zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \quad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

$$\dot{\mathcal{R}}_{it} = -e_{it}^f \quad q_t^f = \bar{\nu}_t (e_{it}^x / \mathcal{R}_{it})^{\nu_i}$$

Household problem: Pontryagin Maximum Principle

$$\begin{aligned} \mathcal{H}^{hh}(s, \{c, k, e^f, e^c, e^r\}, \{\lambda\}) = & e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} u(c_i, \tau_i) + e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \lambda_{it}^w \left((r_t^* - (n_i + \bar{g}_i))w_{it} + \mathcal{D}_i^y(\tau_{it})z_i F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) \right. \\ & + \pi_i^f(q_t^f, \mathcal{R}_{it}) - (r_t^* + \delta)k_{it} - q_{it}^f e_{it}^f - q_{it}^c e_{it}^c - q_{it}^r e_{it}^r - c_{it} \Big) \\ & + e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \lambda_{it}^S (\mathcal{E}_t - \delta_s \mathcal{S}_t) \\ & + e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \lambda_{it}^\tau (\zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0}))) \end{aligned}$$

Choice of controls:

$$\begin{aligned}
[c_t] \quad & u_c(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it})u'(\mathcal{D}(\tau_{it})c_{it}) = \lambda_{it}^w \\
[k_t] \quad & MPk_{it} = r_t^* + \delta \\
[x_t] \quad & MPe_{it}^x = \mathcal{D}_i^y(\tau_{it})z_i \partial_x F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = q_{it}^x \quad \text{for } x \in \{f, c, r\} \\
[e_t^x] \quad & q_t^f = \nu_i^f(e_{it}^x, \mathcal{R}_{it})
\end{aligned}$$

The Pontryagin maximum principle for the states $\{w_{it}, \tau_{it}, \mathcal{S}_t\}$

$$\begin{aligned}
[w_t] \quad & \dot{\lambda}_{it}^w = \lambda_{it}^w(\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}_w(\cdot) = \lambda_{it}^w[(\rho - n_i - (1-\eta)\bar{g}_i) - (r_t^* - n_i - \bar{g}_i)] \\
& \Rightarrow \quad \dot{\lambda}_{it}^w = \lambda_{it}^w(\rho + \eta\bar{g}_i - r_t^*) \\
[\tau_{it}] \quad & \dot{\lambda}_{it}^\tau = \lambda_{it}^\tau(\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + \underbrace{\gamma^y(\tau_{it} - \tau_i^*)\mathcal{D}_i^y(\tau_{it})z_i F(k_{it}, e_{it})\lambda_{it}^w}_{-\partial_\tau \mathcal{D}^y} + \underbrace{\gamma^u(\tau_{it} - \tau_i^*)\mathcal{D}_i^u(\tau_{it})u'(\mathcal{D}^u(\tau_{it})c_{it})}_{-\partial_\tau \mathcal{D}^u} c_{it} \\
[\mathcal{S}_t] \quad & \dot{\lambda}_{it}^S = \lambda_{it}^S(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s) - \zeta \chi \Delta_i \lambda_{it}^\tau
\end{aligned}$$

Recall if $\rho + \eta\bar{g}_i < r_t^*$, then λ_{it}^w decreases (and consumption increases) over time.

Solving the ODE for the Local Cost of Carbon

$$\begin{aligned}
& \lambda_{it}^S = - \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s)(s-t)} \zeta \chi \Delta_i \lambda_{is}^\tau ds \\
\text{with} \quad & \lambda_{it}^\tau = \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \zeta)(s-t)} (\tau_{is} - \tau_i^*)(1 + (\alpha^\gamma - 1)\mathbb{1}_{\{\tau_{is} < \tau_i^*\}}) [\gamma^y y_{is} + \gamma^u c_{is}] \lambda_{is}^w ds \\
& \lambda_{it}^S \xrightarrow{\zeta \rightarrow \infty} - \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s)(s-t)} \chi \Delta_i (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \lambda_{is}^w ds,
\end{aligned}$$

with output is $y_{it} = z_i \mathcal{D}_i^y(\tau_{it})F(k_{it}, e_{it})$ and $\lambda_{it}^w = \mathcal{D}_i^u(\tau_{it})u'(\mathcal{D}_i^u(\tau_{it})c_{it})$. Use the Euler equation, or costate dynamics:

$$\dot{\lambda}_{it}^w = \lambda_{it}^w(\rho + \eta\bar{g}_i - r_t^*) \quad \Rightarrow \quad \lambda_{it}^w = \lambda_{is}^w e^{-\int_t^s (\rho + \eta\bar{g}_i - r_u^*) du}$$

for $s > t$, which gives the Local Cost of Carbon:

$$\begin{aligned}
LCC_{it} &= - \frac{\lambda_{it}^S}{\lambda_{it}^w} \rightarrow \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s)(s-t)} \chi \Delta_i (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] e^{+\int_t^s (\rho + \eta\bar{g}_i - r_u^*) du} ds, \\
LCC_{it} &= \int_t^\infty e^{-\delta_s(s-t) - \int_t^s (r_u^* - n_i - \bar{g}_i) du} \chi \Delta_i (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] ds
\end{aligned}$$

This implies that the future damages are discounted faster if $r_{it}^* > n_i + \bar{g}_i$. Conversely, if growth rate of population n_i and TFP $\bar{g}_i$ are high compared to the world interest rate – think of developing economies – then they would put more weights on future damages on output and consumption per capita.

D Quantitative model - First-Best

First-Best allocation results from the global welfare maximization of the planner, who has access to all the instruments:

$$\mathcal{W}_{t_0} = \max_{\{c, k, e^f, e^c e^r, e^x, \bar{e}^c, \bar{e}^r\}} \sum_{i \in \mathbb{I}} \mathcal{P}_i \omega_i \int_{t_0}^{\infty} e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) dt$$

subject to the good and energy resource constraints and the climate system:

$$\sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \left[c_{it} + (\dot{k}_{it} + (n_i + \bar{g}_i + \delta)k_{it}) + \nu_i^f(e_{it}^x, \mathcal{R}_{it}) + \kappa_i^c e_{it}^c + \kappa_{it}^r e_{it}^r \right] = \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) \quad [\phi_t^w]$$

$$E_{it}^f = \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} e_{it}^f = \sum_{i \in \mathbb{I}} e^{(n_i + \bar{g}_i)t} e_{it}^x \quad [\mu_t^f] \quad \bar{e}_i^c = e_i^c \quad [\mu_{it}^c] \quad \bar{e}_i^r = e_i^r \quad [\mu_{it}^r]$$

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \quad \mathcal{E}_t := \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) \quad [\phi_t^S]$$

$$\dot{\tau}_{it} = \zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \quad [\phi_{it}^\tau]$$

Let us define several objects: total population $\mathcal{P}_t = \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{n_i t}$, $\mathcal{P} = \mathcal{P}_{t_0} = \sum_i \mathcal{P}_i$ and global population growth rate:

$$n_t = \frac{1}{\mathcal{P}_t} \sum_{i \in \mathbb{I}} n_i \mathcal{P}_i e^{n_i t}$$

and the welfare-relevant discount rate:

$$\mathcal{P} e^{-\int_{t_0}^t \bar{\rho}_s ds} = \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}$$

$$\bar{\rho}_t = \frac{1}{\mathcal{P}} \sum_{i \in \mathbb{I}} (\rho - n_i - (1-\eta)\bar{g}_i) \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} = \frac{\sum_{i \in \mathbb{I}} (\rho - n_i - (1-\eta)\bar{g}_i) \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}}{\sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}}$$

Note, all the analysis can accomodate time-varying population growth rate n_{it} and time-varying TFP growth $\bar{g}_{it}$.

First-Best Optimal Control Problem – Pontryagin Principle

The Hamiltonian for the Social planner is:

$$\begin{aligned} \mathcal{H}^{fb}(\mathbf{s}, \mathbf{c}, \boldsymbol{\phi}) = & \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) \\ & + e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^w \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \left(\mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) - (n_i + \bar{g}_i + \delta)k_{it} - e^{\nu_i(n_i + \bar{g}_i)t} \nu_i^f(e_{it}^x, \mathcal{R}_{it}) - \kappa_i^c e_{it}^c - \kappa_{it}^r e_{it}^r - c_{it} \right) \\ & + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (e_{it}^x - e_{it}^f) \\ & + e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^S \left\{ \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) - \delta_s \mathcal{S}_t \right\} \\ & + \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \phi_{it}^\tau \zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \end{aligned}$$

Pontryagin maximum principle, optimality conditions, first for controls $\{c, e^f, e^c, e^r, e^x\}$

- Consumption $[c_{it}]$

$$\omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} u_c(\mathcal{D}_i^u(\tau_{it}) c_{it}) = e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^w \mathcal{P}_i e^{(n_i + \bar{g}_i)t}$$

$$\omega_i e^{-(\rho + \eta \bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \mathcal{D}_i^u(\tau_{it}) u'(\mathcal{D}_i^u(\tau_{it}) c_{it}) = \phi_t^w$$

- Energy sources $[e_{it}^f], [e_{it}^c], [e_{it}^r]$

$$\phi_t^w MPE_{it}^f = \mathcal{D}_i(\tau_{it}) z_{it} F_{ef}(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = \mu_t^f - \xi^f \phi_t^S$$

$$MPE_{it}^c = \mathcal{D}_i(\tau_{it}) z_{it} F_{ec}(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = \kappa_i^c - \xi^c \phi_t^S \quad MPE_{it}^r = \kappa_i^r$$

- Energy extraction $[e_{it}^x]$

$$\phi_t^w e^{\nu_i(n_i + \bar{g}_i)t} \nu_i^f(e_{it}^x, \mathcal{R}_{it}) = \mu_t^f$$

$$e^{\nu_i(n_i + \bar{g}_i)t} \bar{\nu}_i \left(\frac{e_{it}^x}{\mathcal{R}_{it}} \right)^{\nu_i} = \frac{\mu_t^f}{\phi_t^w}$$

note that e^x is the extraction rate per effective capita: with population/TFP growth, the marginal cost become larger.

- Note that we simplify the problem by avoiding treating e_{it}^c and $\bar{e}_{it}^c$ and e_{it}^r and $\bar{e}_{it}^r$ as separate variables.

Pontryagin maximum principle, optimality conditions, second for states $\{k_{it}, \mathcal{S}_t, \tau_{it}\}_{it}$

- Capital $[k_i]$

$$\dot{\phi}_t^w = \phi_t^w \bar{\rho}_t - \mathcal{H}_k^{fb}(\mathbf{s}, \mathbf{c}, \boldsymbol{\lambda}) = \phi_t^w \bar{\rho}_t + \phi_t^w (n_i + \bar{g}_i) - \phi_t^w (MPk_{it} - \delta)$$

$$\dot{\phi}_t^w = \phi_t^w (\bar{\rho}_t + n_i + \bar{g}_i - (MPk_{it} - \delta))$$

Note if that there is only one country, we get $\bar{\rho}_t = \rho - n_i - (1-\eta)\bar{g}_i$ and then we obtain the standard Euler equation $\dot{\phi}_t^w = \phi_t^w (\rho + \eta \bar{g}_i - r_{it}^k)$, with $r_{it}^k = MPk_{it} - \delta$

- Carbon concentration in atmosphere $[\mathcal{S}_t]$

$$\dot{\phi}_t^S = \phi_t^S (\bar{\rho}_t + \delta_s) - \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \zeta \Delta_i \chi \phi_{it}^\tau$$

- Temperature $[\tau_{it}]$, normalized by $e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}$ local discounting

$$\dot{\phi}_{it}^\tau = \phi_{it}^\tau (\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}_\tau^{fb}(\mathbf{s}, \mathbf{c}, \boldsymbol{\lambda})$$

$$\dot{\phi}_{it}^\tau = \phi_{it}^\tau (\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + \omega_i u'(\mathcal{D}_i^u(\tau_{it}) c_{it}) \mathcal{D}_i^u(\tau_{it}) (\tau_{it} - \tau_i^*) \gamma^u c_{it} + e^{-\int_{t_0}^t \bar{\rho}_s ds + (n_i + g_i)t + (\rho - n_i - (1-\eta)\bar{g}_i)t} \phi_t^w (\tau_{it} - \tau_i^*) \gamma^y y_{it}$$

Proof of theorem 6

Solving for the shadow value of temperature ϕ_{it}^τ and carbon ϕ_t^S

$$\begin{aligned}\dot{\phi}_{it}^\tau &= \phi_{it}^\tau(\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho + \eta\bar{g}_i)t} \phi_t^w(\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}] \\ \phi_{it_0}^\tau &= \int_{t_0}^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \zeta)(t-t_0)} e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho + \eta\bar{g}_i)t} \phi_t^w(\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}] \\ \phi_{it_0}^\tau &= \int_{t_0}^\infty e^{-\int_{t_0}^t \bar{\rho}_s ds - (\zeta - n_i - \bar{g}_i)(t-t_0)} \phi_t^w(\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}]\end{aligned}$$

Solving for the SCC_i , we get:

$$\begin{aligned}\phi_{t_0}^S &= - \int_{t_0}^\infty e^{-\int_{t_0}^t \bar{\rho}_s ds - \delta_s(t-t_0)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0) + \int_{t_0}^t \bar{\rho}_s ds} \zeta \Delta_i \chi \phi_{it}^\tau dt \\ \phi_{t_0}^S &= - \int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0)} \zeta \Delta_i \chi \phi_{it}^\tau dt \\ \phi_{t_0}^S &\xrightarrow{\zeta \rightarrow \infty} - \int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0)} \chi \Delta_i (\tau_{it} - \tau_i^*) [\gamma^y y_{it} + \gamma^u c_{it}] \phi_t^w dt ,\end{aligned}$$

Realizing that the marginal value of wealth:

$$\dot{\phi}_t^w = \phi_t^w (\bar{\rho}_t + n_i + \bar{g}_i - \underbrace{(MPk_{it} - \delta)}_{=r_{it}^k}) \quad \Rightarrow \quad \phi_{t_0}^w = \phi_t^w e^{-\int_{t_0}^t (\bar{\rho}_s + n_i + \bar{g}_i - r_{is}^k) ds}$$

This implies that the Social Cost of Carbon defined as $SCC_{t_0} = -\frac{\phi_{t_0}^S}{\phi_{t_0}^w}$ can be rewritten as:

$$\begin{aligned}-\phi_{t_0}^S &\xrightarrow{\zeta \rightarrow \infty} \int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0)} \chi \Delta_i (\tau_{it} - \tau_i^*) [\gamma^y y_{it} + \gamma^u c_{it}] \phi_{t_0}^w e^{-\int_{t_0}^t (r_{is}^k - \bar{\rho}_s - n_i - \bar{g}_i) ds} dt , \\ SCC_{t_0} &\rightarrow \int_{t_0}^\infty \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-[(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0) - \int_{t_0}^t \bar{\rho}_s ds]} e^{-\delta_s(t-t_0) - \int_{t_0}^t (r_{is}^k - n_i - \bar{g}_i) ds} \chi \Delta_i (\tau_{it} - \tau_i^*) [\gamma^y y_{it} + \gamma^u c_{it}] dt ,\end{aligned}$$

The aggregate discount factor is defined as $\sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(t-t_0)} = \mathcal{P} e^{-\int_{t_0}^t \bar{\rho}_s ds}$, and given that c_{it} , y_{it} and τ_{it} are bounded, we can simplify the expression. Moreover, changing the order of the sum and integral by Fubini's theorem, we obtain:

$$\begin{aligned}SCC_{t_0} &\rightarrow \int_{t_0}^\infty \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-\delta_s(t-t_0) - \int_{t_0}^t (r_{is}^k - n_i - \bar{g}_i) ds} \chi \Delta_i (\tau_{it} - \tau_i^*) [\gamma^y y_{it} + \gamma^u c_{it}] dt , \\ SCC_{t_0} &\rightarrow \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i \int_{t_0}^\infty e^{-\delta_s(t-t_0) - \int_{t_0}^t (r_{is}^k - n_i - \bar{g}_i) ds} \chi \Delta_i (\tau_{it} - \tau_i^*) [\gamma^y y_{it} + \gamma^u c_{it}] dt , \\ SCC_{t_0} &\rightarrow \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i LCC_{it}\end{aligned}$$

where the Local Cost of Carbon LCC_{it} is given in Section C. Given that, in the competitive equilibrium, we have free capital flows and frictionless borrowing, it implies $r_{it}^k = MPk_{it} - \delta = r_t^*$, which gives the results of theorem 6. $\square$

E Quantitative model - Second-Best

Second-Best allocation results from the global welfare maximization of the planner, subject to choice of a global carbon tax, t^ε , and local lump-sum rebate: $t_{it}^{ls} = t^\varepsilon(\xi^f e_{it}^f + \xi^c e_{it}^c)$.

$$\mathcal{W}_{t_0} = \max_{\{c, k, e^f, e^c, e^r, e^x, \bar{e}^c, \bar{e}^r\}} \sum_{i \in \mathbb{I}} p_i \omega_i \int_{t_0}^{\infty} e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) dt$$

subject to the good and energy resource market clearing and the climate system:

$$\begin{aligned} \dot{w}_{it} &= (r_t^* - (n_i + \bar{g}_i))w_{it} + \pi_i^f(q_t^f, \mathcal{R}_{it}) + \mathcal{D}_i^y(\tau_{it})z_i F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) - (r_t^* + \delta)k_{it} \\ &\quad - (q_t^f + \xi^f t_{it}^\varepsilon) e_{it}^f - (q_{it}^c + \xi^c t_{it}^\varepsilon) e_{it}^c - q_{it}^r e_{it}^r - c_{it} + t_{it}^{ls}, \quad [\psi_{it}^w] \\ E_{it}^f &= \sum_{i \in \mathbb{I}} p_i e^{(n_i + \bar{g}_i)t} e_{it}^f = \sum_{i \in \mathbb{I}} e^{(n_i + \bar{g}_i)t} e_{it}^x \quad [\mu_t^f] \quad B_t = \sum_{i \in \mathbb{I}} p_i e^{(n_i + \bar{g}_i)t} (w_{it} - k_{it}) = 0 \quad [\mu_t^b] \\ \bar{e}_i^c &= e_i^c \quad [\mu_{it}^c] \quad \bar{e}_i^r = e_i^r \quad [\mu_{it}^r] \\ \dot{\mathcal{S}}_t &= \mathcal{E}_t - \delta_s \mathcal{S}_t \quad \mathcal{E}_t := \sum_{i \in \mathbb{I}} p_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) \quad [\psi_t^S] \\ \dot{\tau}_{it} &= \zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \quad [\psi_{it}^\tau] \end{aligned}$$

as well as the optimality conditions of the agents of the Competitive equilibrium

$$\begin{aligned} [k_t] \quad MPk_{it} &= r_t^* + \delta \quad [v_{it}^k] \\ [x_t] \quad MPe_{it}^x &= \mathcal{D}_i^y(\tau_{it})z_i \partial_x F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = q_{it}^x \quad \text{for } x \in \{f, c, r\} \quad [v_{it}^x] \\ [e_t^x] \quad q_t^f &= \nu_{i_{e^x}}^f(e_{it}^x, \mathcal{R}_{it}) \quad [\theta_{it}^x] \end{aligned}$$

Using the Primal approach, we can write the Hamiltonian, where the states are $\{w_{it}, \mathcal{S}_t, \tau_{it}\}_{it}$, and the controls are $\{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x, \bar{e}_{it}^c, \bar{e}_{it}^r\}_{it}$, and prices $\{r_t^*, q_t^f, w_{it}, q_{it}^c, q_{it}^r\}_{it}$ and where each

country i variable is discounted by $\rho - n_i - (1 - \eta)\bar{g}_i$:

$$\begin{aligned}
\mathcal{H}^{sb}(\mathbf{s}, \mathbf{c}, \boldsymbol{\psi}) = & \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} u(\mathcal{D}_i^u(\tau_{it}) c_{it}) \\
& + \sum_{i \in \mathbb{I}} \psi_{it}^w \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \left((r_t^* - (n_i + \bar{g}_i)) w_{it} + \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) \right. \\
& \quad + \pi_i^f(q_t^f, \mathcal{R}_{it}) - (r_t^* + \delta) k_{it} - (q_t^f + \xi^f \mathfrak{t}_{it}^\varepsilon) e_{it}^f - (q_{it}^c + \xi^c \mathfrak{t}_{it}^\varepsilon) e_{it}^c - q_{it}^r e_{it}^r - c_{it} + \mathfrak{t}_{it}^{ls} \Big) \\
& + \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} [\mu_{it}^c (\bar{e}_{it}^c - e_{it}^c) + \mu_{it}^r (\bar{e}_{it}^r - e_{it}^r)] \\
& + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (e_{it}^x - e_{it}^f) + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^b \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (w_{it} - k_{it}) \\
& + e^{-\int_{t_0}^t \bar{\rho}_s ds} \psi_t^S \left\{ \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) - \delta_s \mathcal{S}_t \right\} \\
& + \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \psi_{it}^\tau \zeta(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \\
& + \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \left[v_{it}^f (q_t^f + \xi^f \mathfrak{t}_{it}^\varepsilon - MP e_{it}^f) + v_{it}^c (q_{it}^c + \xi^c \mathfrak{t}_{it}^\varepsilon - MP e_{it}^c) + v_{it}^r (q_{it}^r - MP e_{it}^r) + v_{it}^k (r_t^* + \delta - MP k_{it}) \right] \\
& + \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \left[\theta_{it}^x (\nu_{iex}^f (e_{it}^x, \mathcal{R}_{it}) - q_t^f) + \theta_{it}^c (\kappa_{it}^c - q_{it}^c) + \theta_{it}^r (\kappa_{it}^r - q_{it}^r) \right]
\end{aligned}$$

PMP: Optimality conditions for the controls $\{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x, \bar{e}_{it}^c, \bar{e}_{it}^r\}_{it}$ are:

- Consumption:

$$\omega_i \mathcal{P}_i u(\mathcal{D}_i^u(\tau_{it}) c_{it}) = \psi_{it}^w \omega_i \mathcal{P}_i$$

- Capital choice:

$$\begin{aligned}
& \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \psi_{it}^w [MP k_{it} - \delta - r_t^*] - e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^b \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \\
& - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} [v_{it}^f \partial_k MP e_{it}^f + v_{it}^c \partial_k MP e_{it}^c + v_{it}^r \partial_k MP e_{it}^r + v_{it}^k \partial_k MP k_{it}] = 0 \\
& \mu_t^b = -e^{-(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} \omega_i [v_{it}^f \partial_k MP e_{it}^f + v_{it}^c \partial_k MP e_{it}^c + v_{it}^r \partial_k MP e_{it}^r + v_{it}^k \partial_k MP k_{it}]
\end{aligned}$$

The multiplier μ_t^b represents the shadow value of liquidity of aggregate bonds. If we increased bond supply B_t , it would decrease the interest rate and improve the ability of firms to borrow and invest, decreasing the marginal value of capital. This redistributive effect has an impact on the firm inputs optimality conditions for input x , written with $v_{it}^x \partial_k MP x_{it}$. As a result, μ_t^b is the equilibrium value equalizing these different redistributive/distortive effects.

- Energy extraction – Oil-gas (Fossil) $[e_{it}^x]$

$$\begin{aligned}
& \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \psi_{it}^w [q_t^f - \nu_{iex}^f (e_{it}^x, \mathcal{R}_{it})] + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \\
& + \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t} \theta_{it}^x \mathcal{P}_i \nu_{iex}^f (e_{it}^x, \mathcal{R}_{it}) \\
& \mu_t^f = -e^{-(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} \omega_i \theta_{it}^x \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \nu_{iex}^f (e_{it}^x, \mathcal{R}_{it})
\end{aligned}$$

The multiplier μ_t^f is the shadow value of liquidity of aggregate oil-gas supply. If we increased supply E_t^f , it would decrease the oil-gas price rate q_t^f going down the supply curve, as denoted by the curvature $\nu_{i e^x}^f$ and weighted by the shadow value of optimality of the fossil firm's extraction. Note also that we scaling the curvature of the cost by population $\mathcal{P}_i e^{(n_i + \bar{g}_i)t}$ – population growth push extraction along the supply curve.

- Energy production (Coal and renewable)

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \mu_{it}^c &= 0 \\ \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \mu_{it}^r &= 0\end{aligned}$$

There's no redistribution effects across countries through the market clearing, due to the fact that (i) the coal (and renewable) are traded locally, and (ii) there are no profits from coal and renewable production.

- Energy consumption – Oil-gas (Fossil)

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [MPe_{it}^f - q_t^f] - e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \mathcal{P}_i e^{(n_i + \bar{g}_i)t} + \xi^f e^{-\int_{t_0}^t \bar{\rho}_s ds} \psi_t^S \mathcal{P}_i e^{(n_i + \bar{g}_i)t} + \\ - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [v_{it}^f \partial_{ef} MPe_{it}^f + v_{it}^c \partial_{ef} MPe_{it}^c + v_{it}^r \partial_{ef} MPe_{it}^r + v_{it}^k \partial_{ef} MPk_{it}] = 0 \\ \omega_i \mathcal{P}_i e^{-(\rho + \eta\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \psi_{it}^w \xi_t^f = \mathcal{P}_i \mu_t^f - \mathcal{P}_i \xi_t^f \psi_t^S + \omega_i \mathcal{P}_i e^{-(\rho + \eta\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} [v_{it}^f \partial_{ef} MPe_{it}^f + v_{it}^c \partial_{ef} MPe_{it}^c + v_{it}^r \partial_{ef} MPe_{it}^r + v_{it}^k \partial_{ef} MPk_{it}] = 0\end{aligned}$$

- Energy consumption – Coal

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [MPe_{it}^c - q_{it}^c] - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \mu_{it}^c + \xi^c e^{-\int_{t_0}^t \bar{\rho}_s ds} \psi_t^S \mathcal{P}_i e^{(n_i + \bar{g}_i)t} + \\ - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [v_{it}^f \partial_{ec} MPe_{it}^f + v_{it}^c \partial_{ec} MPe_{it}^c + v_{it}^r \partial_{ec} MPe_{it}^r + v_{it}^k \partial_{ec} MPk_{it}] = 0 \\ \omega_i \mathcal{P}_i e^{-(\rho + \eta\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \psi_{it}^w \xi_t^c = -\mathcal{P}_i \xi_t^c \psi_t^S + \omega_i \mathcal{P}_i e^{-(\rho + \eta\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} [v_{it}^f \partial_{ec} MPe_{it}^f + v_{it}^c \partial_{ec} MPe_{it}^c + v_{it}^r \partial_{ec} MPe_{it}^r + v_{it}^k \partial_{ec} MPk_{it}] = 0\end{aligned}$$

- Energy consumption – Renewable

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [MPe_{it}^c - q_{it}^r] - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \mu_{it}^r + \\ - \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [v_{it}^f \partial_{er} MPe_{it}^f + v_{it}^c \partial_{er} MPe_{it}^c + v_{it}^r \partial_{er} MPe_{it}^r + v_{it}^k \partial_{er} MPk_{it}] = 0 \\ \Rightarrow [v_{it}^f \partial_{ec} MPe_{it}^f + v_{it}^c \partial_{ec} MPe_{it}^c + v_{it}^r \partial_{ec} MPe_{it}^r + v_{it}^k \partial_{ec} MPk_{it}] = 0\end{aligned}$$

PMP: Optimality conditions for the controls over prices $\{r_t^*, q_t^f, w_{it}, q_{it}^c, q_{it}^r\}_{it}$

- Interest rate $[r_t^*]$

$$\sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [w_{it} - k_{it}] + \sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} v_{it}^k = 0$$

The redistributive effect on agents' budget, weighted by ψ_{it}^w should compensate for the distortionary effect on firms' optimality of capital, weighted by shadow value v_{it}^k .

- Fossil energy price: $[q_t^f]$

$$\sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [e_{it}^x - e_{it}^f] + \sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [v_{it}^f - \theta_{it}^x] = 0$$

- Coal energy price: $[q_{it}^c]$

$$\begin{aligned} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [\bar{e}_i^c - e_{it}^c] + \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [v_{it}^c - \theta_{it}^c] &= 0 \\ \Rightarrow v_{it}^c &= \theta_{it}^c \end{aligned}$$

- Renewable energy price: $[q_{it}^r]$, similarly:

$$\Rightarrow v_{it}^r = \theta_{it}^r$$

For coal and renewable, since the price is local, the "distortive/redistributive" effect on the supply equals the one of its demand.

- Wages w_{it} are determined directly by the Marginal Product of Labor $MP\ell_{it}$, since the labor supply is inelastic and normalized to 1.
- Carbon tax/Carbon price $[t^\epsilon]$

$$\sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [\xi^f v_{it}^f + \xi^c v_{it}^c] = 0$$

The optimal carbon tax level balances out all the distortions for each country, for oil-gas and coal, according to shadow values v_{it}^f and v_{it}^c .

PMP: Optimality conditions for the states $\{w_{it}, \mathcal{S}_t, \tau_{it}\}_{it}$.

- Wealth $[w_{it}]$

$$\begin{aligned} \dot{\psi}_{it}^w &= \psi_{it}^w (\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}_w^b(\mathbf{s}, \mathbf{c}, \boldsymbol{\psi}) \\ \psi_{it}^w &= \psi_{it}^w (\rho + \eta\bar{g}_i - r_t^*) + \frac{1}{\omega_i \mathcal{P}_i} e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho + \eta\bar{g}_i)t} \mu_t^b \\ \mu_t^b &= e^{-[(\rho + \eta\bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds]} \omega_i [v_{it}^f \partial_k MP e_{it}^f + v_{it}^c \partial_k MP e_{it}^c + v_{it}^r \partial_k MP e_{it}^r + v_{it}^k \partial_k MP k_{it}] \\ \dot{\psi}_{it}^w &= \psi_{it}^w (\rho + \eta\bar{g}_i - r_t^*) + e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho + \eta\bar{g}_i)t} e^{-[(\rho + \eta\bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds]} \frac{1}{\mathcal{P}_i} [\dots] \\ \dot{\psi}_{it}^w &= \psi_{it}^w (\rho + \eta\bar{g}_i - r_t^*) + \frac{1}{\mathcal{P}_i} [v_{it}^f \partial_k MP e_{it}^f + v_{it}^c \partial_k MP e_{it}^c + v_{it}^r \partial_k MP e_{it}^r + v_{it}^k \partial_k MP k_{it}] \end{aligned}$$

This implies time-varying liquidity motives for the marginal value of wealth. Abstracting from discounting $\rho - n_i - (1-\eta)\bar{g}_i$ and $\bar{\rho}_t$, if μ_t^b is positive (the planner would like to increase the supply of bond, decreasing return), it needs to be compensated for higher marginal value of wealth ψ_{it}^w in the future ($\dot{\psi}_{it}^w$ is higher if $\mu_t^b > 0$) which implies higher consumption, today at time t .

- Temperature $[\tau_{it}]$

$$\dot{\psi}_{it}^\tau = \psi_{it}^\tau(\rho - n_i - (1 - \eta)\bar{g}_i) - \mathcal{H}_\tau^{sb}(\mathbf{s}, \mathbf{c}, \psi)$$

$$\dot{\psi}_{it}^\tau = \psi_{it}^\tau(\rho - n_i - (1 - \eta)\bar{g}_i + \zeta) + \underbrace{\gamma^y(\tau_{it} - \tau_i^*)\mathcal{D}_i^y(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} z_i F(k_{it}, e_{it}) \psi_{it}^w + \underbrace{\gamma^u(\tau_{it} - \tau_i^*)\mathcal{D}_i^u(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau_{it})c_{it})c_{it}$$

$$\dot{\psi}_{it}^\tau = \psi_{it}^\tau(\rho - n_i - (1 - \eta)\bar{g}_i + \zeta) + (\tau_{it} - \tau_i^*)[\gamma^y y_{it} + \gamma^u c_{it}] \psi_{it}^w$$

- Carbon concentration $[\mathcal{S}_t]$

$$\dot{\psi}_t^S = \psi_t^S \bar{\rho}_t - \mathcal{H}_\tau^{sb}(\mathbf{s}, \mathbf{c}, \psi)$$

$$\dot{\psi}_t^S = \psi_t^S(\bar{\rho}_t + \delta_s) - \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \zeta \Delta_i \chi \psi_{it}^\tau$$

Proof of theorem 8

Solving for the differential equations for the marginal value of temperature ψ_{it}^τ and carbon ψ_{it}^S .

$$\psi_{it}^\tau = \int_t^\infty e^{-(\rho - n_i - (1 - \eta)\bar{g}_i + \zeta)(s - t)} (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \psi_{is}^w ds$$

$$\psi_t^S = - \int_t^\infty e^{-\delta_s(s - t) - \int_t^s \bar{\rho}_s ds} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)(s - t) + \int_t^s \bar{\rho}_s ds} \zeta \Delta_i \chi \psi_{is}^\tau ds$$

$$\psi_t^S = - \int_t^\infty e^{-\delta_s(s - t)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)(s - t)} \zeta \Delta_i \chi \psi_{is}^\tau ds$$

$$\psi_t^S \xrightarrow{\zeta \rightarrow \infty} - \int_t^\infty e^{-\delta_s(s - t)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1 - \eta)\bar{g}_i)(s - t)} \Delta_i \chi (\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \psi_{is}^w ds$$

Using the dynamics for the marginal value of wealth;

$$\begin{aligned} \psi_{it}^w &= \int_t^T e^{-(\rho + \eta \bar{g}_i)(s - t) + \int_t^s r_u^* du} \bar{v}_{is}^k ds + e^{-(\rho + \eta \bar{g}_i)(T - t) + \int_t^T r_u^* du} \psi_{iT}^w \\ \bar{v}_{it}^k &= \frac{1}{\mathcal{P}_i} [v_{it}^f \partial_k M P e_{it}^f + v_{it}^c \partial_k M P e_{it}^c + v_{it}^r \partial_k M P e_{it}^r + v_{it}^k \partial_k M P k_{it}] \end{aligned}$$

First, let us assume that $\bar{v}_{it}^k \approx 0$ there are no liquidity effects, giving $\psi_{it}^w = e^{-(\rho + \eta \bar{g}_i)(T - t) + \int_t^T r_u^* du} \psi_{iT}^w$.

I define the social welfare weights:

$$\begin{aligned} \bar{\psi}_t^w &= \frac{1}{\bar{\mathcal{P}}_t} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-[(\rho - n_i - (1 - \eta)\bar{g}_i)t - \int_{t_0}^t \rho_s ds]} \psi_{it}^w \\ \hat{\psi}_{it}^w &= \frac{\omega_i \mathcal{P}_i \psi_{it}^w}{\bar{\psi}_t^w} \end{aligned}$$

This allow to simplify the marginal value of carbon:

$$\begin{aligned}\psi_t^S &\rightarrow - \int_t^\infty e^{-\delta_s(s-t)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(s-t)} \Delta_i \chi(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \psi_{is}^w ds \\ \psi_t^S &\rightarrow - \int_t^\infty e^{-\delta_s(s-t)} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(s-t)} \Delta_i \chi(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] e^{+(\rho + \eta \bar{g}_i)(s-t) - \int_t^s r_u^* du} \psi_{it}^w ds\end{aligned}$$

This implies the Social Cost of Carbon:

$$\begin{aligned}SCC_t &= -\frac{\psi_t^S}{\psi_t^w} = \int_t^\infty e^{-\delta_s(s-t) - \int_t^s r_u^* du} \sum_{i \in \mathbb{I}} e^{(n_i + \bar{g}_i)(s-t)} \Delta_i \chi(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] \frac{\omega_i \mathcal{P}_i \psi_{it}^w}{\psi_t^w} ds \\ SCC_t &= \sum_{i \in \mathbb{I}} \hat{\psi}_{it}^w LCC_{it} \quad \hat{\psi}_{it}^w = \frac{\omega_i \mathcal{P}_i \psi_{it}^w}{\psi_t^w} \\ LCC_{it} &= \int_t^\infty e^{-\delta_s(s-t) - \int_t^s (r_u^* - n_i - \bar{g}_i) du} \Delta_i \chi(\tau_{is} - \tau_i^*) [\gamma^y y_{is} + \gamma^u c_{is}] ds\end{aligned}$$

which gives the results of theorem 8. $\square$

Proof of theorem 9

Solving for the optimal carbon tax involves for solving for the objects in the optimality condition for energy choices. Take the energy choice for fossil-fuels:

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho + \eta \bar{g}_i)t} \psi_{it}^w \xi^f t_t^\varepsilon &= e^{-\int_{t_0}^t \bar{\rho}_s ds} \mathcal{P}_i \mu_t^f - e^{-\int_{t_0}^t \bar{\rho}_s ds} \mathcal{P}_i \xi^f \psi_t^S \\ &+ \omega_i \mathcal{P}_i e^{-(\rho + \eta \bar{g}_i)t} [v_{it}^f \partial_{ef} M P e_{it}^f + v_{it}^c \partial_{ef} M P e_{it}^c + v_{it}^r \partial_{ef} M P e_{it}^r + v_{it}^k \partial_{ef} M P k_{it}]\end{aligned}$$

We need to solve each of the objects in turn: (i) the marginal value of carbon, ψ_t^S , related to the Social Cost of Carbon, as seen in the previous proposition, (ii) the marginal value of oil supply μ_t^f , related to the energy supply redistribution, (iii) the marginal value of firms' inputs optimality conditions $v_{it}^f, v_{it}^c, v_{it}^r, v_{it}^k$.

Supply redistribution

We want to solve for μ_t^f . First, take the optimality for $[q_t^f]$.

$$\sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \theta_{it}^x = \sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [e_{it}^x - e_{it}^f] + \sum_i \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} v_{it}^f$$

The redistribution effect on oil-gas quantity between exporter and importers $e_{it}^x - e_{it}^f$ exactly compensate – for the planner – the equilibrium effect on demand v_{it}^f and supply θ_{it}^x .

Second, using the optimality of $[e_{it}^x]$:

$$\begin{aligned}\mu_t^f &= -e^{-(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} \omega_i \theta_{it}^x \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \nu_{ie^x}^f(e_{it}^x, \mathcal{R}_{it}) \\ \omega_i \mathcal{P}_i \theta_{it}^x &= -\frac{\mathcal{P}_i e^{-(n_i + \bar{g}_i)t}}{\nu_{ie^x}^f(e_{it}^x, \mathcal{R}_{it})} e^{-(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} \mathcal{P}_i \mu_t^f\end{aligned}$$

As a result, we obtain:

$$\begin{aligned} \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \theta_{it}^x &= -\mu_t^f \sum_i \frac{\mathcal{P}_i e^{(n_i+\bar{g}_i)t}}{\nu_{ie^x e^x}^f(e_{it}^x, \mathcal{R}_{it})} e^{(\rho+\eta\bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \\ \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \theta_{it}^x &= -e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \sum_i \frac{1}{\nu_{ie^x e^x}^f(e_{it}^x, \mathcal{R}_{it})} \\ \Rightarrow e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f &= -\left[\sum_i \nu_{ie^x e^x}^f(e_{it}^x, \mathcal{R}_{it})^{-1} \right]^{-1} \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \theta_{it}^x \end{aligned}$$

$$\Rightarrow e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f = \left[\sum_i \nu_{ie^x e^x}^f(e_{it}^x, \mathcal{R}_{it})^{-1} \right]^{-1} \left\{ \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \psi_{it}^w[e_{it}^f - e_{it}^x] - \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \nu_{it}^f \right\}$$

Demand distortion

We use a Nested CES framework for production. I express the formula without time indices for simplicity.

$$\text{Energy} \quad e_i = \left(\sum_\ell (\omega^\ell)^{\frac{1}{\sigma_e}} (e_i^\ell)^{\frac{\sigma_e-1}{\sigma_e}} \right)^{\frac{\sigma_e}{\sigma_e-1}} \quad \text{Output} \quad y_i = z_i \left((1-\varepsilon)^{\frac{1}{\sigma}} (z_i^e e_i)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}} (k_i^\alpha \ell_i^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Optimality for fossil energy demand features this complicated term, which we can simplify using the CES structure. The curvature of production, represented by terms like $\partial_{ef} MP e_i^f$ are related the the elasticity of energy use.

$$\begin{aligned} \bar{v}_i^f &= [v_i^f \partial_{ef} MP e_i^f + v_i^c \partial_{ef} MP e_i^c + v_i^r \partial_{ef} MP e_i^r + v_i^k \partial_{ef} MP k_i] \\ &= \frac{1}{e_i^f} \left[-v_i^f (q^f + \xi^f t^\varepsilon) \left[\frac{1-s^f}{\sigma^e} + s^f \frac{1-s^e}{\sigma^y} \right] + v_i^c (q_i^c + \xi^f t^\varepsilon) s_i^f \left[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y} \right] + v_i^r q_i^r s_i^f \left[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y} \right] + v_i^k (r^\star + \delta) \frac{s_i^{e^r/y}}{\sigma^y} \right] \end{aligned}$$

with Energy share in production: $s_i^e = \frac{e_i q_i^e}{y_i}$, Fossil share in energy mix $s_i^f = \frac{e_i^f q_i^f}{e_i q_i^e}$ and similarly $s_i^c = \frac{e_i^c q_i^c}{e_i q_i^e}$ and $s_i^r = \frac{e_i^r q_i^r}{e_i q_i^e}$.

When we normalize by $\bar{\psi}$, we obtain:

$$\hat{v}_i^f = \frac{1}{e_i^f} \left[-\hat{v}_i^f (q^f + \xi^f t^\varepsilon) \left[\frac{1-s^f}{\sigma^e} + s^f \frac{1-s^e}{\sigma^y} \right] + \hat{v}_i^c (q_i^c + \xi^f t^\varepsilon) s_i^f \left[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y} \right] + \hat{v}_i^r q_i^r s_i^f \left[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y} \right] + \hat{v}_i^k (r^\star + \delta) \frac{s_i^{e^r/y}}{\sigma^y} \right]$$

We can obtain similar formulas for $\hat{v}_{it}^f$, $\hat{v}_{it}^c$, $\hat{v}_{it}^r$, and $\hat{v}_{it}^k$.

Rewriting the optimal carbon tax

Take the optimality condition for energy choice, where I replaced $\bar{v}_{it}^f$. Sum over countries i and normalize by world population $\mathcal{P}_t$

$$\begin{aligned}\omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \psi_{it}^w \xi^f t_t^\varepsilon &= e^{-\int_{t_0}^t \bar{\rho}_s ds} \mathcal{P}_i e^{n_i+\bar{g}_i} \mu_t^f - e^{-\int_{t_0}^t \bar{\rho}_s ds} \mathcal{P}_i \xi^f \psi_t^S + \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \bar{v}_{it}^f \\ \xi^f t_t^\varepsilon \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \psi_{it}^w &= e^{-\int_{t_0}^t \bar{\rho}_s ds} \underbrace{\sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t}}_{=\mathcal{P}_t} [\mu_t^f - \xi^f \psi_t^S] + \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \bar{v}_{it}^f \\ \xi^f t_t^\varepsilon \bar{\psi}_t^w &= [\mu_t^f - \xi^f \psi_t^S] + \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \bar{v}_{it}^f\end{aligned}$$

We define:

$$\begin{aligned}\text{Supply Redistribution}_t &= \frac{\mu_t^f}{\bar{\psi}_t^w} = \left[\sum_i \nu_i^f e^x (e_{it}^x, \mathcal{R}_{it})^{-1} \right]^{-1} \left\{ \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \frac{\psi_{it}^w}{\bar{\psi}_t^w} [e_{it}^x - e_{it}^f] \right. \\ &\quad \left. + \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \frac{v_{it}^f}{\bar{\psi}_t^w} \right\} \\ &= \left[\sum_i (\nu_i^f e^x)^{-1} \right]^{-1} \left\{ \sum_i \hat{\psi}_{it}^w [e_{it}^x - e_{it}^f] + \sum_i \hat{v}_{it}^f \right\} \\ \text{Demand Distortion}_t &= \frac{1}{\bar{\psi}_t^w} \sum_i \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \bar{v}_{it}^f = \sum_i \hat{v}_{it}^f\end{aligned}$$

We obtain:

$$\begin{aligned}\xi^f t_t^\varepsilon &= \xi^f SCC_t + \left[\sum_i (\nu_i^f e^x)^{-1} \right]^{-1} \left\{ \sum_i \hat{\psi}_{it}^w [e_{it}^x - e_{it}^f] + \sum_i \hat{v}_{it}^f \right\} + \sum_i \hat{v}_{it}^f \\ \xi^f t_t^\varepsilon &= \xi^f SCC_t + \text{Supply Redistribution}_t + \text{Demand Distortion}_t\end{aligned}$$

This implies the formula in theorem 9 $\square$.