Climate Change, Inequality and Optimal Climate Policy

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Abstract

What is the optimal policy to fight climate change? Taxation of carbon and fossil fuels has strong redistributive effects across countries: (i) curbing energy demand is costly for developing economies, which are the most affected by climate change in the first place (ii) carbon taxation has strong general equilibrium effects through energy markets and fossil fuel rents. Through the lens of an Integrated Assessment Model (IAM) with heterogeneous countries, I show that the optimal taxation of carbon depends crucially on the availability of redistribution instruments. After characterizing the Social Cost of Carbon (SCC), I provide formulas for the Second-Best carbon tax in the presence of inequalities in incomes and climate damages, and redistributive and distortionary effects on energy markets. I show that a uniform carbon tax should be increased by approximately 45% in the presence of inequality compared to First-Best where cross-country transfers are available. If country-specific carbon taxes are available, the distribution of carbon prices is proportionally related to the level of income: poor and hot countries should pay lower energy taxes than rich and cold countries. These qualitative results are general, and I propose a dynamic quantitative model to provide recommendations for the optimal path of carbon taxes.

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1 Introduction

Greenhouse gas emissions generated by economic activity are causing climate change, and global atmospheric temperatures have increased by almost $1.5°C$ since the Industrial Revolution. The sources of these emissions are unequally distributed: developed economies account for over 65% of cumulative greenhouse gas (GHG) emissions with $\sim$ 25% each for the European Union countries and the United States, while some developing countries have barely emitted anything compared to their population level. Moreover, carbon emissions and energy consumption tend to correlate highly with development and income (e.g. GDP per capita).

Moreover, the consequences of global warming are also unequal: the increase in temperatures disproportionately affects developing countries where the climate is already warm. Most emerging and low-income economies lie geographically closer to the tropics and the equator and tend to be most vulnerable to global warming, e.g. Burke et al. (2015), Carleton et al. (2022).

Finally, implementing climate policy in the form of Pigouvian carbon taxation has strong redistributive effects as well. Countries that consume a large share of their energy mix in oil, gas, and coal will be affected more by distortionary carbon taxation. Moreover, phasing out fossil fuels reduces energy prices, lowering energy rents and hurting disproportionally exporters of fossil fuels.

These three layers of inequalities raise the following question: what is the optimal carbon policy in the presence of climate externality and inequality? Should the optimal tax on carbon and fossil fuels account for these different dimensions of heterogeneity?

To answer this question, I develop a simple yet general Integrated Assessment Model with countries' heterogeneity. Individual countries are heterogeneous in many dimensions, including (i) income, (ii) damages from climate change, and (iii) exposure to energy markets through differences in energy mix and fossil-fuel exports. Since the quantitative framework is very general, I first provide an extremely simple model to provide the main theoretical intuitions, keeping the same features regarding climate externalities and energy markets.

In both models, I study the design of optimal taxation of carbon and the characterization of the Social Cost of Carbon (SCC), which summarizes the costs of climate change for one additional ton of carbon emitted. I show that the optimal policy depends crucially on the availability of redistributive instruments – such as lump-sum transfers – across countries. In the First-Best, the optimal tax follows the Pigouvian benchmark and equals the Social Cost of Carbon. However, when cross-country transfers are not allowed, the optimal tax needs to account for inequality and redistributive considerations and adjust the level of the uniform carbon tax. Moreover, when choosing country-specific carbon taxes, the optimal policy is to lower the carbon tax for poorer countries, and higher for more advanced economies.

I first show these two main results in a simple "toy model", where I summarize these lessons in a four-equations static model. Differences in TFP, and thus income, in the impacts of climate change, and in the costs of extracting fossil fuels provide a rationale for redistribution. The unconstrained planner – in the First-Best – uses transfers to offset the redistributive effects of the carbon tax itself. However, in the Second-Best, when these transfers are assumed away, we see that the uniform carbon should account for three effects: (i) the climate externality represented by the Social Cost of Carbon (SCC), (ii) a supply redistribution that summarizes the equilibrium effect on the price of fossil-fuel energy which redistribution between exporter and importers, scaled by the inverse elasticity of the energy supply, and (iii) a demand distortion term that scale with energy inputs choices, shares, and demand elasticity. Moreover, (iv) these three terms are the aggregation of these effects for each country, weighted by the "*social welfare weights*", which is the product of the marginal utility of consumption and the Pareto weights. The planner puts more weight on poorer countries than on more advanced ones. To summarize, the world's optimal carbon policy differs from the standard *Carbon tax* $=$ *Social Cost of Carbon*, and the taxation should be adapted to the specific situation of each country.

Second, in this static model, I show how to choose country-specific carbon taxes. In that case, two motives – the Social Cost of Carbon and Supply Redistribution – remain, and Demand Distortion disappears thanks to the ability of the planner to act independently in each country. However, the carbon itself is now inversely proportional to the social welfare weights: the planner strongly reduces the carbon tax required for countries with high Pareto weights or high marginal utility of consumption – i.e. relatively poorer countries. I show in this simple setting how to implement such policies in emission trading systems or cap-and-trade. Moreover, I demonstrate that a direct mapping exists between price instruments – like carbon taxes or carbon prices – and quantity regulations – as discussed in Weitzman (2003), and Weitzman (2015).

I then build a quantitative dynamic model to provide policy recommendations to these questions. I consider an Integrated Assessment Model extending the standard Neoclassical Growth Model with multiple countries and many dimensions of heterogeneity. First, in each country, a representative firm produces a final good using capital, labor, and energy inputs. Countries differ in total productivity and energy efficiency, which implies differences in incomes or GDP/capita and total energy demand. Second, a representative household makes consumption, capital, and borrowing decisions over time. Third, there are three energy firms in each country: oil-gas, coal, and renewable, which are energy inputs used in the final good production. Countries differ in energy mix due to differences in costs of energy production. Moreover, differences in endowments in oil-gas – and dynamically depleting reserves – lead to countries being exporters or importers of fossil fuels. Finally, global fossil-fuel consumption – from oil-gas and coal – emits carbon into the atmosphere, which then feeds back into the climate system. This affects temperatures across regions and has heterogeneous damages across countries for firms and households. I calibrate the model to economic, climate, and energy data for a sample of 68 countries to match the dimensions of heterogeneity at the heart of countries' vulnerability to climate change and climate policies.

Despite the richness of the model and the many market forces and general equilibrium effects, the main result on the optimal carbon policy carries through. First, relying on the continuous time formulation of the model, I provide an analytical characterization of the Social Cost of Carbon (SCC) – which depends on the climate system and damage parameters but also the differences in social welfare weights. I show that in the First-Best allocation, in the absence of redistributive motives, the Social Cost of Carbon is the sum of the Local Costs of Carbon for each country, weighed by the planner's Pareto weights. This result aligns with models that can be perfectly aggregated where redistribution motives are absent. However, in the Second-Best, in cases where the planner cannot undermine preexisting inequality or offset redistributive effects, the Social Cost of Carbon is the sum of Local Costs of Carbon for each country, weighed by the *social welfare weights* which now integrate differences in marginal utility of consumption. If poorer countries are the most affected by climate change, the Social Cost of Carbon would be higher. However, in this class of models, the Local Costs of Carbon scale with consumption and income, which implies that the Social Cost of Carbon is lower than in the First-Best or the Representative Agent economy.

Then, I characterize the optimal carbon policy. In the First-Best allocation, we recover the Pigouvian benchmark, where the carbon tax equals the Social Cost of Carbon. This is because the planner redistributes across countries using lump-sum transfers, for example, taxing lump-sum European and American countries and transferring to South Asian and African economies. In the Second-Best Ramsey policy, when the planner is unable to redistribute freely across countries due to limitations on lump-sum transfers, the world optimal carbon tax needs to be adjusted. As before, it needs to account for Supply Redistribution – depending on energy supply curve elasticities – and Demand Distortion – which is a function of the energy demand elasticities. Finally, I also show that country-specific carbon taxes are scaled by the inverse of the social welfare weights, such that the poorer or warmer the country, the lower the carbon tax it needs to pay. Moreover, in the quantitative model, these terms are slightly more involved as they depend on the path of temperature – as the Social Cost of Carbon increases over time, as is common in carbon taxation in IAM – and the substitution patterns between energy sources and change dynamically. Similarly, the social welfare weights used for aggregating these effects across countries also change over time.

In addition, this framework is general, and I develop a method inspired by the Heterogeneous Agents literature and Mean-Field Games to solve this class of model globally in continuous time. This relies on the sequential formulation of the optimal control problem, which allows to follow the trajectories of each country/agent. As a result, it allows us to consider an arbitrary number of dimensions of ex-ante heterogeneity and a larger number of dimensions of time-varying states than what is typically covered in the literature using dynamic programming methods.

The main quantitative result is that accounting for inequality implies changing the optimal carbon tax in three ways. First, computing the Social Cost of Carbon with the social welfare weights results in a SCC of $\$50/tCO_2$ instead of $\$100/tCO_2$ when simply doing the simple sum of Local Cost of Carbon. Second, implementing the carbon mitigation does reduce the Social Cost of Carbon – which is an equilibrium object depending on climate damage. I show that in the First-Best, the planner would use large transfers to offset inequality. In the Second Best, when redistribution instruments are absent, the optimal uniform carbon tax is approximately 45% higher, from \$75 to \$110. This results from the fact that the carbon tax puts more weight on poorer countries that have a higher marginal value of wealth which. Finally, it accounts for redistribution motives in the energy markets – supply redistribution and demand distortion. It implies, on the one hand, a lower carbon tax to avoid hurting fossil-fuel exporters and, on the other hand, a higher tax since the richest countries are the major consumers of fossil fuels. Quantitatively, the second effect on demand dominates largely. This implies a carbon tax slightly above 110, which is in the range of estimates for the optimal carbon tax. Forthcoming results would show how these effects would change over time with climate change dynamics, the change in the valuation of fossil reserves, and the growth dynamics of developing economies.

Related literature

This paper stands at the intersection of several subfields of macroeconomics, climate economics, and computational and mathematical economics.

First, I develop an Integrated Assessment model (IAM) with heterogeneous countries, and this naturally relates to the classical approach of IAM by Nordhaus. I use a neoclassical model with a climate system and damage of temperatures, as in the DICE model, Nordhaus (1993, 2017), recently revisited in Barrage and Nordhaus (2024). In this representative agent framework, as in the rest of the literature, the standard Pigouvian result holds: the optimal taxation of carbon equals the Social Cost of Carbon (SCC). As a result, it is enough to measure the marginal cost of climate change to know the full path of the carbon tax. Golosov, Hassler, Krusell and Tsyvinski (2014) develop a complete study of the optimal taxation of fossil fuels in a class of models inspired by DICE models and derive the first-best policy and a closed-form formula as a function of the climate and economic parameters for the optimal carbon tax and Social Cost of Carbon.

Second, I extend this class of model to handle country heterogeneity. I build on the literature that started with the RICE – the multi-regions version of the DICE model – with Nordhaus and Yang (1996); Nordhaus (2011). As studied in Hillebrand and Hillebrand (2019), the optimal carbon tax should be the Social Cost of Carbon, i.e. the sum of Local Damages, and the heterogeneity across regions determines the optimal transfer policies. When transfers are unrestricted, there is no need to adjust the carbon tax or the Social Cost of Carbon for inequality. More recently, frameworks with more realistic heterogeneity have been developed to study the impact of climate change and design optimal policies such as Krusell and Smith (2022), Hassler, Krusell, Olovsson and Reiter (2020), Kotlikoff, Kubler, Polbin and Scheidegger (2021*b*), Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021) or Belfiori (2018). I show how to solve for the optimal carbon taxation, which features of the heterogeneity matter, in this class of model.

Related, the spatial-economic geography literature has made important advances in studying the heterogeneous impact of climate change. Cruz and Rossi-Hansberg (2021), Cruz and Rossi-Hansberg (2022*a*), Rudik et al. (2021) or Bilal and Rossi-Hansberg (2023*b*) are rich frameworks that incorporate migration, agglomeration and congestion externality, and meaningful spatial heterogeneity. The design of optimal policies in such models is still being explored in the literature, and the approach in this article is well-suited for this body of work.

Moreover, a blooming literature has been developed to study the redistributive effects of carbon taxation within countries and the heterogeneous impacts of climate change across households. For example, Belfiori, Carroll and Hur (2024) provides similar theoretical and quantitative results for the optimal policy in the First-Best and Second-Best without transfers. In similar heterogeneous agents frameworks, Le Grand, Oswald, Ragot and Aurélien (2023); Wöhrmüller (2024); van der Ploeg, Rezai and Tovar (2024); Fried, Novan and Peterman (2024); Benmir and Roman (2022); Kuhn and Schlattmann (2024); Schlattmann (2024); Douenne, Hummel and Pedroni (2023); Douenne, Dyrda, Hummel and Pedroni (2024) have made significant contributions to understand the redistributive effects – due to differences in goods, car and durables, housing, or adaptation mechanisms along the wealth distribution.

I also relate to the literature closer to climate sciences, reexamining the empirical performances of Integrated Assessment Models, such as in Dietz, van der Ploeg, Rezai and Venmans (2021), Dietz and Venmans (2019), Ricke and Caldeira (2014) or Folini et al. (2021). Following this literature, I consider a simple climate system that allows me to both match larger IAMs and derive closed-form expressions for the social cost of carbon and optimal carbon tax.

Moreover, I also relate to a thriving literature that studies optimal policy design in Heterogeneous Agents models. Solving Ramsey policies, Le Grand et al. (2021), Bhandari et al. (2021*a*), Davila and Schaab (2023) or McKay and Wolf (2022) propose different approaches to conduct monetary and fiscal policy in HANK models. In my framework, I solve the Ramsey policy sequentially and solve climate externalities and Pigouvian taxation in the presence of heterogeneity rather than managing business cycle fluctuations.

The method developed here is flexible enough to handle aggregate uncertainty, such as climate risk and business cycle fluctuation, and results in this dimension are work-in-progress. The Stochastic DICE model of Cai and Lontzek (2019) and Lontzek, Cai, Judd and Lenton (2015) or the general approach to study model uncertainty and ambiguity aversion applied to climate change in Barnett, Brock and Hansen (2020, 2022) are particularly related. If the inclusion of aggregate risk is preliminary in the present paper, I provide intuitions in the toy model and will integrate this in forthcoming works.

Lastly, this work also relates to advances in the mathematical literature on Mean Field Games. Indeed, if the literature has leveraged approaches studying the PDE system, following Lasry-Lions' contribution, Cardaliaguet (2013/2018), or Achdou et al. (2022), they usually rely on dynamic programming methods. However, the Pontryagin maximum principle – used for solving the neoclassical model – extends to the stochastic case, as in Yong and Zhou (1999), or the case with a distribution of agents or – Mean-Field / McKean Vlasov dynamics – as in Carmona et al. (2015), Carmona and Delarue (2018) or Carmona and Laurière (2022). Using this approach in the deterministic case in large dimensions, I solve the model globally, compute the social cost of carbon analytically, and design optimal policy. For the case with aggregate risk, I borrow intuitions from Carmona et al. (2016), Bourany (2019), and Carmona and Delarue (2018) to solve the Stochastic FBSDE system in future work Bourany (2023).

The remainder of this paper is organized as follows. In Section 2, I study the optimal taxation carbon in a simple model to provide most of the intuitions. In Section 3, I lay out the Integrated Assessment Model that we study in the policy analysis. In Section 4 I derive the optimal carbon tax – First-Best and Second-Best Ramsey policy – in this context. In Section 5, I present how I match the model to the data. In Section 6, I present the main result of the quantitative analysis.

2 Toy model

In this section, I develop the simplest climate economy model to highlight the intuition behind the design of the optimal climate policy. The goal is to provide intuitions on the effects of heterogeneity across countries, the source of climate externality related to energy markets, and how it change the level of carbon taxation. In the next section, I develop a more general quantitative model that will be used for policy recommendation.

The model is static and all the decisions are taken in one period. Consider I countries $i \in \mathbb{I}$, heterogeneous in three dimensions that will be detailed below. A unique household in each country consumes the good c_i, produced by the representative firm with labor ℓ_i and energy e_i.[1] In each of these countries, an energy producer extracts energy and sells this input at price q^e on international markets. It earns profits and is owned by the household. Moreover, the countries are subject to climate damage on production. I describe each agent's problem in turn. Finally, a government, whose objective is specified in the next section, imposes a tax on emission t_i^ε and distributes lump-sum transfers t_i^{ls} in each country.

First, a representative household consumes their labor income $w_i\bar{\ell}_i$, where the exogenous labor supply is normalized to $\bar{\ell}_i = 1$, the profit of the energy firm of its country π_i^e and the lump-sum transfers given by the government t_i^{ls}.

$$
\begin{aligned}
&\mathcal{V}_i = U(c_i) \\
&c_i = w_i\bar{\ell}_i + \pi_i^e + \mathrm{t}_i^{ls} \ .
\end{aligned}
$$

Second, a representative firm produces a homogeneous good[2] using energy e_i and household labor ℓ_i with a constant return to scale technology. Since the labor supply is normalized to 1, e_i represents the energy use per capita. The production function $\widetilde{F}(\ell_i, e_i)$ is concave in (ℓ_i, e_i), and $\widetilde{F}_e(\ell, e) > 0$ and $\widetilde{F}_{ee}(\ell, e) < 0$ and features Inada conditions. This firm maximizes profits:

$$
\max_{\ell_i, e_i} \ \mathcal{D}_i(\mathcal{S}) z_i \widetilde{F}(\ell_i, e_i) - (q^e + \mathrm{t}_i^\varepsilon) e_i - w_i \ell_i \quad , \tag{1}
$$

where t_i^ε is a carbon tax paid per unit of energy.

Both countries are subject to climate damages $\mathcal{D}_i(\mathcal{S})$ caused by climate externalities related to the world fossil-fuel energy consumption that release greenhouse gas emissions in the atmosphere:

$$
\mathcal{S} = \mathcal{S}_0 + \overbrace{\textstyle\sum_{i \in \mathbb{I}} e_i}^{\text{GHG emissions } \mathcal{E}} \quad ,
$$

where energy use and emissions are measured in (metric) tons of Carbon or CO_2. This depends on the mix between fossil fuels and renewable energies, taken as given in this static model. The quantitative model introduces this endogenous channel of energy substitution.

[1] Generalization of this model, with differing population $\mathcal{P}_i$, endowments of inputs in the production function (e.g. capital k_i), do not change the qualitative implication of this framework, as we will see in the quantitative model.

[2] This good is traded costlessly across countries and its price is the numeraire, and hence normalized to 1.

The global carbon emission stock is not internalized by households in their energy consumption decision, leading to damage $\mathcal{D}_i(\mathcal{S})$ that affects country i's effective productivity, as in standard Integrated Assessment models, e.g. Nordhaus DICE models.

In each country, a competitive energy producer extracts energy e_i^x – for example oil, gas, or coal – maximizing its profit, subject to convex cost $\mathcal{C}(e^x)$, i.e. $\mathcal{C}'(e^x) > 0$ and $\mathcal{C}''(e^x) > 0$ that is paid in the homogenous good.

$$\pi_i^e = \max_{e_i^x} q^e e_i^x - \mathcal{C}_i(e_i^x) \quad ,$$

$$\Rightarrow \qquad q^e = \mathcal{C}_i'(e_i^x) \qquad \Rightarrow \qquad \begin{cases} e_i^x &= \mathcal{E}_i(q_i^e) = \mathcal{C}_i'^{-1}(q^e) \\ \pi_i^e &:= q^e \mathcal{E}(q_i^e) - \mathcal{C}_i(\mathcal{E}_i(q_i^e)) \end{cases} \quad ,$$

subject the energy price q^e. This corresponds to a decreasing-return-to-scale extraction technology, and implies positive profits $\pi_i^e > 0$. Since energy is traded without friction on international markets, this price is set to clear the supply and demand:

$$E = \sum_{i \in \mathbb{I}} e_i = \sum_{i \in \mathbb{I}} e_i^x \ .$$

Since the good firm's technology is constant return to scale (CRS), define $F(e_i) = \widetilde{F}(1, e_i)$ to aggregate firms and household budgets into a single constraint:

$$c_i + (q^e + \mathrm{t}_i^\varepsilon) e_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q_i^e e_i^x - \mathcal{C}_i(e_i^x) + \mathrm{t}_i^{ls} \qquad [\lambda_i] \ . \tag{2}$$

with λ_i the shadow value of that constraint, which plays an important role in redistribution motives.

Heterogeneity. The countries $i \in \mathbb{I}$ are symmetric in all regards, except for differences in three parameters. To fix ideas, consider two regions, *North* and *South*, to give qualitative predictions of the policy results. First, countries differ in terms of productivity z_i. Here, I consider a wide definition of z_i that accounts for technology, efficiency, market frictions, and institutions. This results in some countries – e.g. the North – producing more and being richer, leading to inequality in consumption.[3] Second, energy reserves endowments are unequally distributed, which results in differences in costs of extraction. I assume that northern countries, e.g. US, Canada, Russia, Norway, etc. have lower costs of extraction $\mathcal{C}_N'(e) < \mathcal{C}_S'(e)$, implying larger production and energy rents $\pi_N^e > \pi_S^e$. Third, some countries, e.g., Southern Hemisphere, are more vulnerable to the damages of climate, $\mathcal{D}_S(\mathcal{S}) < \mathcal{D}_N(\mathcal{S})$ for all $\mathcal{S}$ the stock of carbon. In this sense, the damage parameter $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$ is higher in the South, $\gamma_S > \gamma_N$. All these differences yield heterogeneity in consumption in the competitive equilibrium and motives for redistribution, e.g. $c_N > c_S$.

[3]Indeed, assuming $F(e)$ is Cobb Douglas $F(e) = \bar{\ell}^{1-\alpha} e^\alpha$, with $\bar{\ell} = 1$, we obtain $\alpha \mathcal{D}_i(\mathcal{S}) z_i e_i^{\alpha-1} = q^e$ leading to

$$e_i = \big(\alpha \mathcal{D}_i(\mathcal{S}) z_i / q^e\big)^{1/(1-\alpha)} \qquad y_i - q^e e_i = \big(\mathcal{D}_i(\mathcal{S}) z_i\big)^{1/(1-\alpha)} (q^e)^{-\alpha/(1-\alpha)} \big[\alpha^{\alpha/(1-\alpha)} - \alpha^{1/(1-\alpha)}\big]$$

which is increasing in z_i and $\mathcal{D}_i(\mathcal{S})$

Definition 2.1 (Competitive Equilibrium). *Given a carbon policy* t^{ε}, *a competitive equilibrium (CE) is an allocation* $\{c_i, e_i, e_i^x\}_i$ *and energy price* q^e *such that (i) the good firm chooses input* e_i *maximizing profit, and (ii) the energy firm chooses extraction* e_i^x *maximizing profit, and both goods and energy markets clear:*

$$\sum_{i\in\mathbb{I}} c_i + \mathcal{C}_i(e_i^x) = \sum_{i\in\mathbb{I}} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad\qquad E = \sum_{i\in\mathbb{I}} e_i = \sum_{i\in\mathbb{I}} e_i^x \quad .$$

This results in the following optimality conditions. First, for consumption, the multiplier λ_i represents the marginal value of wealth or the marginal utility of consumption.

$$\lambda_i = U'(c_i) \qquad \textit{with} \qquad c_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e(e_i^x - e_i) - \mathcal{C}_i(e_i^x) + \mathrm{t}_i^{ls}$$

where consumption depends on production, energy cost, and net energy export.

The second and third optimality for energy use and energy extraction write as follow:

$$\begin{aligned} MPe_i &= q^e + \mathrm{t}_i^{\varepsilon} \qquad \textit{with} \qquad MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(e_i)\ , \\ q^e &= \mathcal{C}'(e_i^x)\ , \end{aligned}$$

and this corresponds to the standard condition Marginal Product = Marginal Cost for Energy.

This competitive equilibrium is inefficient: climate damages $\mathcal{D}_i(\mathcal{S})$ are not internalized, and energy consumption is too high in view of the economic costs of global warming. Moreover, economic inequality results from the heterogeneity in productivity, energy endowment, and climate damage. In our two regions example, $c_N > c_S$ results in $\lambda_S > \lambda_N$. Redistribution from the North to the South could be desirable from a utilitarian point of view. This inequality in consumption and damages arises despite trade openness.[4] I explore how the social planner allocates consumption and energy in such an environment.

2.1 First-Best: Social planner allocation with full transfers

Consider a Social Planner choosing the agent's decisions, subject to the resource constraints in goods and energy as well as the climate externality.

$$\begin{aligned} \mathcal{W} = \max_{\{c_i, e_i, e_i^x\}_{i\in\mathbb{I}}} & \sum_{i\in\mathbb{I}} \omega_i U(c_i) \\ \sum_{i\in\mathbb{I}} c_i + \mathcal{C}_i(e_i^x) &= \sum_{i\in\mathbb{I}} \mathcal{D}_i(\mathcal{S}) z_i F(e_i) \qquad [\phi] \\ E = \sum_{i\in\mathbb{I}} e_i &= \sum_{i\in\mathbb{I}} e_i^x \qquad [\mu^e] \\ \mathcal{S} &:= \mathcal{S}_0 + \sum_{i\in\mathbb{I}} e_i \end{aligned} \tag{3}$$

[4] We could consider trade and financial autarky preventing production from being exported to other countries. This would strengthen heterogeneity and redistributive motives.

where ϕ is the shadow value of the good market clearing and μ^e the one of the energy market clearing. The welfare function is the weighted sum of countries' utilities, with Pareto weights ω_i. In the following, I denote the social planner allocation $\{\hat{c}_i, \hat{e}_i\}_{i\in\mathbb{I}}$ to distinguish it from the competitive equilibrium.

Choosing the consumption on behalf of the agents yields a redistribution motive:

$$[c_i] \qquad \phi = \omega_i U'(\hat{c}_i) \qquad \Rightarrow \qquad \omega_i U'(\hat{c}_i) = \omega_j U'(\hat{c}_j) \qquad \forall i,j \in \mathbb{I}\ .$$

Depending on the Pareto weights, there is a motive for transferring consumption across countries.

Regarding the choice of energy inputs:

$$[\hat{e}_i]\ \&\ [\hat{e}_i^x] \qquad c'(\hat{e}_i^x) = \frac{\mu^e}{\phi} = \mathcal{D}_i(\mathcal{S}) z_i F'(\hat{e}_i) + \underbrace{\sum_{j\in\mathbb{I}} \mathcal{D}_j'(\mathcal{S}) z_j F(\hat{e}_j)}_{=:\overline{SCC}}$$

with an additional term that represents the cost of emitting one ton of carbon in terms of forgone production. This term is the *Social Cost of Carbon (SCC)* in the social planner allocation and represents the marginal global damage of climate change. With $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{S}\mathcal{D}_i(\mathcal{S})}$, which is constant with Nordhaus' Damage function $\mathcal{D}_i(\mathcal{S}) = e^{-\frac{\gamma_i}{2}(\mathcal{S}-\mathcal{S}_0)^2}$, I redefine it as $\overline{SCC} = \sum_{j\in\mathbb{I}} \mathcal{S}\gamma_j y_j$.

I turn to the decentralization of such allocation. I consider a planner who has access to all instruments $\{\mathrm{t}_i^e, \mathrm{t}_i^{ls}\}_i$, and, in particular, lump-sum transfers t_i^{ls} across countries.

Proposition 1 (**First-Best Policy and Decentralization**).
The optimal policy decentralizing the First-Best allocation is a uniform carbon tax, $\mathrm{t}_i^\varepsilon = \mathrm{t}^\varepsilon$, *equal to the Social Cost of Carbon* $\mathrm{t}^\varepsilon = SCC$, *and uses lump-sum transfers for redistribution. Indeed, optimality writes:*

$$MPe_i := \mathcal{D}_i(\mathcal{S}) z_i F'(\hat{e}_i) = q^e + \mathrm{t}^\varepsilon \qquad q^e = c'(\hat{e}_i^x)$$

$$\textit{with} \qquad \mathrm{t}^\varepsilon = \overline{SCC} := -\sum_{j\in\mathbb{I}} \mathcal{D}_j'(\mathcal{S}) z_j F(\hat{e}_j) = \sum_{j\in\mathbb{I}} \mathcal{S}\gamma_j y_j = I\mathbb{E}_j\left[\mathcal{S}\gamma_j y_j\right]$$

and transfers $\{\mathrm{t}_i^{ls}\}_i$ *which are implicitly defined by*

$$\omega_i U'(\hat{c}_i) = \omega_j U'(\hat{c}_j)\ ,$$
$$\hat{c}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\hat{e}_i) + q^e(\hat{e}_i^x - \hat{e}_i) - c_i(\hat{e}_i^x) + (\mathrm{t}_i^{ls} - \mathrm{t}^\varepsilon \hat{e}_i)\ .$$

For arbitrary Pareto weights $\boldsymbol{\omega}$, *lump-sum transfers are redistributive:* $\exists\, i,j\ s.t.\ \mathrm{t}_i^{ls} > \mathrm{t}^\varepsilon \hat{e}_i, \mathrm{t}_j^{ls} < \mathrm{t}^\varepsilon \hat{e}_j$.

To see this last point, summing the budget constraints yields:

$$\sum_{i\in\mathbb{I}} \mathrm{t}_i^{ls} = \mathrm{t}^\varepsilon \sum_{i\in\mathbb{I}} \hat{e}_i$$

implying there is lump-sum redistribution from richer countries to poorer ones in the presence of heterogeneity across countries.[5] In our North-South example, with $z_S < z_N$, $c_N' < c_S'$ or $\mathcal{D}_S' > \mathcal{D}_N'$,

[5]Given that $\mathrm{t}_i^{ls} = \mathrm{t}^\varepsilon \hat{e}_i + U'^{-1}(\frac{\phi}{\omega_i}) - \mathcal{D}_i(\mathcal{S}) z_i F(\hat{e}_i) - q^e(\hat{e}_i^x - \hat{e}_i)$.

and for arbitrary Pareto weights[6], we obtain that $\mathrm{t}_S^{ls} > \mathrm{t}^\varepsilon \hat{e}_S$, and $\mathrm{t}_N^{ls} < \mathrm{t}^\varepsilon \hat{e}_N$. This implies that some funds are taxed from the North and redistributed lump-sum to the South. However, there exists a unique set of Pareto weights $\omega_i = 1/U'(\hat{c}_i)$ – the so-called Negishi weights – such that this motive disappears $\mathrm{t}_S^{ls} = \mathrm{t}^\varepsilon \hat{e}_S$ and $\mathrm{t}_N^{ls} = \mathrm{t}^\varepsilon \hat{e}_N$ and there are no transfers across countries.

In the following, I rule out this flexible lump-sum transfers assumption: if development aid exists, in practice, full redistribution with lump-sum taxes and transfers to cover the differences in technology, market frictions, and institutions is politically unfeasible.

2.2 Second Best: Ramsey problem, uniform carbon tax, and limited transfers

Consider now a social planner designing the optimal climate policy, taking into account the constraints preventing transfers across countries. Subject to the competitive equilibrium optimality conditions, the climate externality and the absence of financial instruments for full lump-sum redistribution, the planner takes the decisions of consumption and energy to maximize global welfare. I denote the Ramsey allocation $\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i$ to distinguish it from the competitive equilibrium $\{c_i, e_i, e_i^x\}$ and the First-best allocation $\{\hat{c}_i, \hat{e}_i, \hat{e}_i^x\}$.

$$\mathcal{W} = \max_{\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i, q^e, \mathrm{t}^\varepsilon} \sum_{i \in \mathbb{I}} \omega_i U(\tilde{c}_i) \tag{4}$$

I consider a uniform carbon tax for all countries $\mathrm{t}_i^\varepsilon = \mathrm{t}^\varepsilon$, and this, for several reasons. First, the goal is to provide a direct comparison to the standard Pigouvian framework, where the natural outcome is a global uniform tax on fossil energy. Second, following the arguments of Weitzman (2015), the uniform carbon tax or price of carbon serves as a "focal point", where the social-planner policy is a representation of the bargaining outcome in an agreement coming from all the countries in the world. Third, in the next section, I consider different tax rates for each country.

In both cases – uniform tax or country-specific taxes – I assume away cross-country transfers: as the revenue of the tax is redistributed lump-sum to the household $\tilde{\mathrm{t}}_i^{ls} = \mathrm{t}^\varepsilon \tilde{e}_i$. Moreover, in the Second-Best Ramsey policy, the planner internalizes the optimality conditions of the competitive equilibrium. Using the Primal Approach in public finance, the Ramsey problem accounts for the countries' budgets and the firms' optimality conditions and is written as:

$$\begin{aligned}
&\mathcal{W} = \max_{\{\tilde{c}_i, \tilde{e}_i, \tilde{e}_i^x\}_i, q^e, \mathrm{t}^\varepsilon} \sum_{i \in \mathbb{I}} \omega_i U(c_i) \\
&s.t \quad \tilde{c}_i + (q^e + \mathrm{t}^\varepsilon)\tilde{e}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\tilde{e}_i) + q^e \tilde{e}_i^x - \mathcal{C}_i(\tilde{e}_i^x) + \mathrm{t}^\varepsilon \tilde{e}_i && [\phi_i] \\
&\qquad q^e = \mathcal{C}_i'(\tilde{e}_i^x) \qquad\qquad q^e + \mathrm{t}^\varepsilon = MPe_i && [\upsilon_i] \\
&\qquad \mathcal{S} := \mathcal{S}_0 + \sum_i e_i \qquad\qquad E = \sum_i e_i = \sum_i e_i^x && [\mu^e]
\end{aligned} \tag{5}$$

with the Lagrange Multipliers $\phi_i, \upsilon_i, \mu^e$, respectively for the budget constraint, the energy choice of the good firm, and the energy market clearing.

[6]In particular, this is the case if the Pareto weights are large enough, i.e. $\omega_S \geq \phi / U'(c_S)$ i.e. more than the weight imposed by the shadow value of good discounted by South' marginal utility

The Lagrangian of this problem, as well as the detailed optimality conditions, are derived in detail in Appendix A. They yields the following optimality conditions for consumption c_i, demand e_i and supply e_i^x for energy:

$$\omega_i U'(\tilde{c}_i) = \phi_i = \text{marginal value of income/wealth}$$

$$\phi_i \mathrm{t}^\varepsilon = \underbrace{\upsilon_i \, \mathcal{D}_i(\mathcal{S}) z_i F''(\tilde{e}_i)}_{=\text{demand distortion}} + \underbrace{\mu^e}_{=\text{supply redistribution}} \underbrace{- \sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(\tilde{e}_j)}_{\propto \text{ Social Cost of Carbon}}$$

The planner chooses a single carbon policy instrument and thus accounts for several redistribution channels across countries through (i) the marginal value of income ϕ_i, (ii) the distortion of energy demand symbolized by the shadow value of energy choice υ_i, (iii) the redistributive effects on energy market with the market clearing multiplier μ^e, and (iv) the Social Cost of Carbon summarizing the marginal cost of climate change. There, before deriving the main formula for the optimal carbon tax t^ε and explaining the economic intuition behind it, let us introduce these key objects.

First, I define the "social welfare weight" $\widehat{\phi}_i = \phi_i / \overline{\phi}$ that represents the relative weight that the planner uses for global policy. I define it as a ratio, rescaling the multiplier ϕ_i, of the shadow value of relaxing the budget constraint for country i.

$$\widehat{\phi}_i = \frac{\phi_i}{\overline{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{I} \sum_{i \in \mathbb{I}} \omega_i U'(c_i)} \lesseqgtr 1$$

where $\overline{\phi} = \partial \mathcal{W} / \partial c = \frac{1}{I} \big(\sum_{i \in \mathbb{I}} \omega_i U'(c_i) \big)$ is the average marginal utility. $\overline{\phi}$ is the "money↔welfare" conversion factor for the social planner. When there is no full redistribution, this factor $\widehat{\phi}_i$ is high for relatively poorer countries or countries with a high Pareto weight ω_i.

Second, climate change affects countries differently according to their marginal damages $\mathcal{D}_j'$. The social cost of carbon scales those damages by the social welfare weights/inequality factor $\widehat{\phi}_i \propto \omega_i U'(c_i)$ given that the planner does not have access to full redistribution. Rescaled in monetary unit, with the conversion factor $\overline{\phi}_i$, the SCC writes:

$$SCC := -\frac{\partial \mathcal{W} / \partial \mathcal{S}}{\partial \mathcal{W} / \partial c} = -\frac{1}{\overline{\phi}} \sum_j \phi_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) = -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) = \sum_{j \in \mathbb{I}} \widehat{\phi}_j \, \mathcal{S} \gamma_j y_j$$

with the definition $\gamma_i = -\frac{\mathcal{D}_j'(\mathcal{S})}{\mathcal{D}_j(\mathcal{S}) \mathcal{S}}$, the slope of the climate damage function, as in Nordhaus' DICE model. In particular, in this heterogeneous countries model with limited redistribution, the Social Cost of Carbon integrates the distribution of consumption/income under the factor $\widehat{\phi}_i$:

$$SCC := \sum_{j \in \mathbb{I}} \widehat{\phi}_j \, \mathcal{S} \gamma_j y_j = I \mathbb{E}_j \Big(\widehat{\phi}_j \, \mathcal{S} \gamma_j y_j \Big)$$

$$SCC = I \, \mathbb{E}_j [\mathcal{S} \gamma_j y_j] + I \, \mathbb{C}\mathrm{ov}_j \Big(\widehat{\phi}_j, \mathcal{S} \gamma_j y_j \Big) \quad \lesseqgtr \quad I \, \mathbb{E}_j [\mathcal{S} \gamma_j y_j] =: \overline{SCC}$$

with the damage slope $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{D}_i(\mathcal{S}) \mathcal{S}}$, and country i's output $y_i = \mathcal{D}_i(\mathcal{S}) z_i F(e_i)$. The $\overline{SCC} =$

$I\mathbb{E}_j[-\mathcal{D}_j'(\mathcal{S})z_jF(e_j)] = I\mathbb{E}_j[\mathcal{S}\gamma_j y_j]$ is the Social Cost of Carbon when full redistribution is available, or equivalently in representative agent models where redistributive concerns are absent. Since $\mathbb{E}_j(\cdot)$ is a mean[7] over countries j, we need to multiply by the number of countries (I here) to obtain the sum of local damages.

Is the SCC higher in the model with inequality compared to the one-agent setting? Take the North-South economy as an example. First, low-income countries have a lower consumption and hence higher marginal utility of consumption, e.g. $c_S < c_N$ and $\widehat{\phi}_S > \widehat{\phi}_N$. Second, South is suffering from stronger damages $\mathcal{D}_S' > \mathcal{D}_N'$. However, third, productivity is higher in the North $z_N > z_S$ implying $F(e_N) > F(e_S)$ and income $\bar{y}_N{:=}z_N F(e_N) > \bar{y}_S$. Therefore, the covariance between $\widehat{\phi}_i$, $\bar{y}_i$ and $\gamma_i \propto \mathcal{D}_i'$ is ambiguous. Quantitatively, in a large class of Integrated Assessment models, the local cost of climate change $\mathcal{D}_i'(\mathcal{S})y_i$ is strongly correlated with income y_i, as there larger production loss of climate change in richer countries. In such cases, the covariance $\mathbb{C}\text{ov}_j\Big(\hat{\phi}_j, \gamma_j y_i\Big)$ is negative, and as a result:

$$SCC = I\,\mathbb{E}_j[\mathcal{S}\gamma_i y_i] + I\,\mathbb{C}\text{ov}_j\Big(\widehat{\phi}_j, \mathcal{S}\gamma_j y_j\Big) < I\mathbb{E}_j[\gamma_j y_j] = \overline{SCC} \qquad \text{if} \quad \mathbb{C}\text{ov}_j\Big(\widehat{\phi}_j, \gamma_j y_j\Big) < 0$$

Third, I explore the redistributive effect of carbon taxation on the energy supply. Changing the price affects the market clearing, with shadow value μ^e. I formulate this supply side channel as a redistribution between energy importers and exporters, weighted by a factor representing the curvature of aggregate energy supply:

$$\begin{aligned}
\textit{Supply Redistribution} \; &= \frac{\mu^e}{\bar{\phi}} = \mathcal{C}_{EE}\,\frac{1}{I}\sum_j \frac{\phi_j}{\bar{\phi}}(e_j - e_j^x) \qquad \text{with} \qquad \mathcal{C}_{EE} = \Big(\sum_j c_j''(e_j^x)^{-1}\Big)^{-1}\\
&= \; \mathcal{C}_{EE}\;\mathbb{E}_j\Big(\widehat{\phi}_j(e_j - e_j^x)\Big)\\
&= \mathcal{C}_{EE}\;\mathbb{C}\text{ov}_j\Big(\widehat{\phi}_j,\, e_j - e_j^x\Big) \lesseqgtr 0
\end{aligned}$$

What is the sign of this covariance? In our two regions example, we assumed that the North had a larger endowment in energy resources and hence higher net energy exports $e_N - e_N^x < e_S - e_S^x$. Therefore, since the net import of energy correlates with lower consumption, and hence a higher marginal value of consumption $U'(c_i)$, the covariance term is positive. Moreover, the magnitude of this terms-of-trade redistribution ultimately depends on the aggregate supply elasticity:

$$\mathcal{C}_{EE} = \Big(\sum_j c_j''(e_j^x)^{-1}\Big)^{-1} = q^e\,\frac{\bar{\nu}}{E} \qquad \text{with} \qquad \bar{\nu} = \big(\textstyle\sum_j \lambda_j^x \nu_j^{-1}\big)^{-1}$$

with ν_j the inverse supply elasticity, constant in the iso-elastic case $q^e = c_i'(e) = \bar{\nu}_i e^{\nu_i}$ and the share of country i in energy production $\lambda_j^x = e_i^x/E$. As a result, this Social "Supply Redistribution" is positive. It is larger when the energy supply is inelastic – price and profits vary a lot for small changes in quantity produced. It is null when the energy production is Constant Return to Scale (CRS) when $\nu_j = 0$, and therefore, no energy rents are redistributed.

[7]The formula of the expectation of a product writes $\mathbb{E}_i[x_i y_i] = \mathbb{E}_i[x_i]\mathbb{E}_i[y_i] + \mathbb{C}\text{ov}_i[x_i y_i]$

Third, carbon taxation distorts energy choice across users and changes the equilibrium energy price along the demand curve. We derive the Social "Demand Distortion" term as:

$$\textit{Demand Distortion} = \frac{1}{I}\sum_j \frac{\upsilon_j}{\bar{\phi}}\mathcal{D}_j(\mathcal{S})z_jF''(e_j) = \mathbb{E}_j\left(\widehat{\upsilon}_j\,\mathcal{D}_j(\mathcal{S})z_jF''(e_j)\right)$$

$$= \mathbb{C}\text{ov}_j\Big(\frac{\upsilon_j}{\bar{\phi}}, \mathcal{D}_j(\mathcal{S})z_jF''(e_j)\Big) \lesseqgtr 0$$

with υ_j is the multiplier on the energy demand optimality condition: positive value implies that the planner would like to relax the constraint, increase the quantity e_i, lower the MPe_i, and conversely for negative values. I define $\widehat{\upsilon}_j = \upsilon_j/\bar{\phi}$ as the rescaled shadow value of country j's energy demand.

There is no aggregate distortion, only redistributive distortions across countries. This comes from the optimality of the tax t^{ε}, as $\mathbb{E}_j(\widehat{\upsilon}_j) = 0$, and this yields the last line as a covariance. How to determine its sign? In our North-South example, lower-income economies have energy demand more sensitive to price distortions since $z_iF''(e_i)$ relates to the energy share and demand elasticity:

$$\mathcal{D}_i(\mathcal{S})z_iF''(e_i) = -\frac{q^e}{e_i\sigma_i^e}(1-s_i^e) \qquad\qquad \Rightarrow \qquad\qquad \mathcal{D}_S(\mathcal{S})z_SF''(e_S) < \mathcal{D}_N(\mathcal{S})z_NF''(e_N)$$

where $s_i^e = \frac{e_iq^e}{y_i} < 1$ is the energy share in production and σ_i^e is country i's energy demand elasticity. The North relies "more" on energy – since $z_N > z_S$ implies that $e_N > e_S$: more productive countries have higher energy demand ceteris paribus. It would also be the case if energy is more substitutable in richer countries, i.e. large σ^e, and demand varies a lot with price. The covariance would then be negative: if the planner values more the production, high υ_j, of the most inelastic countries, lower $F''(e_i)$ then the tax would be lower. Moreover, that term is null if the energy demand/production function is constant return in energy such that $s_i^e = 1$, or if energy is perfectly substitutable $\sigma^e \to \infty$, or if we are in a representative agent economy $\mathcal{D}_N(\mathcal{S})z_NF''(e_N) = \mathcal{D}_S(\mathcal{S})z_SF''(e_S)$ and there is no heterogeneity in demand across countries.

Proposition 2 (**Second-Best Ramsey Policy with limited transfers**).
The optimal Second-Best carbon tax accounts for three distributional motives when setting a single uniform level*: (i) climate damage in the* ***Social Cost of Carbon (SCC)****, (ii)* ***Supply Redistribution*** *in energy markets through terms-of-trade and energy rents and (iii)* ***Demand Distortion*** *through distorted firms' energy choices. This includes redistribution motives due to the presence of inequality through the* ***social welfare weights*** $\widehat{\phi}_j = \phi_j/\bar{\phi} = \omega_jU'(c_j)/\frac{1}{I}\sum_i \omega_iU'(c_i)$. *The optimal energy tax writes:*

$$\mathrm{t}^{\varepsilon} = \textit{SCC} \;+\; \textit{Supply Redistribution} \;+\; \textit{Demand Distortion}$$

$$\mathrm{t}^{\varepsilon} = -\sum_j \widehat{\phi}_j\mathcal{D}'_j(\mathcal{S})z_jF(e_j) \;+\mathcal{C}_{EE}\,\frac{1}{I}\sum_j \widehat{\phi}_j(e_j - e_j^x) \;+\; \frac{1}{I}\sum_j \widehat{\upsilon}_j\mathcal{D}_j(\mathcal{S})z_jF''(e_j)$$

$$\mathrm{t}^{\varepsilon} = I\mathbb{E}_j\Big(\mathcal{S}\gamma_jy_j\Big) + I\mathbb{C}\text{ov}_j\Big(\widehat{\phi}_j, \mathcal{S}\gamma_jy_j\Big) + q^e\frac{\bar{\nu}}{E}\mathbb{C}\text{ov}_j\Big(\widehat{\phi}_j, e_j - e_j^x\Big) - q^e\,\mathbb{C}\text{ov}_j\Big(\widehat{\upsilon}_j, \tfrac{1-s_i^e}{\sigma_i^e e_i}\Big)\;,$$

for the carbon tax such that $MPe_i = c'(e^x) + \mathrm{t}^\varepsilon$, where $\gamma_i = -\frac{\mathcal{D}_i'(\mathcal{S})}{\mathcal{D}_i(\mathcal{S})\mathcal{S}}$ is the marginal damage of climate change[8], *$y_i = \mathcal{D}_i(\mathcal{S})z_iF(e_i)$ is total production, $\bar{\nu} = \left(\sum_j \lambda_j^x \nu_j^{-1}\right)^{-1}$ the average inverse energy supply elasticity, s_i^e the energy cost shares, and σ_i^e the energy demand elasticity. We see these three motives matter with a single tax and lump-sum rebate. Compared to the economy with full redistribution, the carbon tax is* ***smaller*** *if (i) the cost of climate $\gamma_j y_j$ is concentrated in richer countries, with low $\widehat{\phi}_i$, (ii) the net energy imports are high, with higher $e_i - e_i^x$, in richer, low $\widehat{\phi}_i$ countries, (iii) the energy is more essential, with low demand elasticity and high $\frac{1-s_e^i}{\sigma_i^e e_i}$, in poorer, high distortion $\widehat{\upsilon}_i$, countries. For (ii) and (iii), carbon taxation is isomorphic to an energy terms-of-trade manipulation between the exporters and the importers in trade theory.*

In the next corollary, we reexpress the carbon tax as a function of observable sufficient statistics. Indeed, given its dependence on the multipliers for individual demand υ_i, the Demand Distortion term can be quite opaque.

Corollary 3 (**Second-Best Ramsey Policy with limited transfers, sufficient statistics**). *The optimal Second-Best uniform carbon tax can be rewritten as:*

$$\mathrm{t}^\varepsilon = \frac{1}{1 + \mathbb{C}\mathrm{ov}_j\left(\widehat{\phi}_j, \widehat{e}_j^{s,\sigma}\right)}\left[SCC + \textit{Supply Redistribution}\right] \qquad \text{with} \qquad \widehat{e}_j^{s,\sigma} := \frac{\frac{\sigma_j^e e_j}{1-s_j^e}}{\sum_i \frac{\sigma_i^e e_i}{1-s_i^e}}$$

$$\mathrm{t}^\varepsilon = \frac{1}{1 + \mathbb{C}\mathrm{ov}_j\left(\widehat{\phi}_j, \widehat{e}_j^{s,\sigma}\right)}\left[I\mathbb{E}_j\left(\mathcal{S}\gamma_j y_j\right) + I\mathbb{C}\mathrm{ov}_j\left(\widehat{\phi}_j, \mathcal{S}\gamma_j y_j\right) + q^e\frac{\bar{\nu}}{E}\mathbb{C}\mathrm{ov}_j\left(\widehat{\phi}_j, e_j - e_j^x\right)\right]$$

for a uniform carbon tax t^ε, with marginal climate damage γ_i, output y_i, average inverse energy supply elasticity $\bar{\nu}$, energy cost shares s_i^e, energy demand elasticity σ_i^e. Note that $\widehat{e}_i^{s,\sigma} = \widehat{e}_i = e_i/E$ if the production function has a Cobb-Douglas form.[9] *Demand distortion amplifies or dampens the carbon taxation motives, i.e. the Social Cost of Carbon (SCC) and Supply Redistribution. The carbon tax is* ***lower*** *if the largest energy consumers e_i, with high energy share s_i^e and demand elasticity σ_i^e have low consumption c_i and thus high social welfare weights $\widehat{\phi}_j = \omega_j U'(c_j)/\frac{1}{I}\sum_i \omega_i U'(c_i)$.*

Note, in representative agent models, as in Nordhaus (2017), or Golosov et al. (2014), there is no heterogeneity across countries, making all the covariances trivially null. Hence, we obtain the standard Pigouvian result $\mathrm{t}^\varepsilon = SCC = I\mathbb{E}_j[\mathcal{S}\gamma_j y_j] = \overline{SCC}$. Moreover, models with unconstrained transfers, which can be aggregated, yield $\widehat{\phi}_j = \widehat{\phi}_i = 1$, also reducing these covariances to zero. Models with heterogeneity in income, climate damage, or country size, like Nordhaus and Yang (1996) or Krusell and Smith (2022), but no heterogeneity in energy demand nor energy rent redistribution also yield the Pigouvian result $\mathrm{t}^\varepsilon = SCC$, where the Social Cost of Carbon is adjusted for inequality.

These Second-Best carbon tax formulas hold for a single uniform carbon tax. If the planner has access to a distribution of carbon tax rates (or carbon prices), the *distribution* of the tax changes with the presence of inequality, as we see in the next section.

[8] The parameter γ_i is constant in the damage function used in the DICE model $\mathcal{D}_i(\mathcal{S}) = e^{-\gamma_i(\mathcal{S}-\mathcal{S}_0)^2}$.

[9] If all countries i have the same Cobb-Douglas production of the form $F(\ell_i, e_i) = e_i^\alpha \ell_i^{1-\alpha}$, we get $e_j^{s,\sigma} := \frac{\sigma_i^e e_i}{1-s_i^e} = \frac{1 \times e_i}{1-\alpha}$ and hence $\widehat{e}_j^{s,\sigma} := e_j^{s,\sigma}/\sum_i e_i^{s,\sigma} = e_i/E$

2.3 Ramsey Problem with heterogeneous carbon tax & limited transfers

We consider a case where the Social Planner implements a distribution of *country-specific* carbon taxes t_i^ε. I again assume away cross-country transfers, and the revenue of the carbon tax is rebated lump-sum $\mathrm{t}_i^{ls} = \mathrm{t}_i^\varepsilon e_i$. The welfare objective and the constraints internalized by the planner are the same as before: (i) the budget $\tilde{c}_i = \mathcal{D}_i(\mathcal{S}) z_i F(\tilde{e}_i) + q^e(\tilde{e}_i^x - \tilde{e}_i) - \mathcal{C}_i(\tilde{e}_i^x)$ with multiplier ϕ_i, (ii) the energy firm optimality $q^e = \mathcal{C}'(e_i^x)$, (iii) the energy demand optimality $q^e + \mathrm{t}_i^\varepsilon = MPe_i$ with multiplier υ_j, and (iv) the energy market clearing $E = \sum_i e_i = \sum_i e_i^x$. The only exception is that the carbon tax is country-specific t_i^ε. The planner optimality conditions become:

$$\omega_i U'(c_i) = \phi_i \qquad \qquad \widehat{\phi}_i = \frac{\phi_i}{\bar{\phi}} = \frac{\omega_i U'(c_i)}{\frac{1}{I}\sum_{i\in\mathbb{I}} \omega_i U'(c_i)} \lesseqgtr 1$$

the same as before, as the planner keeps the same motive for redistribution. The social welfare weights, or inequality factor, come for heterogeneity in the marginal value of income ϕ_i. However, when the planner can choose one instrument per country, the distortion of demand is absent:

$$\upsilon_j = 0 \qquad \Rightarrow \qquad \textit{Demand Distortion} = 0$$

Proposition 4 (Second-Best Ramsey Policy, heterogeneous taxes & limited transfers). *The optimal Second-Best energy taxation policy with heterogeneous taxes when transfers are absent becomes:*

$$\mathrm{t}_i^\varepsilon = \frac{1}{\widehat{\phi}_i} \underbrace{\sum_j \widehat{\phi}_j\big(- \mathcal{D}_j'(\mathcal{S}) z_j F(\tilde{e}_j)\big)}_{\propto \text{ SCC}} + \frac{1}{\widehat{\phi}_i} \underbrace{\mathcal{C}_{EE}\, \frac{1}{I}\sum_j \widehat{\phi}_j (e_j - e_j^x)}_{= \textit{Supply Redistribution}}$$

$$\mathrm{t}_i^\varepsilon = \frac{1}{\widehat{\phi}_i}\Big[SCC + \textit{Supply Redistribution}\Big]$$

$$\mathrm{t}_i^\varepsilon = \frac{1}{\widehat{\phi}_i}\Big[I\mathbb{E}_j\Big(\mathcal{S}\gamma_j y_j\Big) + I\mathbb{C}\mathrm{ov}_j\Big(\widehat{\phi}_j, \mathcal{S}\gamma_j y_j\Big) + q^e \frac{\bar{\nu}}{E}\,\mathbb{C}\mathrm{ov}_j\Big(\widehat{\phi}_j, e_i - e_i^x\Big)\Big]$$

where γ_i is the marginal damage of climate change, y_i is total output and $\bar{\nu}_i$ the average inverse energy supply elasticity, and $\widehat{\phi}_j = \phi_j/\bar{\phi} \propto \omega_j U(c_j)$ are the social welfare weights

The planner accommodates country-specific levels of inequality for the distribution of carbon prices. Indeed, for a given – potentially arbitrary – distribution of Pareto weights ω_i, the optimal carbon tax is relatively lower for poorer countries. The two motives for carbon taxation, (i) the Pigouvian Social Cost of Carbon and (ii) the supply redistribution, changing terms-of-trade and energy rent in general equilibrium, both need to be *discounted* by the country level of inequality $\widehat{\phi}_i \propto \omega_i U'(c_i)$. The tax is reduced for countries with low consumption – due to inherently low income (due to TFP) or climate damage – or high Pareto weight in the global welfare. Lastly, the energy demand is not affected by this country-specific tax.

These main findings – that the *level* and the *distribution* of carbon taxes change with inequality – are general and hold in a dynamic quantitative model that I develop in the next sections.

2.4 From carbon taxation to carbon pricing in emissions markets

In this section, I investigate how to implement the optimal climate policy when the planner chooses to design a "cap-and-trade" emission market, such as the European Union's "emission trading system (ETS)". Emissions markets are a privileged policy solution as they simultaneously provide the strict regulation of a cap and the market efficiency of emission trading.

Consider a social planner designing the cap-and-trade system, choosing the number of allowances, or "quotas" or "permits", $\overline{\mathcal{E}}$ to be auctioned in a world market. Moreover, it also chooses the number of permits that are given "for free" to each country $\overline{\varepsilon}_i$. As a result, it controls the total supply of permits $\overline{\mathcal{E}} + \sum_i \overline{\varepsilon}_i$, which are traded on a global market at price q^ε. Finally, the representative firm in each country chooses how many permits to purchase ε_i to cover its emissions from energy use. The country i's representative household/firm faces the following problem:

$$\max_{c_i, e_i, \varepsilon_i} U(c_i) \qquad s.t \qquad \begin{cases} c_i &= \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e(e_i^x - e_i) - \mathcal{C}_i(e_i^x) + q^\varepsilon(\overline{\varepsilon}_i - \varepsilon_i) + \mathrm{t}_i^{ls} \\ e_i &\leq \varepsilon_i \end{cases}$$

where the firm pay for ε_i carbon permits at price q^ε. This yields the optimality condition for energy e_i and carbon permits ε_i:

$$MPe_i = q^e + q^\varepsilon$$

which implies that the implicit carbon tax is $MPe_i - q^e = \tilde{\mathrm{t}}^\varepsilon = q^\varepsilon$. The complete treatment of this example is detailed in Appendix A.3. Below, I summarize several lessons on the optimal design of this type of policy.

First, the unconstrained distribution of "free carbon permits" $\overline{\varepsilon}_i$ acts as implicit money transfers across countries. Indeed, if the planner "offers" the ownership of carbon permits, countries/firms can sell them on the market to get $\overline{\varepsilon}_i q^\varepsilon$ and redistribute that money to households. Moreover, due to the government budget constraints, the revenue and cost of those permits are redistributed lump-sum to the household $\sum_i \mathrm{t}_i^{ls} = q^\varepsilon \sum_i(\varepsilon_i - \overline{\varepsilon}_i)$. If these implicit transfers are allowed, the planner can achieve full redistribution $\omega_i U'(c_i) = \omega_j U'(c_j)$. As a result, we can recover the First-Best allocation with $q^\varepsilon = SCC$, exactly as in Proposition 1 and Section 2.1.

Second, if we prevent both explicit and implicit transfers, i.e. $\bar{\varepsilon}_i = 0$, the planner again needs to adjust the carbon price for the presence of inequality, as represented by the social welfare weights $\widehat{\phi}_i = \omega_j U(c_j)/\frac{1}{I}\sum_i \omega_i U(c_i)$. This implies that the total quantity $\bar{\mathcal{E}}$ to be auctioned on a global market should be chosen to target a carbon price of q^ε as in the Second-Best in Section 2.2.

Corollary 5 (Global carbon price, cap-and-trade system with limited transfers). *The carbon price target on a global cap-and-trade emission market needs to target the following level, accounting for redistribution motives, exactly as in Proposition 2:*

$$\begin{aligned} q^\varepsilon &= SCC \ + \ \textit{Supply Redistribution} \ + \ \textit{Demand Distortion} \\ q^\varepsilon &= -\sum_j \widehat{\phi}_j \mathcal{D}_j'(\mathcal{S}) z_j F(e_j) \ + \mathcal{C}_{EE}\,\frac{1}{I}\sum_j \widehat{\phi}_j (e_j - e_j^x) \ + \ \frac{1}{I}\sum_j \widehat{\upsilon}_j \mathcal{D}_j(\mathcal{S}) z_j F''(e_j) \\ q^\varepsilon &= I\mathbb{E}_j\Big(\mathcal{S}\gamma_j y_j\Big) + I\mathbb{C}\mathrm{ov}_j\Big(\widehat{\phi}_j, \mathcal{S}\gamma_j y_j\Big) + q^e\frac{\bar{\nu}}{E}\mathbb{C}\mathrm{ov}_j\Big(\widehat{\phi}_j, e_j - e_j^x\Big) - q^e\,\mathbb{C}\mathrm{ov}_j\Big(\widehat{\upsilon}_j, \tfrac{1-s_i^e}{\sigma_i^e e_i}\Big)\,. \end{aligned}$$

Third, if the planner wants to achieve additional redistribution, it needs to design *segmented markets* with $I = \#\mathbb{I}$ different prices q_i^ε. In each market, the planner needs to set the supply $\bar{\mathcal{E}}_i = e_i$ to achieve the target prices $q_i^\varepsilon \equiv \mathrm{t}_i^\varepsilon = (1/\widehat{\phi}_i)(SCC + \textit{Supply redistribution})$, which are the same level as Proposition 4 in Section 2.3. In that context, the carbon permits/allowances are not tradeable across countries unless against the exchange rate $q_i^\varepsilon / q_j^\varepsilon$ for one ton of CO_2.

We see that the question of the number of instruments prevails over the *nature* of the instruments – price or quantity – as I explain in the next section.

2.5 Prices vs. Quantity

I now discuss the pros and cons of a global carbon tax or country-specific carbon taxes, or *price instruments*, in comparison to quantity targets in a cap-and-trade system, either with a global carbon budget or country-specific carbon targets. In that sense, I explore the arguments made in Weitzman (2003) and Weitzman (2015) on these two types of instruments.

Weitzman (2015) argues strongly against quantity targets and emissions cap-and-trade systems. He defends that a universal carbon tax ideally possesses (1) cost-effectiveness, (2) a natural one-dimensional *focal point* (as in Schelling), and (3) a built-in self-enforcement mechanism that internalizes the externality, balancing out countervailing forces and heterogeneous interests. According to him, n different quantity assignments, as in the top-down approach of the Kyoto Protocol, fail on the second and third points and are harder to bargain upon in international agreements. It spurs free-riding as each nation has a self-interest to negotiate a low cap on their own carbon emissions – lower than socially optimal and "disagreements over the subdivision of an aggregate world cap into n national quantity targets" prevent the achievement of an efficient solution.

I showed above that (i) the availability of transfers and (ii) the choice of instruments – whether a global policy or country-specific ones, are the most important determinants of the policy effectiveness, rather than the choice of quantity or price instruments. Indeed, as noted in Section 2.4, there is a direct mapping between a quantity of carbon permits $\bar{\mathcal{E}}$ along with a distribution of "free allowances" $\{\bar{\varepsilon}_i\}_i$ and a carbon tax t^ε along with cross-countries transfers t_i^{ls}, and both replicate the First-Best allocation. If, like Weitzman, we note that allocating carbon permits appears as "visible transfer payments across national borders", we could instead advocate for $\bar{\varepsilon}_i = 0$ to refrain from transfers that are undermined by free-riding incentives.

In that context, there is also a direct link between choosing a total quantity of permits $\bar{\mathcal{E}}$ and a single carbon price $q^\varepsilon = \mathrm{t}^\varepsilon$ that replicates Weitzman's idea of an "internationally harmonized but nationally retained carbon tax" exactly as detailed in Section 2.2, Proposition 2 and corollary 5. Regarding policy uncertainty and the volatility of carbon price in cap-and-trade systems for a given quantity target $\mathcal{E}$, one could note that policymakers could control the quantity of permits supplied and perform "open-market operations" to reach a stable carbon price target. Finally, country-specific quantity $\bar{\mathcal{E}}_i = e_i$ and country-specific carbon taxes $\{\mathrm{t}_i^\varepsilon\}$ both suffer from the same pitfalls because of free-riding, bargaining frictions, and transaction costs in international negotiations. These considerations are examined in Bourany (2024*a*).

2.6 Extensions and implication for carbon taxation

In the previous sections, I showed how the carbon tax should differ from the representative agent Pigouvian framework in the simplest model with heterogeneous countries. In Bourany (2024*a*), I analyze the case with two important extensions. First, international trade has redistributive "carbon leakage" effects that dampen the effectiveness of carbon policy. For example, imposing a carbon tax on energy in country i reallocates production toward other countries j that now may export toward i. In Bourany (2024*a*), I show that the presence of inequality and trade create a fourth motive that differentiates the optimal carbon tax from the social cost of carbon.

Second, the carbon policy is chosen by a social planner at the world level. However, free-riding – when individual countries deviate without implementing the socially optimal policies – create participation constraints that limit what can be achieved by the planner. In Bourany (2024*a*), I study how to optimally design climate agreements accounting for such free-riding incentives.

2.7 Climate Risk and Social Cost of Carbon

Until now, the present analysis is deterministic: both the future gains and losses of climate policies and climate change are known to all agents. In Bourany (2023), I study how uncertainty about future climate damage and economic opportunities affect carbon policies. I show a summary of those results here.

Let us consider an extension of the previous where the damage caused by climate change and productivity is affected by a random variable $\epsilon \in \mathcal{E}$ following the probability distribution $\epsilon \sim \varphi(\epsilon)$, such that the productivity becomes stochastic: $\mathcal{D}_i(\mathcal{S}|\epsilon)$. This is a simple example of "climate risk", where for a given amount of carbon emissions, the economic damage of climate change is unknown. The model is set up in two stages: (i) the agents and the planner take the energy – and thus emissions – decisions ex-ante, and (ii) the productivity/climate shock is realized and production and consumption $c_i(\epsilon)$ adjust ex-post to satisfy the budget and resources constraints.

In the competitive equilibrium, the representative agents maximize their expected utility:

$$\max_{e_i, e_i^x} \int_{\mathcal{E}} \max_{c_i(\epsilon)} U\big(c_i(\epsilon)\big) \; d\varphi(\epsilon)$$

In that case, the energy input decision is taken in expectation but does not imply fundamentally different choice. The optimality condition writes:

$$\int_{\mathcal{E}} MPe_i(\epsilon) \; d\varphi(\epsilon) = q^e$$

as before the households and firms do not internalize the climate externality caused by emissions.

I now consider the case of a social planner maximizing the world's welfare. In the two-step decision process, the planner would choose the decisions in energy inputs, extraction, and a uniform carbon tax t^ε for all countries:

$$\max_{\{e_j, e_j^x\}_j, q^e, \mathrm{t}^\varepsilon} \int_{\mathcal{E}} \max_{\{c_j(\epsilon)\}_j} \sum_{i \in \mathbb{I}} \omega_i U\big(c_i(\epsilon)\big) d\varphi(\epsilon)$$

As before, we consider the Social welfare weights, that are now adjusted for the ex-post allocation of risk:

$$\widehat{\phi}_i(\epsilon) = \frac{\phi_i(\epsilon)}{\mathbb{E}_{i,\epsilon}[\phi_i(\epsilon)]} = \frac{\omega_i U'(c_i(\epsilon))}{\frac{1}{I}\sum_i \int_\epsilon \omega_j U'(c_j(\epsilon))\ d\varphi(\epsilon)}$$

The planner's optimality condition for the energy choice gives us an analytical formulation for the carbon tax, which consist of four terms: the three terms we considered above: (i) the Social Cost of Carbon, (ii) the Supply redistribution, and (iii) the Demand Distortion, all of them adjusted for the presence of ex-post heterogeneity due to risk. There is also a new additional term: (iv) the effect of risk on the the energy choice. This last term correct for the redistributive motives caused by the presence of risk and is expressed as a covariance term: if the countries with the highest need for energy – i.e. with a high marginal product of energy – are also the poorest – with high social weights and marginal utility – this implies an energy subsidy ex-ante to provide redistribution in the absence of insurance markets.

$$\mathbb{E}_\epsilon\big(MPe_i(\epsilon)\big) = q^e + \mathbb{E}_\epsilon\Big[SCC(\epsilon) + \textit{Supply Redistribution}(\epsilon) + \textit{Demand Distortion}(\epsilon)\Big] - \underbrace{\mathbb{C}\text{ov}_\epsilon\Big(\widehat{\phi}_i(\epsilon), MPe_i(\epsilon)\Big)}_{\substack{\text{effect of agg. risk } \epsilon \\ \text{on energy choice}}}$$

The three terms seen above are taken in expectation and the carbon taxation accounts for the presence of risk. Take the example of the Social Cost of Carbon: following the approach above to decompose how the heterogeneity ex-ante and ex-post affect the level of the Pigouvian motive. The Social Cost of Carbon, in expectation $\mathbb{E}_\epsilon[SCC(\epsilon)]$ is written:

$$\begin{aligned}\mathbb{E}_\epsilon\big[SCC(\epsilon)\big] &= \sum_{i\in\mathbb{I}} \int_{\mathcal{E}} \widehat{\phi}_i(\epsilon) LCC_i(\epsilon)\, d\varphi(\epsilon) \\ &= \mathbb{E}_j\Big[\underbrace{\mathbb{C}\text{ov}_\epsilon\Big(\widehat{\phi}_i(\epsilon), LCC_j(\epsilon)\Big)}_{=\text{effect of aggregate risk } \epsilon}\Big] + \underbrace{\mathbb{C}\text{ov}_j\Big[\mathbb{E}_\epsilon\big(\widehat{\phi}_i(\epsilon)\big), \mathbb{E}_\epsilon\big(LCC_j(\epsilon)\big)\Big]}_{=\text{effect of heterogeneity across } j} + \underbrace{\mathbb{E}_{j,\epsilon}\big[LCC_j(\epsilon)\ \big]}_{=\text{average exp. damage}}\end{aligned}$$

The second and third terms are the same as in the previous sections: the presence of inequality implies a covariance term for *ex-ante heterogeneity* that can increase or decrease the carbon tax. However, this time, it is taken in expectations, for the local cost of carbon $\mathbb{E}_\epsilon\big(LCC_j(\epsilon)\big)$ and the social welfare weights $\mathbb{E}_\epsilon\big(\widehat{\phi}_i(\epsilon)\big)$. Similarly, the last terms is the average, ex-ante and ex-post, of the local cost of carbon.

The novelty is in the first term, representing the effect of risk, and hence *ex-post heterogeneity* on the cost of climate change. If the climate damages $LCC_i(\epsilon)$ are highest in the poorest countries, i.e. with the highest $\widehat{\phi}_i(\epsilon)$, the covariance is positive and it can then increases the SCC. This create insurance motives for climate policy, as the Social Cost of Carbon is heightened due to climate risk when the poorest countries are the most affected by climate change.

We will see that these general lessons also hold in a richer dynamic setting.

3 Quantitative model

I develop a neoclassical framework with rich heterogeneity across regions. Time is continuous $t \in [t_0, \infty)$, and the countries are indexed by $i \in \mathbb{I}$. They can be heterogenous in an arbitrary number of dimensions as summarized in the state variables s_{it}[10].

In each country, we consider five representative agents: (i) a household making consumption and saving decisions, (ii) a firm using capital, labor, and different energy sources to produce the final good, and three energy firms that (iii) extract fossil fuels (oil and gas), (iv) produce coal, and (v) produce renewable/non-carbon energy. Finally, each country has a government that collects taxes and redistributes lump-sum rebates.

3.1 Household

Each region $i \in \mathbb{I}$ is populated by a representative household of size $\mathcal{P}_{it}$ at time t. This population is increasing at an exogenous constant growth rate n_i, and $\dot{\mathcal{P}}_{it} = n_i \mathcal{P}_{it}$. As a result, the population is given as $\mathcal{P}_{it} = \mathcal{P}_{i0} e^{n_i t}$. The household owns all the different firms, including the representative firm that produces goods with total factor productivity z_{it}, which also grows with growth rate $\bar{g}_i$, implying $z_{it} = z_{i0} e^{\bar{g}_i t}$. In the tradition of the Neoclassical model, I normalize all the economic variables of the model per "effective capita", dividing by the trend $e^{(n_i + \bar{g}_i)t}$.

The household consumes the homogeneous final good c_{it} and is affected by the region's temperature τ_{it}. They can save and borrow in a liquid financial asset b_{it} at a world interest rate $r_t^\star$. Moreover, they can invest and hold that wealth in capital k_{it} to be rented to the homogeneous good producer at rate r_{it}^k. Households supply locally their inelastic labor $\bar{\ell}_i = 1$ to the final good firm, receiving the wage income $\bar{\ell}_i \mathrm{w}_{it}$. Moreover, the household receives the profit that the fossil firm generates π_i^f, as will be detailed below. They maximize the per-capita present discounted utility with discount rate ρ, and solve the following intertemporal problem.

$$\mathcal{V}_{it_0} = \max_{\{c_{it}, b_{it}, k_{it}\}} \int_{t_0}^{\infty} e^{-(\rho - n_i)t} \; u_i(c_{it}, \tau_{it}) \, dt \tag{6}$$

The utility that households receive from consumption is also scaled by a damage function $\mathcal{D}_i^u(\tau)$, which represents the direct impact of temperature τ_{it}. I consider standard CRRA preference with the intertemporal rate of substitution $1/\eta$.

$$u_i(c_{it}, \tau_{it}) = u\Big(\mathcal{D}_i^u(\tau_{it}) c_{it}\Big) \qquad\qquad u(\mathcal{D}\, c) = \frac{(\mathcal{D} c)^{1-\eta}}{1-\eta} \;. \tag{7}$$

We aggregate the bond and capital of the individual country as a single wealth variable $w_{it} = k_{it} + b_{it}$ and rescale income and wealth per effective unit of labor, accounting for TFP and

[10] More precisely, state variables of heterogeneity can be split in two, $s_{it} = \{\underline{s}_i, \bar{s}_{it}\}$, where $\underline{s}_i$ represents ex-ante heterogeneity and states variables $\bar{s}_{it}$ represent ex-post heterogeneity that changes over time. In practice, the dimensionality of $\underline{s}$ can be arbitrarily large, as I explain in the computational section below.

population growth $\bar{g}_i + n_i$, it yields the dynamics of wealth w_{it} accumulation:

$$\dot{w}_{it} = (r_t^\star - (n_i + \bar{g}_i))w_{it} + \mathrm{w}_{it}\bar{\ell}_i + \pi_{it}^f + \mathrm{t}_{it}^{ls} \ , \tag{8}$$

starting from initial condition $w_{t_0} = k_0 + b_0$. The return on capital is r_{it}^k is equalized to the bond return $r_{it}^k = r_t^\star$ in the absence of other financial market frictions. Furthermore, the household receives the energy sector profits, and, more specifically, the profit from the oil-gas firm π_{it}^f that generates meaningful energy rents. Finally, the household also receives lump-sum transfers t_i^{ls} from the government. Wealth w_{it} is the first dimension of ex-post heterogeneity.

3.2 Final good firm

In each country $i \in \mathbb{I}$, a representative firm produces a homogeneous final good using a constant-return-to-scale technology $F(\cdot)$ and different inputs: labor $\bar{\ell}_i$ at wage w_{it}, capital per effective capita k_{it} at the rental rate $r_t^\star$, and energy per effective capita e_{it} at price q_{it} as detailed below.[11] The firm maximizing profit, i.e. output per capita $y = \mathcal{D}^y(\tau)zF(\cdot)$ net of input costs:

$$\max_{k_{it}, e_{it}} \ \mathcal{D}_i^y(\tau_{it})z_iF_i(k_{it}, \bar{\ell}_i, e_{it}) - \mathrm{w}_{it}\bar{\ell}_i - q_{it}^e e_{it} - (r_t^\star + \delta)k_{it} \ . \tag{9}$$

The firm's productivity first differs across countries due to institutional and efficiency time-invariant factors summarized in z_i. Second, temperatures τ_{it} affect output through climate damages $\mathcal{D}_i^y(\tau_{it})$, which is the source of climate externality which will be detailed below. The production function has a constant elasticity of substitution between the capital-labor bundle $k^\alpha \ell^{1-\alpha}$ and energy e:

$$F_i(k_{it}, \ell_i, e_{it}) = \left[(1-\varepsilon)^{\frac{1}{\sigma}}\left(k_{it}^\alpha \ell_i^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}}\left(z_{it}^e e_{it}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\sigma < 1$, such as energy is complementary in production[12] and where directed technical change z_t^e is exogenous and deterministic. This directed – energy augmenting – technical change increases in output for a given energy consumption mix.[13] This exogenous trend implies $z_{it}^e = \bar{z}_i^e \, e^{g_e t}$.

The firm optimal inputs decision for capital, k_{it}, labor ℓ_i, and energy e_{it} is such that the marginal product of the inputs equals its price. With the marginal product of capital, labor and energy are defined as: $MPx_{it} = \partial_x[\mathcal{D}_i^y(\tau_{it})z_iF_i(k_{it}, \ell_i, e_{it})]$ for $x \in \{k, \ell, e\}$, we obtain the first-order conditions:

$$r_{it}^k = MPk_{it} - \delta = r_t^\star \qquad \mathrm{w}_{it} = MP\ell_{it} \qquad q_{it}^e = MPe_{it} \tag{10}$$

Energy demand

[11]The original – unnormalized – production function:

$$Y_t = \widetilde{F}(K_t, L_t, E_t) = \mathcal{D}(\tau_t)z_t\left[(1-\varepsilon)^{\frac{1}{\sigma}}\left(K_t^\alpha L_t^{1-\alpha}\right)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}}\left(z_t^e E_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

I normalize output Y_t by the trends in TFP $e^{\bar{g}_i t}$ and population $L_t \equiv \mathcal{P}_0 e^{n_i t}$ to obtain output per effective capita.

[12]If $\sigma = 1$, we have the Cobb Douglas : $F(k, \ell, e) = \bar{\varepsilon} z_t^{e\,\varepsilon}(k^\alpha \ell^{1-\alpha})^{1-\varepsilon}e^\varepsilon$

[13]An upward trend in such technology is sometimes argued to be behind the "relative decoupling" of developed economies: an increase in production and value-added simultaneous to a decline in energy consumption.

Given the demand for energy inputs e_t in each country, the firm has the choice among three sources of energy: fossil fuels e^f_{it}, coal e^c_{it} and low-carbon/renewables e^r_{it}. These three sources are substitutable, and total energy e_{it} has constant elasticity of substitution σ_e.

$$e_{it} = \left(\omega_f^{\frac{1}{\sigma_e}} (e^f_{it})^{\frac{\sigma_e - 1}{\sigma_e}} + \omega_c^{\frac{1}{\sigma_e}} (e^c_{it})^{\frac{\sigma_e - 1}{\sigma_e}} + \omega_r^{\frac{1}{\sigma_e}} (e^r_{it})^{\frac{\sigma_e - 1}{\sigma_e}}\right)^{\frac{\sigma_e}{\sigma_e - 1}}$$

subject to the budget for energy expenditures, which implies the price of the energy bundle q_{it}, for $\sigma_e \in (0, \infty)$:

$$q^e_{it} e_{it} = e^f_{it}(q^f_t + \xi^f \mathrm{t}^\varepsilon_{it}) + e^c_{it}(q^c_{it} + \xi^c \mathrm{t}^\varepsilon_{it}) + e^r_{it} q^r_{it}$$

$$q^e_{it} = \left(\omega_f (q^f_t + \xi^f \mathrm{t}^\varepsilon_{it})^{1-\sigma_e} + \omega_c (q^c_{it} + \xi^c \mathrm{t}^\varepsilon_{it})^{1-\sigma_e} + \omega_r (q^r_{it})^{1-\sigma_e}\right)^{\frac{1}{1-\sigma_e}}$$

where q^f_t is the international price of oil and gas, q^c_{it} the local price of coal energy, and q^r_{it} the local price of low-carbon energy. Similarly, the representative final good firm choose the energy inputs according to the first-order condition:

$$q^f_t + \xi^f \mathrm{t}^\varepsilon_{it} = MPe^f_{it} = q^e_{it}\ \omega_f^{\frac{1}{\sigma_e}} \left(\frac{e^f_{it}}{e_{it}}\right)^{-\frac{1}{\sigma_e}} \tag{11}$$

and similarly for the other energy inputs $q^c_{it} + \xi^c \mathrm{t}^\varepsilon_{it} = MPe^c_{it}$ and $q^r_{it} = MPe^c_{it}$.

Energy from oil-gas, e^f_i, and coal, e^c_i, differ from renewable in the sense that they emit greenhouse gases, with respective carbon concentration ξ^f and ξ^c, as we will see in Section 3.4. As a result, there is a motive for taxing oil, gas, and coal energy with the carbon tax $\mathrm{t}^\varepsilon_{it}$, which is a tax per ton of CO_2. I discuss the choice of this tax in the next sections.

3.3 Energy markets

The final good firm consumes three kinds of energy sources – oil and gas, coal, or renewable (non-carbon) – supplied by three representative energy firms in each country $i \in \mathbb{I}$. Oil and gas sources are traded internationally, while coal and renewable sources are both traded locally.

3.3.1 Fossil firm

A competitive energy producer extracts fossil fuels oil and gas e^x_i from its pool of resources $\mathcal{R}_{it}$. The energy is extracted with convex production cost $\nu^f_i(e^x_{it}, \mathcal{R}_{it})$, where these costs are paid in units of the final good, and the oil and gas are sold in international markets at a price q^f.

The fossil-fuel reserves $\mathcal{R}_{it}$ are depleted with extraction e^x_{it} such that $\dot{\mathcal{R}}_{it} = -e^x_{it}$. We assume that neither the fossil firms nor the social planners internalize the scarcity of these resources. Internalizing the resource depletion would imply a more involved dynamic Hotelling problem – with stock effects – with the Hotelling rent rising over time, dampening the extraction rate e^x_i. I suggest an extension in appendix XX to see how these motives would change the taxation of fossil fuel and carbon theoretically. However, in the interest of keeping the framework simple, I refrain from considering this extension in the quantitative analysis. Richer models developed in Bornstein, Krusell and Rebelo (2023), Heal and Schlenker (2019), and Asker, Collard-Wexler, De Canniere, De Loecker and Knittel (2024) study the dynamic aspects of the oil market and the considerations for carbon emissions.

As a result, the static maximization problem of the fossil firm is given by :

$$\begin{aligned} \pi_{it}^f &= \max_{e_{it}^x} q_t^f e_{it}^x - \nu_i^f(\mathcal{P}_{it} e_{it}^x, \mathcal{R}_{it})/\mathcal{P}_{it} \ , \\ \dot{\mathcal{R}}_{it} &= -\mathcal{P}_{it} e_{it}^x \end{aligned} \tag{12}$$

where $\nu_i^f(\mathcal{P}_{it} e_{it}^x, \mathcal{R}_{it})/\mathcal{P}_{it}$ is the extraction cost per capita, which is convex in e_{it}^x, and $\mathcal{R}_{it_0} = \mathcal{R}_{i0}$ the initial condition for reserves. Since the extraction costs are convex, the production function has decreasing return to scale.[14] As a result, a positive energy rent π_{it}^f exists, even if the competitive firm takes the fossil price q_t^f as given. Moreover, I abstract from market power in the oil market, for example with the OPEC as a cartel – even though this framework could easily allow for such an extension. Any sources of misallocation – in the sense of Hsieh and Klenow (2009) – are accounted for in the calibration of the cost function $\nu_i^f(\cdot)$ as we will see in the quantification Section 5. I consider a functional form for cost that yields isoelastic supply curves for fossil energy extraction.

$$\nu_i^f(e_{it}^x, \mathcal{R}_{it}) = \frac{\bar{\nu}_i}{1+\nu_i} \Big(\frac{e_{it}^x}{\mathcal{R}_{it}}\Big)^{1+\nu_i} \mathcal{R}_{it} \tag{13}$$

which is homogeneous of degree one in $(e_i^x, \mathcal{R}_i)$ and where the elasticity $\nu_i = \frac{\nu_i^{f\,''}(e^x,\mathcal{R})}{\nu_i^{f\,'}(e^x,\mathcal{R})e^x}$ is constant.

Naturally, the optimal extraction decision for the fossil firm follows from the optimality condition:

$$q_t^f = \nu_{i\,e^x}^f(\mathcal{P}_{it} e_{it}^x, \mathcal{R}_{it}) = \bar{\nu}_i \Big(\frac{\mathcal{P}_{it} e_{it}^x}{\mathcal{R}_{it}}\Big)^{\nu_i} \tag{14}$$

which yields the implicit function $e_{it}^{x\star} = e_i^x(q_t^f) = \nu_i^{f\prime\,-1}(q_t^f) = \mathcal{R}_{it}\ (q_t^f/\bar{\nu}_i)^{1/\nu_i}$ for the optimal oil and gas extraction.

Finally, energy rent comes from fossil firms' profits $\pi^f(q^f, \mathbb{P}_i) = q^f e^x(q^f) - \nu_i^f(e^x(q^f), \mathcal{R}) > 0$, and the per-capita profit function writes:

$$\pi_i^f(q_t^f, \mathcal{R}_{it}) = q_t^f e_{it}^x - \nu_i^f(\mathcal{P}_{it} e_{it}^x, \mathcal{R}_{it})/\mathcal{P}_{it} = \frac{\nu_i \bar{\nu}_i}{1+\nu_i} \Big(\frac{\mathcal{P}_{it} e_i^x}{\mathcal{R}_i}\Big)^{1+\nu_i} \frac{\mathcal{R}_{it}}{\mathcal{P}_{it}} = \frac{\nu_i \bar{\nu}_i^{-1/\nu_i}}{1+\nu_i} \mathcal{R}_{it} (q_t^f)^{1+\frac{1}{\nu_i}} \ . \tag{15}$$

As we will see below, the profit $\pi_i^f(q^f, \mathcal{R})$ and its share in income $\eta_{it}^{\pi f} = \frac{\pi_{it}^f}{y_i + \pi_{it}^f}$ are key to determine the exposure of a country to carbon taxation. Indeed, reducing carbon emissions by phasing out of fossil fuels reduces energy demand and its price q^f and hence affects energy profit π_i^f and the welfare of large oil and gas exporters.

[14] We can also define a fossil production function with inputs x_i^f such that $e^x = g(x_i^f)$ and profit $\pi = q^f g(x) - x$ instead of $\pi = q^f e^x - \nu(e^x)$, in which case $g(x) = \nu^{-1}(x)$

3.3.2 International fossil energy markets

Oil and gas are traded frictionlessly in international markets.[15] The market clears such that

$$E^f_{it} = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\, e^f_{it} = \sum_{i\in\mathbb{I}} e^{(n_i+\bar{g}_i)t} e^x_{it} \ . \tag{16}$$

Countries have different exposure to this fossil energy market. As country i consumes fossil fuels in total quantity $\mathcal{P}_{it}e^f_{it} = \mathcal{P}_i e^{(n_i+\bar{g}_i)t} e^f_{it}$, and produces total quantity $\mathcal{P}_{it}e^x_{it} = \mathcal{P}_i e^{(n_i+\bar{g}_i)t} e^x_{it}$, its net exports of oil and gas per effective capita are $e^x_i - e^f_i \lesseqgtr 0$.

3.3.3 Coal firm

A representative firm produces coal that is consumed by the final good firm. I differentiate coal from other fossil fuels like oil and gas because coal production typically does not generate large energy rents for producing countries as a share of GDP. Moreover, large coal producers also consume a large fraction of that coal locally, as trade costs for coal transportation are larger. Hence, I make this empirically grounded assumption that coal is not traded internationally. Moreover, I assume that production is not subject to the finiteness of the stock of reserves. Indeed, the scarcity of coal sources is not a concern since the ratio of reserve/production is above a hundred for most large world producers.

The production $\bar{e}^c_{it}$ has constant returns to scale and uses final good inputs. The profit maximization problem is analogous to the fossil problem:

$$\pi^c_{it} = \max_{\bar{e}^c_{it}} q^c_i \bar{e}^c_{it} - \kappa^c_i \bar{e}^c_{it} \ ,$$

where the marginal cost κ^c_i is constant. This implies that there is no coal profit[16] in equilibrium, i.e. $\pi^c_{it} = 0$. The price for coal and the market clearing condition are given by:

$$q^c_i = \kappa^c_i \ , \qquad\qquad \bar{e}^c_i = e^c_i \ . \tag{17}$$

This implies a perfectly elastic supply curve for coal energy, something we observe in practice as coal production is easily scalable in response to oil and gas price fluctuations.

3.3.4 Low-carbon, renewable, firm

The final good firm also uses renewable and other low-carbon energy sources, such as solar, wind, or nuclear electricity. This provides a way of substituting away from fossil fuels in the production function $F(\cdot)$.

A representative firm produces renewable or non-carbon energy, and this supply, $\bar{e}^r_{it}$, is not traded. This assumption is verified by the fact that electricity is rarely traded across countries

[15] For the sake of simplicity, I make the simplifying assumption that fossil fuels produced in different countries are not distinguishable – crude oil or natural gas from Nigeria, Saudi Arabia, or Russia are not differentiated varieties.

[16] This is motivated by evidence that even the largest coal producers do not have coal rents above 1% of GDP.

– and when it is, it is only the result of temporary differences in electricity production due to intermittency rather than large structural imbalances. The production $\bar{e}^r_{it}$ also has constant returns to scale, and this input is paid in units of the final good. Hence, the renewable firm maximization problem is:

$$\pi^r_{it} = \max_{\bar{e}^r_{it}} q^r_{it}\bar{e}^r_{it} - \kappa^r_{it}\bar{e}^r_i \ ,$$

where κ^r_{it} is the marginal cost of producing renewables, resulting in zero profits $\pi^r_i = 0$. The price of renewable and the market clearing are given by:

$$q^r_{it} = \kappa_{it} = \bar{\kappa}_i e^{-g_r t} \ , \qquad\qquad e^r_{it} = \bar{e}^r_{it} \tag{18}$$

where I assume that the marginal cost κ_{it} decreases exogenously at rate g_r such that $\kappa_{it} =$ is lower over time. Given those marginal costs, this returns a perfectly elastic supply curve. This is a slightly stronger assumption in the context of renewable energy: In the short run, renewable energy requires investments in capacity, implying a fairly inelastic supply curve. This is especially true considering the intermittency problems of wind and solar energy, c.f. Gentile (2024). I take the conservative assumption that the supply curve is flat in the medium run.

However, in the long run, technological progress and learning-by-doing create positive externalities, substantially decreasing the cost of clean energy, resulting in a decreasing supply curve. To allow this learning-by-doing effect, as in Arkolakis and Walsh (2023), I consider an extension where the marginal cost depends on power capacity:

$$q^r_{it} = \kappa_{it} = \bar{\kappa}_i \left(\mathcal{C}^r_{it}\right)^{-\kappa^r} \qquad \text{where} \qquad \dot{\mathcal{C}}^r_{it} = \max\{\dot{\bar{e}}^r_{it}, 0\}^{\gamma_r} \tag{19}$$

where the renewable capacity $\mathcal{C}^r_{it}$ increases for each additional renewable production $\bar{e}^r_{it}$. The capacity of production, in kW, increases for each additional kWh of production needed $\bar{e}^r_{it}$ at each point in time, and this by a factor γ_r, and decreases production costs by a factor κ^r. The parameter $\gamma_r \in (0, \infty)$ represents a *learning-by-doing* factor. Indeed, $\mathcal{C}^r_{it} = \int_{t_0}^t |\dot{\bar{e}}^r_{it}|^{\gamma_r} dt$, and if $\gamma_r > 1$ capacity increases more than one for one due to increasing return to scale. Note that when $\gamma_r = 1$, and $\dot{\bar{e}}^r_{it} > 0, \forall t$, we simply get that $\mathcal{C}^r_{it} = \bar{e}^r_{it}$, i.e. the capacity is exactly what is needed for production. This would imply a downward-sloping supply curve with (negative) supply elasticity $-\kappa^r$. I assume that neither the firm nor the social planner internalizes this learning-by-doing externality. The study of the optimal policy in the presence of both negative climate externalities and positive Schumperian externalities is beyond the scope of this paper.

3.4 The climate system

The economic activity and fossil fuel consumption of each country create a climate externality by emiting carbon in the atmosphere. This feeds back in the climate system, which increases the temperatures and causes heterogeneous damages over different regions. As in standard Integrated Assessment Models (IAM), it creates a Pigouvian motive for carbon taxation as summarized by the Social Cost of Carbon.

3.4.1 Emissions

The consumption of fossil fuels are emitting carbon dioxide (CO_2) and other greenhouse gas emissions in the atmosphere. Due to oil and gas e^f_{it} and coal e^c_{it} consumptions – expressed in unit per effective capita, subject to growth rate of population n_i and TFP $\bar{g}_i$– we obtain that each country $i \in \mathbb{I}$ releases total CO_2 emissions:

$$\epsilon_{it} = \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\left(\xi^f e^f_{it} + \xi^c e^c_{it}\right) ,$$

where ξ^f and ξ^c denote the carbon content of respectively oil-gas and coal energy. As a result, global emissions aggregate to:

$$\mathcal{E}_t = \bar{\xi}_t \sum_{i\in\mathbb{I}} \epsilon_{it} .$$

I consider that emissions are non-exploding and I follow Krusell and Smith (2022) by assuming that part of emissions $\mathcal{E}_t$ is abated via carbon capture and storage (CCS) modeled by the exogenous parameter $\bar{\xi}_t$, with $\bar{\xi}_{t_0} = 1$. The share of emissions abated grows to 100% in the long-run, implying that $\bar{\xi}_t \to_{t\to\infty} 0$. Increasing CCS allows the system to reach net-zero in several centuries, stabilizing cumulative carbon emissions and temperatures.

3.4.2 Climate system and temperature

Moreover, these emissions are released in the atmosphere, adding up to the cumulative stock of greenhouse gas $\mathcal{S}_t$ – or atmospheric carbon concentration:

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t . \tag{20}$$

A part of these emissions exit the atmosphere and is stored in oceans or the biosphere, discounting the current stock by an amount δ_s. Moreover, these cumulative emissions push the global atmospheric temperature $\mathcal{T}_t$ upward linearly with climate sensitivity χ, with some inertia and delay represented by the parameter ζ.

$$\dot{\mathcal{T}}_t = \zeta(\chi\mathcal{S}_t - (\mathcal{T}_t - \mathcal{T}_{t_0})) . \tag{21}$$

More particularly, the inertia ζ is the inverse of persistence, and modern calibrations set $\zeta \approx 0.5$ is such that the pick of emissions happens after $10 - 15$ years. Dietz et al. (2021) show that classical IAM models such at Nordhaus' DICE tend to generate a too large climate system inertia, as shown in the Figure 2. Conversely, if $\zeta \to \infty$, temperature reacts immediately and we obtain a linear model – which is a good long-run approximation:

$$\mathcal{T}_t = \bar{\mathcal{T}}_{t_0} + \chi\mathcal{S}_t = \bar{\mathcal{T}}_{t_0} + \chi \int_{t_0}^{t} e^{-\delta_s s}\bar{\xi}_s \mathcal{E}_s \, ds$$

This simple two-equations climate system is a good approximation of large-scale climate

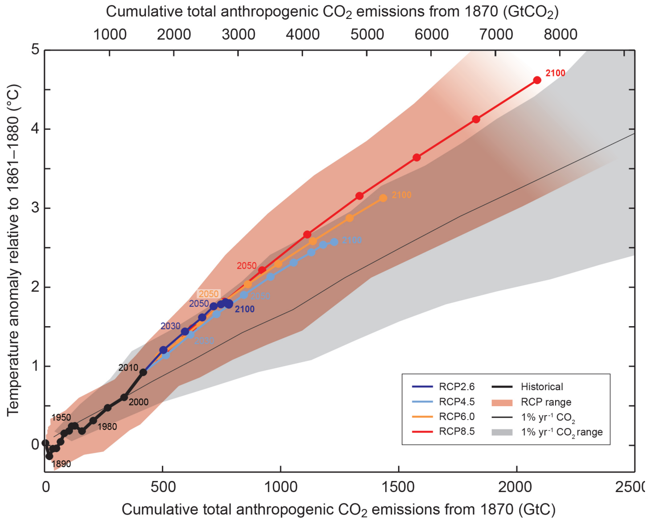


Figure 1: Linear temperature model - IPCC report

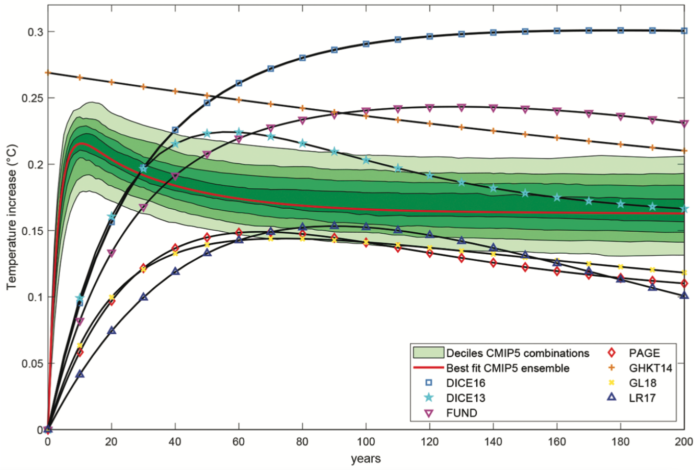


Figure 2: Pulse experiment

models.[17] Indeed, with the appropriate calibration of parameters δ_s for carbon exit, ζ for climate system inertia, and χ for the climate sensitivity, we can match these larger models as represented by the pulse experiment as shown in Figure 2.

3.5 Damage and externality

Climate damages are related to local temperatures: warmer regions are more affected and vulnerable to extreme events and impacts.

The temperature in country i is affected by global warming of the atmosphere $\mathcal{T}_t$ with linear pattern scaling Δ_i

$$\begin{aligned} \dot{\tau}_{it} &= \Delta_i \dot{\mathcal{T}}_t \\ \tau_{it} &= \tau_{i0} + \Delta_i (\mathcal{T}_t - \mathcal{T}_{t_0}) \end{aligned} \tag{22}$$

Atmospheric temperature $\mathcal{T}_t$ translates into local temperature τ_{it} via the sensitivity Δ_i that depends on the geographic properties of country i – like temperature, latitude, longitude, elevation, distance from coasts and water bodies, vegetation, and albedo (sunlight reflexivity due to ice, vegetation and soil properties). Evidence of this temperature scaling is displayed in Section 3.5 from the IPCC report. As shown in Section 5, I estimate this pattern scaling by regressing local temperatures on global temperature.

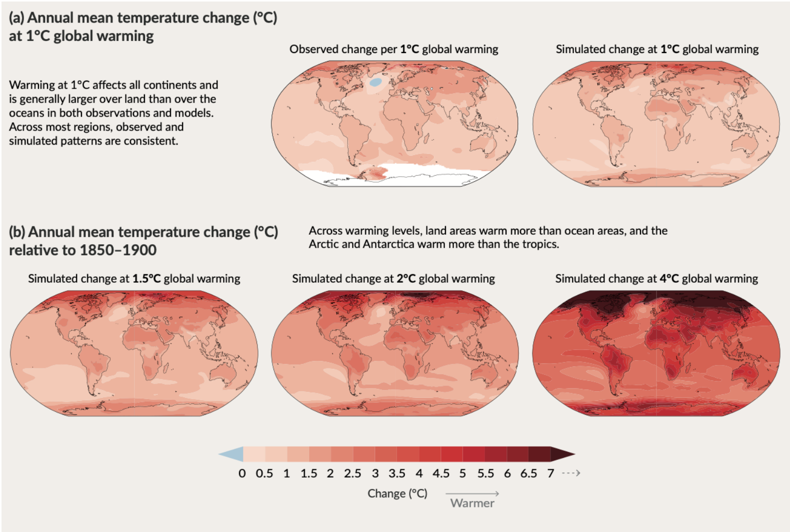
Finally, I consider a period damage function $\mathcal{D}^y_i(\tau_{it}) := \mathcal{D}^y(\tau_{it} - \tau^\star_i)$ for productivity and $\mathcal{D}^u(\tau_{it} - \tau^\star_i)$ for damage to utility. The target $\tau^\star_i$ is the "optimal" temperature for country i. The function $\mathcal{D}^y(\hat{\tau})$ is a reduced-form representation of the economic damage. In the baseline

[17] These climate models have a much more complex climate block, adding 3 to 4 additional state variables, e.g. with $\mathbf{J}$ the vector of carbon "boxes": layers of the atmosphere and sinks such as layers of oceans:

$$\dot{\mathbf{J}}_t = \Phi^J \mathbf{J}_t + \rho^S \sum_{\mathbb{I}} \epsilon_{it}$$

$$F_t = \mathcal{F}\big(\rho^F \mathbf{J}_t\big) \qquad \dot{\mathcal{T}}_t = \Phi^T \mathcal{T} + \rho^F F_t$$

with F_t carbon forcing with $\mathcal{F}(\cdot) \sim \log(\cdot)$, ρ^S, ρ^F, and ρ^T are vectors are Φ^J and Φ^T Markovian transition matrices.



quantification, I assume damages are quadratic, as in standard Integrated Assessment Models such as the DICE framework. This methodology follows Krusell and Smith (2022), Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021) and Burke et al. (2015). Moreover, those damages are such that productivity decays to zero when temperatures are extremely cold or hot.

$$\mathcal{D}_i^y(\tau) = \mathcal{D}^y(\tau - \tau_i^\star) = \exp\left(- \gamma^y \mathbb{1}_{\{\tau > \tau_i^\star\}} (\tau - \tau_i^\star)^2 - \alpha^\gamma \gamma^y \mathbb{1}_{\{\tau < \tau_i^\star\}} (\tau - \tau_i^\star)^2 \right) , \tag{23}$$

where γ_y represents the damage parameter on output for warm temperatures, with an asymmetric impact $\alpha^\gamma \gamma^y < \gamma^y$ for cold temperatures following the quantification in Rudik et al. (2021), who show that productivity impact is much weaker for cold than for hot temperatures. The damage function for utility $\mathcal{D}_i^u(\tau)$ has the same functional form with damage γ^u.

This creates winners and losers: countries warmer than their target temperature $\tau_i^\star$ are extremely affected by global warming. In contrast, regions with negative $\tau_{it} - \tau_i^\star$ benefit – at least in the short-run – from a warmer climate. I deviate from the above articles by assuming that the target temperature $\tau_i^\star$ differs across countries: an already warm regions have different adaptation costs compared to a country which is historically cold. The target temperature $\tau_i^\star = \alpha^\tau \tau^\star + (1-\alpha^\tau)\bar{\tau}_{it_0}$ is more or less tilted toward historical baseline. I discuss this quantification in Section 5.

3.6 Competitive Equilibrium

The final good is freely traded, and, with output y_i, the market clearing holds:

$$\sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \Big[c_{it} + (\dot{k}_{it} + (n_i+\bar{g}_i+\delta)k_{it}) + \nu_i^f(e_{it}^x, \mathcal{R}_{it}) + \kappa_i^c e_{it}^c + \kappa_{it}^r e_{it}^r \Big] = \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, \ell_i, e_{it}) \,. \tag{24}$$

Similarly, the bond market, in zero net supply, clears[18] such that $\sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t} b_{it} = 0$

Definition. Competitive equilibrium (C.E.):

[18] This is also equivalent to $\sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t} w_{it} = \sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t} k_{it}$.

For a set of policies $\{\mathrm{t}^{\varepsilon}_{it}, \mathrm{t}^{ls}_{it}\}_{it}$ across countries, a C.E. is a set $\{c_{it}, b_{it}, k_{it}, e^{f}_{it}, e^{c}_{it}, e^{r}_{it}, e^{x}_{it}, \bar{e}^{c}_{it}, \bar{e}^{r}_{it}\}_{it}$ of decisions, and prices $\{r^{\star}_{t}, q^{f}_{t}, \mathrm{w}_{it}, q^{c}_{it}, q^{r}_{it}\}_{it}$, and states $\{\omega_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_{t}, \tau_{it}\}_{it}$ such that:

(i) Households choose consumption, saving and investment, $\{c_{it}, k_{it}, b_{it}\}_{it}$ maximizing utility as in equation (6) subject to the budget constraint and wealth dynamics equation (8).

(ii) Final good firms choose inputs $\{k_{it}, \ell_{i}, e_{it}, e^{f}_{it}, e^{c}_{it}, e^{r}_{it}\}_{it}$ to maximize profits, resulting in input choices following equation (11) and equation (10).

(iii) Fossil energy firms maximize profits as in equation (12) and extract/produce $\{e^{x}_{i}\}_{i}$ given by equation (14)

(iv) Renewable and coal energy firms maximize profits, and supplies $\{\bar{e}^{c}_{i}, \bar{e}^{r}_{i}\}$ are given respectively by equation (17) and equation (18)

(v) Energy markets clears for fossils as in equation (16) and for coal and renewable in equation (17) and equation (18)

(vi) The emissions $\mathcal{E}_{t} = \sum_{i} \epsilon_{it}$ affects the climate system $\{\mathcal{S}_{t}, \mathcal{T}_{t}, \tau_{it}\}_{it}$, following equation (20), equations (21) and (22).

(vii) Good markets clear for final good for each country as in equation (24), and bond market clear by Walras law.

Heterogeneity. This model features many dimensions of heterogeneity, that can be summarized by the state variable $s_{it} = \{\underline{s}_{i}, \bar{s}_{it}\}$, describing the ex-ante dimensions of heterogeneity $\underline{s}_{i}$ – differences across countries that do not change over time – and ex-post heterogeneity $\bar{s}_{it}$ that change endogeneously over time. The states are: $\underline{s}_{i} = \{\mathcal{P}_{i}, n_{i}, \bar{g}_{i}, z_{i}, z^{e}_{i}, \bar{\nu}_{i}, \nu_{i}, \bar{\kappa}^{c}_{i}, \bar{\kappa}^{r}_{i}, \Delta_{i}, \tau^{\star}_{i}, \tau_{i0}, \omega_{i0}, \mathcal{R}_{i0}\}$ and $\bar{s}_{it} = \{\omega_{it}, \tau_{it}, \mathcal{R}_{it}\}$. For solving the model, we need to keep track of this dynamical system with heterogeneity, as we see next.

3.7 Sequential formulation and household decisions

First, since the Household owns the four firms – final good, fossil, coal, and renewable energy – we can aggregate profits and household budget constraint, which gives:

$$\begin{aligned} \dot{\omega}_{it} = (r^{\star}_{t} - (n_{i}+\bar{g}_{i}))\omega_{it} + \pi^{f}_{i}(q^{f}_{t}, \mathcal{R}_{it}) + \mathcal{D}^{y}_{i}(\tau_{it}) z_{it} F(k_{it}, \ell_{i}, e^{f}_{it}, e^{c}_{it}, e^{r}_{it}) - (r^{\star} + \delta) k_{it} \\ - (q^{f}_{t} + \xi^{f} \mathrm{t}^{\varepsilon}_{it}) e^{f}_{it} - (q^{c}_{it} + \xi^{c} \mathrm{t}^{\varepsilon}_{it}) e^{c}_{it} - q^{r}_{it} e^{r}_{it} - c_{it} + \mathrm{t}^{ls}_{it} \ , \end{aligned} \tag{25}$$

and yields a single optimal control problem. The consumption/saving relates to the path of wealth ω_{it}, given that the firms decisions, given by the optimality conditions

To solve for the competitive equilibrium and the optimal decision of the Household, we solve this class of Integrated Assessment Model with the sequential formulation of optimal control problem. This relies on the Pontryagin Maximum Principle, which can be applied in heterogeneous agents settings – with discrete agents in our case, or continuous agents/Mean-Field Games, as in Carmona and Delarue (2018). The household in each country has individual states $\mathbf{s} = \{\underline{s}_{i}, \bar{s}_{it}\}_{it} = \{\underline{s}_{i}, \omega_{it}, \tau_{it}, \mathcal{R}_{it}\}_{it}$, individual controls, $\mathbf{c} = \{c_{it}, b_{it}, k_{it}, e^{f}_{it}, e^{c}_{it}, e^{r}_{it}, e^{x}_{it}\}_{it}$, take prices as given $\mathbf{q} = \{r^{\star}_{t}, q^{f}_{t}, \mathrm{w}_{it}, q^{c}_{it}, q^{r}_{it}\}_{it}$, and has costates or Lagrange multipliers, $\boldsymbol{\lambda} = \{\lambda^{\omega}_{it}, \lambda^{\tau}_{it}, \lambda^{S}_{it}\}$, each of which represents shadow value of the respective states dynamics. The Hamiltonian of the individual country can be written as follow:

$$\mathcal{H}(\mathbf{s}, \mathbf{c}, \mathbf{q}, \boldsymbol{\lambda}) = u(c, \tau) + \lambda^w \dot{w} + \lambda^\tau \dot{\tau} + \lambda^S \dot{\mathcal{S}}$$

for the dynamics $\dot{w}_{it}$ given in equation (25), $\dot{\tau}_{it}$ given by equation (22) and $\dot{\mathcal{S}}_t$ given by equation (20).

As a result, the equilibrium relations for the household consumption/saving problem boil down to the standard neoclassical model dynamics and, for each country $i \in \mathbb{I}$, we obtain a system of coupled ODEs:

$$\begin{cases} \dot{\lambda}^w_{it} &= \lambda^w_{it}(\rho + \eta \bar{g}_i - r^\star_t) \\ \lambda^w_{it} &= u_c(c_{it}, \tau_{it}) \end{cases} \tag{26}$$

where λ^w_{it} is the costate, or "marginal value" of wealth w_{it}, equal to marginal utility $u_c(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it}) u'(\mathcal{D}^u(\tau_{it}) c_{it})$. Using the optimality for c, we obtain the Euler equation:

$$\frac{\dot{c}_{it}}{c_{it}} = \frac{1}{\eta} \left(r^\star_t + \eta \bar{g}_i - \rho \right) + \gamma^c (\tau_{it} - \tau^\star_i) \dot{\tau}_{it}$$

The dynamics of local temperature appear in the Euler equation. Indeed, because the marginal utility of consumption is affected directly by changes in temperature, an increase in temperature in the future triggers substitution from present to future consumption through saving.

To close the control problem, note that the household income is determined by the firms decisions. There, the capital and energy choices simply result from static optimization between the price or cost and the marginal return of those inputs in production. However, the climate variables affect damages, and the country i household internalizes that under the "local social cost of carbon" as we will see now.

3.8 Social and Local Cost of Carbon

The Social Cost of Carbon (SCC) is a measure used by climate scientists and economists to summarize the marginal welfare cost of climate change in monetary terms. The cost of carbon is an equilibrium concept: it depends on the path of temperatures but also on economic variables and policies. In the competitive equilibrium, the climate externality is not internalized and households and firms do not take climate damages into account for choosing consumption, production, and energy decisions. Still, forward-looking agents anticipate perfectly the evolution of climate.

The Local Cost of Carbon (LCC) represents such a welfare valuation, for the cost incurred by country i of one additional ton of CO_2 released in the atmosphere. In continuous time, and using our the Pontryagin Maximum Principle sequential approach, the Local Cost of Carbon (LCC) can be written easily as the ratio of the two costates:

$$LCC_{it} := -\frac{\frac{\partial \mathcal{V}_{it}}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{V}_{it}}{\partial c_{it}}} = -\frac{\lambda^S_{it}}{\lambda^w_{it}} \ . \tag{27}$$

The welfare cost of carbon λ^S_{it} represents the marginal welfare change from an additional ton of carbon $\mathcal{S}_t$. This is normalized in monetary units with the marginal value of wealth, as indeed the monetary value of welfare differs across regions i, $\frac{\partial \mathcal{V}_{it}}{\partial c_{it}} = \lambda^w_{it} = u_c(c_{it}, \tau_{it}) \neq u_c(c_{jt}, \tau_{jt}) = \lambda^w_{jt}$, due to inequality in consumption. This notion is exactly analogous to the Local Cost of Carbon concept developed in Cruz and Rossi-Hansberg (2022*a*), among many others.

As a result, following the dynamics of the LCC amounts to solve for the dynamics of both costates λ^w_{it} and λ^S_{it}. Recalling the dynamics of the climate system:

$$\begin{cases} \mathcal{E}_t &= \sum_{i\in\mathbb{I}} \epsilon_{it} = \sum_{\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}(\xi^f e^f_{it} + \xi^c e^c_{it}) \\ \dot{\mathcal{S}}_t &= \mathcal{E}_t - \delta_s \mathcal{S}_t \\ \dot{\tau}_{it} &= \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big) \end{cases}$$

we can use the Pontryagin principle to pin down the dynamics of the local cost of carbon. First, the shadow value of increasing temperatures is affected by the cost of climate on both the productivity effect $\mathcal{D}^y(\tau)zF(k,e)$ and the utility effect $u(\mathcal{D}^u(\tau)c)$.

$$\dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + \underbrace{\gamma^y(\tau_{it} - \tau^\star_i)\mathcal{D}^y_i(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} z_i F(k_{it}, e_{it})\lambda^w_{it} + \underbrace{\gamma^u(\tau_{it} - \tau^\star_i)\mathcal{D}^u_i(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau_{it})c_{it})c_{it} \ . \tag{28}$$

Indeed, this shadow value increases with marginal damages, scaled by both marginal utility of wealth λ^w_{it} and consumption $u'(\mathcal{D}^u(\tau)c_{it})$. This change in the marginal value of temperature affects directly the shadow value of adding carbon in the atmosphere according to the dynamics of λ^S_{it}:

$$\dot{\lambda}^S_{it} = \lambda^S_{it}(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it} \ . \tag{29}$$

Emitting carbon in the atmosphere has a differential marginal impacts across regions due to heterogeneous costs of temperature and vulnerability to climate synthesized by the pattern scaling Δ_i and marginal damages $\gamma^y(\tau_{it} - \tau^\star_i)$ parameters. Solving the differential equations analytically, we can obtain the general formula – as found in Appendix B. When climate inertia is null $\zeta \to \infty$, this rewrites:

$$\lambda^S_{it} \xrightarrow[\zeta\to\infty]{} -\int_t^\infty e^{-(\rho-n_i-(1-\eta)\bar{g}_i+\delta_s)(s-t)} \chi\Delta_i(\tau_{is} - \tau^\star_i)\big[\gamma^y y_{is} + \gamma^u c_{is}\big]\lambda^w_{is} ds \ ,$$

where output is $y_{it} = z_i\mathcal{D}^y_i(\tau_{it})F(k_{it}, e_{it})$ and $\lambda^w_{it} = \mathcal{D}^u_i(\tau_{it})\,u'\big(\mathcal{D}^u_i(\tau_{it})c_{it}\big)$. Using the Euler equation, or costate dynamics of equation (26), we get $\lambda^w_{it} = \lambda^w_{is} e^{-\int_t^s(\rho+\eta\bar{g}_i - r^\star_s)du}$ for $s > t$, which gives the Local Cost of Carbon:

$$LCC_{it} \to \int_t^\infty e^{-\delta_s(s-t) - \int_t^s (r^\star_u - n_i - \bar{g}_i)du} \chi\Delta_i(\tau_{is} - \tau^\star_i)\big[\gamma^y y_{is} + \gamma^u c_{is}\big] ds \ . \tag{30}$$

Note that the Local Cost Carbon is discounted with the interest rate $r^\star_t$ of the global bond market, and how it compares to growth of population n_i and TFP $\bar{g}_i$.

The Social Cost of Carbon is an aggregate measure that summarizes the global effects of climate change. It depends on the global welfare metrics and aggregates the Local Costs of Carbon from different regions. We introduce it in the next section.

4 Optimal policy

I consider the social planner that design the optimal climate policy to maximize global welfare, following three benchmarks: (i) the First-Best allocation, where the social planner has access to all the instruments, including cross-countries lump-sum transfers, (ii) the Second-Best policy where the social planner only chooses a single carbon tax $\mathrm{t}^{\varepsilon}_t$, and (iii) the Second-Best allocation, choosing country-specific carbon taxes $\mathrm{t}^{\varepsilon}_{it}$. Despite the model being richer than Section 2's model, the theoretical results are analogous: I show how inequality and the lack of redistributive instruments change the path of the carbon tax.

4.1 First-Best

We consider the optimal policy of a social planner can choose the allocation decision of each country, subject to the resource constraints of the economy. They maximize global welfare, which is the weighted sum of household utilities, with Pareto weights[19] ω_i:

$$\mathcal{W}_{t_0} = \max_{\{c,k,e^f,e^c e^r,e^x,\bar{e}^c,\bar{e}^r\}} \sum_{\mathbb{I}} \mathcal{P}_i\, \omega_i \int_{t_0}^{\infty} e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\, u\big(\mathcal{D}^u_i(\tau_{it})\, c_{it}\big)\; dt \tag{31}$$

subject to the good and energy resource constraints and the climate system:

$$\sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\Big[c_{it} + (\dot{k}_{it}+(n_i+\bar{g}_i+\delta)k_{it}) + \nu^f_i(e^x_{it},\mathcal{R}_{it}) + \kappa^c_i e^c_{it} + \kappa^r_{it} e^r_{it}\Big] = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\mathcal{D}_i(\tau_{it}) z_{it} F(k_{it},\ell_i,e_{it}) \qquad [\phi^w_t]$$

$$E^f_{it} = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\, e^f_{it} = \sum_{i\in\mathbb{I}} e^{(n_i+\bar{g}_i)t} e^x_{it} \qquad [\mu^f_t] \qquad \bar{e}^c_i = e^c_i \qquad [\mu^c_{it}] \qquad \bar{e}^r_i = e^r_i \qquad [\mu^r_{it}]$$

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \mathcal{E}_t := \sum_{\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}(\xi^f e^f_{it} + \xi^c e^c_{it}) \qquad [\phi^S_t]$$

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big) \qquad [\phi^\tau_{it}]$$

I associate the Lagrange multipliers ϕ^w_t for the good market clearing, $\mu^f_t, \mu^c_{it}, \mu^r_{it}$ for the market clearing respectively of fossil (oil-gas), coal and renewable (low-carbon) and ϕ^S_t and ϕ^τ_{it} for the shadow value of additional carbon emissions and temperatures increase.

The result is similar to the toy model example, and the complete description is available in Appendix C. The optimality condition for consumption shows a redistribution motive: the planner equalizes marginal utility subject to Pareto weights:

$$\omega_i u_c(c_{it},\tau_{it}) = \phi^w_t = \omega_j u_c(c_{jt},\tau_{jt})$$

with marginal utility $u_c(c_{it},\tau_{it}) = \mathcal{D}^u(\tau_{it})u'(\mathcal{D}^u(\tau_{it})c_{it}) = \mathcal{D}^u(\tau_{it})^{1-\eta}c_{it}^{-\eta}$, with the CRRA func-

[19] Pareto weights should sum to one, s.t. $\frac{1}{\mathcal{P}}\sum_{\mathbb{I}} \mathcal{P}_i\omega_i = 1$

tional form. Despite the possibility, in the competitive equilibrium, to trade in goods, bonds, and energy, strong inequality exists due to differences in productivity, energy rents or climate damage. As a consequence, in the First-Best, the social planner redistributes consumption using lump-sum transfers in the decentralized equilibrium.

4.1.1 First-Best – Social Cost of Carbon

Given the welfare function in Appendix C, the marginal cost of adding one unit of carbon in the atmosphere $\mathcal{S}_t$ can be summarized by the Social Cost of Carbon. It represents the

$$SCC_t^{fb} := -\frac{\frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{W}_t}{\partial c_{it}}} = -\frac{\phi_t^S}{\phi_t^w} \tag{32}$$

where the aggregate welfare change ϕ_t^S is normalized in monetary units with the aggregate marginal value of wealth $\partial \mathcal{W}_t / \partial c_{it} = \omega_i u_c(c_{it}, \tau_{it}) = \phi_t$. The welfare cost of carbon evolves again with the marginal damage of temperature, ϕ_{it}^τ which is the costate of the temperature dynamics, and the marginal value of carbon ϕ_t^S, the costate of carbon concentration dynamics, which aggregates these different costs across all countries. Solving the differential equations for ϕ_{it}^τ, ϕ_t^S and ϕ_t^w, following the same approach as in Section 3.8, we obtain the Social Cost of Carbon:

$$SCC_t^{fb} \to \int_t^\infty \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-\delta_s(t-t_0) - \int_{t_0}^t (r_{iu}^k - n_i - \bar{g}_i) du} \chi \Delta_i (\tau_{is} - \tau_i^\star) [\gamma^y y_{is} + \gamma^u c_{is}] dt \ ,$$

and this implies the following proposition.

Proposition 6 (**Social Cost of Carbon – First-Best Allocation**).
In the First-Best allocation, marginal utilities are equalized $\phi_i = \omega_i u_c(c_{it}, \tau_{it})$, $\forall i \in \mathbb{I}$, and as a result, we obtain that the Social Cost of Carbon is the sum of Local Costs of Carbon of the different locations $i \in \mathbb{I}$.

$$SCC_t^{fb} = \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i LCC_{it}$$

where the Local Cost of Carbon LCC_{it} is defined as in equation (30), i.e.

$$LCC_{it} = \int_t^\infty e^{-\delta_s(s-t) - \int_t^s (r_u^k - n_i - \bar{g}_i) du} \chi \Delta_i (\tau_{is} - \tau_i^\star) [\gamma^y y_{is} + \gamma^u c_{is}] ds \ .$$

Despite the model being substantially richer, the main result of the Section 2 holds. This aligns with models without heterogeneity in marginal value of wealth, which can be aggregated. Models where differences in climate damages do not have redistributive effects on production, consumption and welfare would imply that the Social Cost of Carbon is simply the sum, weighted by population, $\mathcal{P}_i$, and Pareto weights ω_i, of the Local Cost of Carbon LCC_{it}. Such models include Nordhaus and Yang (1996), and Krusell and Smith (2022), where in the later the free mobility of capital and the absence of heterogeneity in TFP or production allows to aggregate the economy.

4.1.2 Decentralization of the First-Best: transfers and carbon tax

I show how this allocation can be decentralized with carbon taxation in the competitive equilibrium. The detailed treatment of this result is in Appendix A.

Proposition 7 (**First-Best Allocation – Carbon taxation and transfers**).

To decentralize the First-Best allocation, we need two set of instruments: (i) a carbon tax, (ii) cross-country lump-sum transfers. First, the optimal First-Best carbon tax equals the Social Cost of Carbon, as defined in Proposition 6:

$$MPe_{it}^f = q_t^f + \xi^f \mathrm{t}_t^\varepsilon \qquad\qquad \mathrm{t}_t^\varepsilon = SCC_t^{fb}$$

A similar optimality condition holds for coal $MPe_{it}^c = q_{it}^r + \xi^c \mathrm{t}_t^\varepsilon$. *In particular, the optimal carbon tax is equal across countries. This results from the equalization marginal value of wealth. To achieve such equalization, the planner uses lump-sum transfers:*

$$\omega_i u_c(c_{it}, \tau_{it}) = \phi_t = \omega_j u_c(c_{jt}, \tau_{jt}) \qquad \Rightarrow \qquad c_{it}^\star = u_c^{-1}(\phi_t | \tau_{it})$$

which pins down the lump-sum transfers needed given the budget constraint:

$$c_{it}^\star = (r_t^\star - (n_i + \bar{g}_i)) w_{it} + \pi_{it}^f + y_{it} - (r^\star + \delta) k_{it} - (q_t^f + \xi^f \mathrm{t}_t^\varepsilon) e_{it}^f - (q_{it}^c + \xi^c \mathrm{t}_t^\varepsilon) e_{it}^c - q_{it}^r e_{it}^r - \dot{w}_{it} + \mathrm{t}_{it}^{ls} \ ,$$

A uniform carbon tax result aligns with the standard policy recommendations in representative agent models – which encompass most Integrated Assessment Models, like DICE, Nordhaus (2017), Barrage and Nordhaus (2024), FUND, PAGE, MERGE, and others – or the optimal carbon taxation result of Golosov, Hassler, Krusell and Tsyvinski (2014). It also reminiscent of models with unrestricted redistribution such as Hillebrand and Hillebrand (2019).

Lump-sum transfers redistribute across countries and across time:

$$\int_{t_0}^{\infty} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \sum_{\mathbb{I}} \mathrm{t}_{it}^{ls} \, dt = 0$$

Situations where income y_{it}, energy rents π_{it}^f, climate damage and temperature τ_{it}, or Pareto weights ω_i are very heterogeneous such that consumption differentials in the equilibrium without policy intervention are large, the First-Best implies that some countries receive positive lump-sum transfers $\exists j, s.t.\ \mathrm{t}_j^{ls} > 0$ and others pay lump-sum taxes $\exists j', s.t.\ \mathrm{t}_{j'}^{ls} < 0$. Therefore, such decentralized allocation features direct lump-sum transfers across countries.

The question is whether such lump-sum transfers are politically feasible. Can a world central planner impose lump-sum transfers to solve world inequality, for example taxing North America and Europe and rebating it lump-sum to Africa or South Asia? In the next sections, I analyze a family of policies where transfers are not allowed.

4.2 Ramsey problem and optimal uniform carbon taxation

I consider the optimal carbon taxation, where the planner is prevented from achieving redistribution. Lump-sum transfers are not available instruments, for political, governance, or economic reasons. This implies to solve a Ramsey taxation problem with imperfect instruments, where the planner internalize the constraints that are impose by the Competitive Equilibrium.

In particular, it maximizes global welfare, choosing a uniform carbon tax for the world $\{\mathrm{t}^{\varepsilon}_t\}_t$ and rebates the revenue of that tax to the household of the country that pays it $\mathrm{t}^{ls} = \mathrm{t}^{\varepsilon}_t \varepsilon_{it}$.

$$\mathcal{W}_{t_0} = \max_{\substack{\{c,b,k,e^f,e^c e^r,e^x,\bar{e}^c,\bar{e}^r\} \\ \{\mathrm{t}^{\varepsilon},r,q^f,q^c,q^r\}}} \sum_{\mathbb{I}} \mathcal{P}_i\, \omega_i \int_{t_0}^{\infty} e^{-(\rho+n_i)t}\, u\big(\mathcal{D}^u_i(\tau_{it})\, c_{it}\big)\; dt \; . \tag{33}$$

The planner takes the agent decisions, $\{c_{it}, b_{it}, k_{it}, e^f_{it}, e^c_{it}, e^r_{it}, e^x_{it}, \bar{e}^c_{it}, \bar{e}^r_{it}\}_{it}$, prices $\{r^{\star}_t, q^f_t, \mathrm{w}_{it}, q^c_{it}, q^r_{it}\}_{it}$, and states $\{w_{it}, \tau_{it}, \mathcal{S}_t\}_{it}$ subject to the constraints of the Competitive equilibrium, which are (i) the budget constraint equation (25), with multiplier ψ^w_{it}, (ii) the climate dynamics for carbon $\mathcal{S}_t$, equation (21), with multiplier ψ^S_t, and temperatures τ_{it}, equation (22), with multiplier ψ^{τ}_{it}, (iii) the firms optimality conditions for energy $e^f_{it}, e^c_{it}, e^r_{it}$, from equation (11), and capital k_{it}, respectively with multipliers $\upsilon^f_{it}, \upsilon^c_{it}, \upsilon^r_{it}, \upsilon^k_{it}$, (iv) the energy firms optimality for extraction and production, from equations (14), (17) and (18), and finally (v) the market clearing for goods equation (24), bonds, fossil, coal and renewable energy, from equations (16) to (18).

We apply the Pontryagin Maximum Principle and the details of the entire system are found in Appendix D. Note that the social planner has full commitment, in the sense that decisions taken in the initial period t_0 binds $\forall t \in (t_0, \infty)$ and there is no time inconsistency. We provide some intuitions of the most important results.

First, before of lack of redistribution, we can define the normalized social welfare weights – as some inequality index – using the marginal value of wealth ψ^w_{it}:

$$\begin{aligned} &\psi^w_{it} = u_c(c_i, \tau_{it}) \qquad\qquad \Rightarrow \qquad \omega_i u_c(c_{it}, \tau_{it}) \neq \omega_j u_c(c_{jt}, \tau_{jt}) \\ &\widehat{\psi}^w_{it} := \frac{\omega_i \mathcal{P}_i \psi^w_{it}}{\overline{\psi}^w_t} = \frac{\omega_i \mathcal{P}_i u_c(c_{it}, \tau_{it})}{\frac{1}{\mathcal{P}} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i u_c(c_{it}, \tau_{it})} \qquad\qquad \overline{\psi}^w_t = \tfrac{1}{\mathcal{P}} \sum_{\mathbb{I}} \omega_i \mathcal{P}_i \psi^w_{it} \; , \end{aligned} \tag{34}$$

with $\overline{\psi}^w_t$ the marginal value of wealth of the "average" agent. If the ratio $\widehat{\psi}^w_{it}$ is higher than 1, the planner sees country i at time t with a lower welfare than the average household.

4.2.1 Second-Best, Social Cost of Carbon

Again, as in the previous section, the Social Cost of Carbon is measured as the ratio of the marginal value of carbon and the marginal value of wealth:

$$SCC^{sb}_t := -\frac{\frac{\partial \mathcal{W}_t}{\partial \mathcal{S}_t}}{\frac{\partial \mathcal{W}_t}{\partial c_t}} = -\frac{\psi^S_t}{\overline{\psi}^w_t} \tag{35}$$

it differs slightly from the definition in equation (32) of the First-Best, because now the planner consider the "average agent" marginal utility $\overline{\psi}_t^w$ to convert the Social Cost of Carbon in monetary unit.

To obtain the Social Cost of Carbon, we again solve the differential equation for the marginal damage to temperature and the marginal value of carbon.

$$SCC_t^{sb} = \int_t^\infty e^{-\delta_s(s-t)-\int_t^s r_u^\star du} \chi \sum_{i\in\mathbb{I}} e^{(n_i+\bar{g}_i)(s-t)} \Delta_i(\tau_{is}-\tau_i^\star)\big[\gamma^y y_{is} + \gamma^u c_{is}\big] \widehat{\psi}_{is}^w ds$$

This now differs from the First-Best because the temperature damages $(\tau_{it} - \tau_i^\star)\big[\gamma^y y_{it} + \gamma^u c_{it}\big]$ are weighted by welfare ψ_{it}^w. We thus obtain an important result in Second-Best economies:

Proposition 8 (**Social Cost of Carbon – Second-Best Allocations**).
In the Second-Best allocation, inequalities across countries persist as measured by the social welfare weights $\widehat{\psi}_{it}^w \neq 1, \forall i \in \mathbb{I}$. When redistribution is constrained, the Social Cost of Carbon is the weighted sum of Local Costs of Carbon of the different locations $i \in \mathbb{I}$, weighted by the social welfare weights $\widehat{\psi}_{it}^w$ as given in equation (34):

$$SCC_t^{sb} = \sum_{i\in\mathbb{I}} \widehat{\psi}_{it}^w \, LCC_{it} \quad \propto \quad \sum_{i\in\mathbb{I}} \mathcal{P}_i \, \omega_i \, u_c(c_{it}, \tau_{it}) \, LCC_{it}$$

where the Local Cost of Carbon LCC_{it} is defined as in equation (30), i.e.

$$LCC_{it} = \int_t^\infty e^{-\delta_s(s-t)-\int_t^s (r_u^\star - n_i - \bar{g}_i)du} \chi \Delta_i(\tau_{is}-\tau_i^\star)\big[\gamma^y y_{is} + \gamma^u c_{is}\big] ds \ .$$

As a result, we can express the Social Cost of Carbon as:

$$\begin{aligned} SCC_t^{sb} &= \sum_{\mathbb{I}} \widehat{\psi}_{it}^w \, LCC_{it} \\ &= \mathcal{P}\,\mathbb{E}^{\mathbb{I}}[\omega_i LCC_{it}] + \mathcal{P}\,\mathbb{C}\mathrm{ov}^{\mathbb{I}}\Big(\widehat{\psi}_{it}^w, LCC_{it}\Big) \qquad \lesseqgtr \quad \mathcal{P}\,\mathbb{E}^{\mathbb{I}}[\omega_i LCC_{it}] =: SCC_t^{fb} \end{aligned}$$

for the mean $\mathbb{E}^{\mathbb{I}}[\cdot]$ and covariance $\mathbb{C}ov^{\mathbb{I}}(\cdot)$ over locations.[20] *Since the Social Cost of Carbon is a sum – and not a mean – one needs to rescale by world population $\mathcal{P}$.*

To summarize, the presence of heterogeneity and the correlation between local damage and income change the Social Cost of Carbon from the Social Planner's perspective.

4.2.2 Second-Best, Uniform Carbon Taxation

As shown in model of Section 2, taxation of carbon and fossil fuels have strong redistributive general equilibrium effects through energy markets.

Fossil energy supply redistribution. First, the implementation of carbon taxation reduces demand for fossil fuels, which has strong redistributive effects on the energy rent and along the

[20] We define them as $\mathbb{E}^{\mathbb{I}}[x_{it}] = \frac{1}{\mathcal{P}} \sum_{i\in\mathbb{I}} \mathcal{P}_i x_{it}$ and $\mathbb{C}\mathrm{ov}^{\mathbb{I}}(x_{it}, y_{it}) = \frac{1}{\mathcal{P}} \sum_{i\in\mathbb{I}} \mathcal{P}_i \big(x_{it} - \mathbb{E}^{\mathbb{I}}[x_{it}]\big) \ \big(y_{it} - \mathbb{E}^{\mathbb{I}}[y_{it}]\big) it$.

supply curve. Recall that υ^f_{it} is the Lagrange multiplier for the optimality condition on fossil fuel demand, and we denote $\widehat{\upsilon}^f_{it} = \upsilon^f_{it}/\overline{\psi}^w_{it}$ the rescaled value by the marginal value of wealth – it would represent the monetary value of marginally relaxing that optimality condition. Moreover, a change in equilibrium quantity of oil-gas relaxes the market clearing, leading to the following mechanism:

$$\textit{Supply Redistribution}^f_t = \Big(\sum_i \nu^f_{i\, e^x e^x(e^x_{it},\mathcal{R}_{it})}{}^{-1}\Big)^{-1}\Big(\sum_i \widehat{\psi}^w_{it}[e^f_{it} - e^x_{it}] - \sum_i \widehat{\upsilon}^f_{it}\Big) \; ,$$

where $\Big(\sum_i \nu^f_{i\, e^x e^x(e^x_{it},\mathcal{R}_{it})}{}^{-1}\Big)^{-1}$ is the aggregate supply elasticity for oil and gas, which depends on the equilbrium extraction e^x_{it} and reserves $\mathcal{R}_{it}$ of each country. Moreover, as we saw above, carbon taxation lowers the energy price q^f_t, leading to a term-of-trade effect. This redistributes energy rents from exporters to importers, and hence scale with $e^f_{it} - e^x_{it}$. This change is weighted by the marginal valuation of wealth $\widehat{\psi}^w_{it}$, and it is higher when the fossil-fuel producers are also relatively poorer or weighted more by the planner. This effect rewrites $\sum_i \widehat{\psi}^w_{it}[e^x_{it} - e^f_{it}] = \mathbb{C}\text{ov}^{\mathbb{I}}\big(\widehat{\psi}^w_{it}(e^x_{it} - e^f_{it})\big)$, and we can measure this covariance empirically as a sufficient statistics. Moreover, the change in energy price also affects the oil and gas price, affecting the demand, as denoted by the term $\sum_i \widehat{\upsilon}^f_{it}$. This term comes from the fact that carbon polity is affecting differentially oil-gas and coal. Recall that, if $\widehat{\upsilon}^f_{it} > 0$, the planner would increase energy prices, moving up along the supply curve, which results in a lower the optimal carbon tax t^ε_t.

<u>*Fossil energy demand distortion*</u>. Second, carbon taxation is a distortionary tax that create a wedge for the demand of energy. This demand distortion differs across countries due to differences in energy mix, productivity and the endowmnent and cost of energy sources. From the planner's optimality for energy, we can define the following term:

$$\begin{aligned}
\textit{Demand Distortion}^f_{it} := \widehat{\widehat{\upsilon}}^f_{it} &= \big[\widehat{\upsilon}^f_{it}\, \partial_{e^f} MPe^f_{it} + \widehat{\upsilon}^c_{it}\, \partial_{e^f} MPe^c_{it} + \upsilon^r_{it}\, \partial_{e^f} MPe^r_{it} + \widehat{\upsilon}^k_{it}\, \partial_{e^f} MPk_{it}\big] \\
&= z_i \mathcal{D}_i(\tau_{it})\Big[\widehat{\upsilon}^f_{it}\, F_{e^f e^f} + \widehat{\upsilon}^c_{it}\, F_{e^f e^c} + \widehat{\upsilon}^r_{it}\, F_{e^f e^r} + \widehat{\upsilon}^k_{it}\, F_{e^f k}\Big] \\
&= \frac{1}{e^f_{it}}\Big[- \widehat{\upsilon}^f_{it}(q^f_t + \xi^f \mathrm{t}^\varepsilon_t)\Big(\frac{1-s^f_{it}}{\sigma^e} + s^f_{it}\frac{1-s^e_{it}}{\sigma^y}\Big) + \widehat{\upsilon}^c_{it}(q^c_{it} + \xi^c \mathrm{t}^\varepsilon_t) s^f_{it}\Big(\frac{1}{\sigma^e} - \frac{1-s^e_{it}}{\sigma^y}\Big) \\
&\qquad + \widehat{\upsilon}^r_{it} q^r_{it} s^f_{it}\Big(\frac{1}{\sigma^e} - \frac{1-s^e_{it}}{\sigma^y}\Big) + \widehat{\upsilon}^k_{it}(r^\star_t + \delta)\frac{s^f_{it} s^e_{it}}{\sigma^y}\Big] \; ,
\end{aligned}$$

with energy share in production, $s^e_{it} = e_{it} q^e_i / y_i$, fossil share in energy mix $s^f_{it} = e^f_{it} q^f_t / e_{it} q^e_{it}$ and similarly $s^c_{it} = e^c_{it} q^c_{it} / e_{it} q^e_{it}$ and $s^r_{it} = e^r_{it} q^r_{it} / e_{it} q^e_{it}$. Moreover, σ^e is the elasticity of substitution between energy sources, σ^y the one between energy and the capital/labor bundle, and $\widehat{\upsilon}^f_{it} := \omega_i \mathcal{P}_i \upsilon^f_{it} / \overline{\psi}^w_{it}, \widehat{\upsilon}^r_{it}$ and $\widehat{\upsilon}^k_{it}$ are rescaled versions of the Lagrange multipliers for fossil energy, renewables, and capital respectively. Note, we can find a similar expression for coal energy, *Demand Distortion*$^c_{it}$, as function of $F_{e^c e^f}, F_{e^c e^c}$, etc.

To understand the intuition behind this term, take the first term, $-\frac{1-s^f_{it}}{\sigma^e} - s^f_{it}\frac{1-s^e_{it}}{\sigma^y}$, as an example. We see that this demand channel of taxation has two effects: the first channel lowers fossil consumption due to direct substitution effect between the three energy inputs, lowering the marginal product of fossil MPe^f_{it} with elasticity σ^e. The second effect is indirect through the total energy use e_{it}, proportionally to the fossil share s^f_{it}. Weighting these different distortions with the shadow values υ^x_{it} for input x and scaling it for the input prices q^f_t, q^c_{it}, etc. we obtain the total distortion caused by taxation of fossil fuels. This term is more involved than in the simpler model of Section 2 and Proposition 2, because of the general substitution patterns across energy and heterogeneity across countries. Moreover, the aggregate level of the carbon tax balances out all these distortions across countries and energies, and the optimality condition for t^ε_t gives:

$$\sum_{i\in\mathbb{I}}(\xi^f\widehat{\upsilon}^f_{it} + \xi^c\widehat{\upsilon}^c_{it}) = 0$$

This minimization of total distortion relates to the standard principles of Ramsey taxation, as in Diamond and Mirrlees (1971); Diamond (1973); Atkinson and Stiglitz (1976).

Optimal carbon tax. We now present our main result for the optimal Second-Best uniform carbon tax. Details on how this formula is derived from the optimality condition for energy choices can be found in Appendix D. The optimal level of carbon taxation integrates the different redistribution motives that we detailed above. As a result, the Ramsey planner accounts for these general equilibrium effects as symbolized by the curvature of demand and supply of energy.

Proposition 9 (Uniform Carbon Taxation – Second-Best Allocation).
*The optimal Second-Best carbon tax accounts for three distributional motives when setting a single uniform level t^ε, in the absence of cross-countries transfers, when revenues of the carbon are rebated locally $\mathrm{t}^{ls}_{it} = \epsilon_{it}\mathrm{t}^\varepsilon_t$: (i) climate damage in the **Social Cost of Carbon (SCC)**, (ii) **Supply Redistribution** in energy markets through terms-of-trade and energy rents and (iii) **Demand Distortion** through distorted firms' energy choices. This includes redistribution motives due to the presence of inequality through the **social welfare weights** $\widehat{\psi}^w_{it} = \omega_i\mathcal{P}_i\psi^w_{it}/\overline{\psi}^w_{it} \propto \omega_i\mathcal{P}_i u_c(c_{it}, \tau_{it})$. The optimal carbon tax writes:*

$$\xi^f\mathrm{t}^\varepsilon_t = \xi^f\, SCC^{sb}_t \;+\; \textit{Supply Redistribution}^f_t \;+\; \sum_{i\in\mathbb{I}}\textit{Demand Distortion}^f_{it}$$

$$SCC_t = \mathcal{P}\,\mathbb{E}^{\mathbb{I}}\big[\omega_i LCC_{it}\big] + \mathcal{P}\,\mathbb{C}\mathrm{ov}^{\mathbb{I}}\Big(\widehat{\psi}^w_{it}, LCC_{it}\Big)$$

$$\textit{Supply Redistribution}^f_t = \Big(\sum_i(\nu^f_{i\,e^x e^x})^{-1}\Big)^{-1}\Big[\mathcal{P}\mathbb{C}\mathrm{ov}^{\mathbb{I}}(\widehat{\psi}^w_{it}, e^x_{it} - e^f_{it}) - \mathcal{P}\mathbb{E}^{\mathbb{I}}[\widehat{\upsilon}^f_{it}]\Big] \tag{36}$$

$$\textit{Demand Distortion}^f_{it} = \sum_{x\in\{e^f,e^c,e^r,k\}}\widehat{\upsilon}^x_{it}\quad\partial_{e^f}MPx_{it}$$

where $\widehat{\upsilon}^x_{it}$ are the rescaled multipliers for the optimality condition $MPx_{it} = q^x_{it}$ for the choice of input x, i.e. $\widehat{\upsilon}^f_{it} := \omega_i\mathcal{P}_i\upsilon^x_{it}/\overline{\psi}^w_{it}$, for x being fossil (oil-gas) e^f, coal e^c, renewable (low-carbon) e^r and capital k. Moreover, the supply redistribution depends on the curvature of the oil-gas supply *with $\nu^f_{i\,e^x e^x}$ the supply elasticity, and the demand distortion depends on $\partial_{e^f} MPx_{it}$ the curvature of the production function, i.e. the own-elasticity in oil-gas $F_{e^f e^f}$ and cross-elasticity, e.g. $F_{e^f e^r}$.*

In addition, even without climate externality $SCC_t = 0$, the carbon tax $\mathrm{t}^{\varepsilon}_t$ could be positive, accounting for energy terms-of-trade manipulations, for example with wealthy exporters and relatively poorer importers, or when richer countries are consuming more fossil-fuels with higher elasticity, following the intuitions of Section 2.

Such a result holds as long as different agents – countries, firms, or households – have different marginal utilities of consumption, i.e. different $\widehat{\psi}^w_{it}$. However, these motives would be absent in models like Golosov et al. (2014) for two reasons: First if the supply curve for energy is perfectly elastic, because of constant return to scale, which yields $\nu_{e^x e^x} = 0$. Second, when the agent/firm is "representative" or multiple agents can be aggregated – at least in the inputs demand decisions – and a single energy tax instrument in the First-Best, the social planner is not "distorting" the energy demand across agents: the planner and the agents would achieve the same optimality condition for fossil fuel demand.

4.3 Country-specific carbon taxation

We now consider an experiment where the social planner design country-specific taxes that would allow to correct some of these redistributive concerns. In that case, not only the *level* but also the *distribution* of the fossil fuel/carbon tax is affected by redistribution motives. The planner maximizes global welfare as in equation (33), choosing a countries-specific carbon taxes $\{\mathrm{t}^{\varepsilon}_{it}\}_t$ over time and rebates the carbon tax revenue to the household of the country that pays it $\mathrm{t}^{ls} = \mathrm{t}^{\varepsilon}_t \varepsilon_{it}$. We solve the planner's problem choosing the agent decisions, prices, and states, subject to the constraints of the Competitive equilibrium.

Proposition 10 (Country-specific Carbon Taxation – Second-Best Allocation).
The optimal Second-Best country-specific carbon taxation, when transfers are absent and revenues rebated locally, can be written as:

$$\mathrm{t}^{\varepsilon}_{it} = \frac{1}{\widehat{\psi}^w_{it}}\Big[SCC_t \;+\; \textit{Supply Redistribution}^f_t \;+\; \textit{Demand Distortion}^f_{it}\Big]$$

$$SCC_t = \mathcal{P}\,\mathbb{E}^{\mathbb{I}}[\omega_i LCC_{it}] + \mathcal{P}\,\mathbb{C}\mathrm{ov}^{\mathbb{I}}\Big(\widehat{\psi}^w_{it}, LCC_{it}\Big)$$

$$\textit{Supply Redistribution}^f_t = \Big(\sum_i (\nu^f_{i\,e^x e^x})^{-1}\Big)^{-1}\Big[\mathcal{P}\mathbb{C}\mathrm{ov}^{\mathbb{I}}(\widehat{\psi}^w_{it}, e^x_{it} - e^f_{it}) - \mathcal{P}\mathbb{E}^{\mathbb{I}}[\widehat{v}^f_{it}]\Big]$$

$$\textit{Demand Distortion}^f_{it} = \sum_{x\in\{e^f,e^c,e^r,k\}} \widehat{v}^x_{it} \quad \partial_{e^f} MPx_{it}$$

The demand distortion is only local since the planner can choose country-specific taxes. However, this distortion is not null, since the planner does not have ***energy-sources****-specific taxes. In particular, the carbon tax affects both oil-gas and coal,*

$$\xi^f \widehat{v}^f_{it} \;+\; \xi^c \widehat{v}^c_{it} = 0$$

These local distortive effects remain if energy demands e^f_{it}, e^c_{it}, energy shares, and elasticities differ.

Following the logic in Section 2, the optimal tax still depend on both the Social Cost of Carbon SCC_t and $\textit{Supply Redistribution}^f_t$ as developed above. These two energy taxation motives – climate externality correction and energy rents redistribution – remains the same. However, we observe that the demand distortion is reduced considerably. We observe that because the planner has access to n different instruments

At the difference with Section 2, we now have only the local distortions, represented by the Lagrange multipliers $\widehat{\upsilon}^f_{it}, \widehat{\upsilon}^c_{it}, \widehat{\upsilon}^r_{it}, \widehat{\upsilon}^k_{it}$ for the input choices. These inputs distortion are no longer null in equilibrium as in the Proposition 2, despite the planner choosing a country-specific tax level. The reason is that now there is a distortion and a reallocation between oil-gas, coal, and other inputs. The optimality for t^ε_i gives:

$$\xi^f \widehat{\upsilon}^f_{it} \; + \; \xi^c \widehat{\upsilon}^c_{it} = 0 \qquad \not\Rightarrow \qquad \widehat{\upsilon}^c_{it} = 0 \quad \& \quad \textit{Demand Distortion}^f_{it} = 0$$

Nevertheless, the tax is country i specific and depends on redistribution motives. Indeed the ratio $1/\widehat{\psi}^w_{it}$ is the inverse of the social welfare weight – the inequality index developed earlier in this section in equation (34). It implies that richer/colder countries, which have higher consumption and lower marginal utilities are charged higher carbon taxes, and conversely poorer countries should be charged a lower tax:

$$\text{low} \;\; c_{it} \;\; \text{high} \;\; \tau_{it} \qquad \Rightarrow \qquad \text{high} \;\; \widehat{\psi}^w_{it} \; \propto \; u_c(c_{it}, \tau_{it}) \qquad \Rightarrow \qquad \text{low} \;\; t^f_{it}$$

everything else being constant, in particular SCC_t, $\textit{Supply Redistribution}^f_t$ and $\textit{Demand Distortion}^f_{it}$.

In particular, this result provides a justification for the use of a "tiered" carbon taxation – where carbon tax would be lower for developing economies and higher for advanced economies. This proposal, that tend to be promoted for moral reasons, is shown here to have an explanation based on efficiency and welfare maximization.

5 Quantification and calibration

The model is calibrated to a sample of 68 countries to provide realistic predictions on the impact of optimal carbon policy. I first describe the data used. I then provide details on the quantification, and how the parameters are calibrated to match the data. I summarize in Table 2 the dimensions of heterogeneity of the model. Table 1 contains the summary table for the calibration described in this section.

5.1 Data

First, I describe briefly the data used to calibrate the model. I use data for the year 2018-2023, taking the average over that period to smooth out the effect of the COVID-19 recession on energy and macroeconomic data.

I use data for GDP per capita, in Purchasing Power Parity (PPP, in 2016 USD) from the World Bank, as collected and processed by the Maddison Project, Bolt and van Zanden (2023). For the energy variables, I use the comprehensive data collected and processed in the Statistical Review of Energy Energy Institute (2024), that includes the production and consumption of various energy sources, including Oil, Gas, Coal and Renewable. It also includes proven reserves of those fossil fuels. For energy rent, I use the World Development Indicators that use national accounts to measure the share of GDP coming from energy (oil, gas and coal) and natural resource rents. Finally, for temperature, I use the same time series as Burke et al. (2015), which use the temperature at country level, averaged over the year and weighted by population across locations.

5.2 Welfare and Pareto weights

The welfare function that the social planner maximize, in Appendix C is the weighted sum of individual utilities in all countries, with $\mathcal{P}_i$ the population size and ω_i the Pareto weights.

I consider two sets of Pareto weights. First, I consider the utilitarian benchmark, where the planner weight every individual in the world equally: $\omega_i = 1$. Second, following the discussion in Anthoff et al. (2009), Nordhaus (2011) and Nordhaus and Yang (1996), one would like to choose Pareto weights that eliminate redistributive effects that are orthogonal to climate change and carbon policy. To that purpose, the "Negishi" Pareto weights make the preexisting competitive equilibrium efficient under that welfare metric. This implies that:

$$\omega_i = \frac{1}{u_c(\bar{c}_{it_0}, \mathcal{T}_{it_0})} \qquad \Leftrightarrow \qquad C.E.(\bar{c}_i) \in \underset{\bar{c}_i}{\operatorname{argmax}} \sum_i \mathcal{P}_i \omega_i u(\bar{c}_{it_0}, \mathcal{T}_{it_0})$$

$$\omega_i u_c(\bar{c}_{it_0}, \mathcal{T}_{it_0}) = \omega_j u_c(\bar{c}_{jt_0}, \mathcal{T}_{jt_0}) \qquad \forall i, j \ \in \mathbb{I}$$

where $\bar{c}_i$ is the consumption level in the present competitive equilibrium – the period 2018-2023 – absent future climate damage. This implies that the carbon policy do not have redistributive motives through energy general equilibrium effects. However, global warming, and the carbon taxation itself have redistributive effects, as they change the distribution of c_{it}. These effects would thus be taken into account in the choice of policies as shown in Proposition 9.

5.3 Macroeconomy, trade and production

For the macroeconomic part of the framework, I consider standard utility and production functions. For utility, as in the equation (7), I calibrate the CRRA/IES parameter to be $\eta = 1.5$, taken from Barrage and Nordhaus (2024).

For production, I use a nested CES framework. The firm combines a Cobb-Douglas bundle of capital k_i and labor ℓ_i[21] with a composite of energy e_i, with elasticity σ^y. Second, the energy e_i aggregates the different energy sources: oil and gas e^f, coal e_i^c, and renewable/non-carbon e_i^r, with elasticity σ^e. To calibrate these functions, I set the capital-labor ratio $\alpha = 0.35$ to match

[21] Labor is inelastically supplied $\ell_i = \bar{\ell}_i$ in each country and normalized to 1 – since the country size $\mathcal{P}_i$ is already taken into account. As a result, all the variables can be seen as input per capita.

the cost share of capital. For the energy, I set $\varepsilon = 0.10$ to match the world average energy cost share $\frac{q_i^e e_i}{y_i} = 6\%$, as measured in Kotlikoff, Kubler, Polbin and Scheidegger (2021*b*) and used in Krusell and Smith (2022). For the elasticity between energy and other inputs, I set $\sigma^y = 0.3$ for all countries, which is in the range of estimates in Papageorgiou et al. (2017), among others.[22] Therefore, capital/labor and energy are complementary in production: an increase in the price of energy has a strong impact on output as it is less productive to "substitute away" to other inputs – capital, labor here. This aligns with other empirical and structural evidence on the impact of energy shocks, e.g. Hassler et al. (2021*a*). Then, I calibrate the energy mix for oil-gas, with $\omega^f = 0.56$, coal $\omega^c = 0.27$, and non-carbon $\omega^r = 0.17$, to match the aggregate shares of each of these energy sources in the data. In the next section, I document how I match the individual countries' energy mix using energy prices/costs. Finally, for the elasticity between energy inputs, I use the value $\sigma_e = 2$, following the rest of the literature, i.e. Papageorgiou et al. (2017), Kotlikoff, Kubler, Polbin and Scheidegger (2021*b*), and Hillebrand and Hillebrand (2019), among others.

I calibrate the productivity z_i of the production function $y_{it_0} = \mathcal{D}_i^y(\tau_{it_0})\, z_i\, \bar{y}_{it_0}$ to match exactly the GDP, y_{it_0}, across countries. This parameter z_i, represents productivity residuals as well as institutional/efficiency differences across countries. In Figure 3, I show the GDP levels, as they replicated with this model.

Table 1: Baseline calibration

Technology & Energy markets			
α	0.35	Capital share in $F(\cdot)$	Capital/Output ratio
ϵ	0.12	Energy share in $F(\cdot)$	Energy cost share (8.5%)
σ^y	0.3	Elasticity capital-labor vs. energy	Complementarity in production (c.f. Bourany 2022)
ω^f	0.56	Fossil energy share in $e(\cdot)$	Oil-gas/Energy ratio
ω^c	0.27	Coal energy share in $e(\cdot)$	Coal/Energy ratio
ω^r	0.17	Non-carbon energy share in $e(\cdot)$	Non-carbon/Energy ratio
σ^e	2.0	Elasticity fossil-coal-non-carbon	Slight substitutability & Study by Stern
δ	0.06	Depreciation rate	Investment/Output ratio
$\bar{g}$	$0.01^\star$	Long run TFP growth	Conservative estimate for growth
Preferences & Time horizon			
ρ	0.03	HH Discount factor	Long term interest rate & usual calib. in IAMs
η	1.5	IES / Risk aversion	Standard calibration
n	0.0035	Long run population growth	Conservative estimate for growth
ω_i	1	Pareto weights	Uniforms / Utilitarian Social Planner
ω_i	$1/u'(c_i)$	Pareto weights	Negishi / Status-quo Social Planner
T	400	Time horizon	Time for climate system to stabilize
Climate parameters			
ξ^f	2.761	Emission factor – Oil & natural gas	Conversion 1 $MTOE$ $\Rightarrow$ 1 MT CO_2
ξ^c	3.961	Emission factor – Coal	Conversion 1 $MTOE$ $\Rightarrow$ 1 MT CO_2
χ	2.3/1e6	Climate sensitivity	Pulse experiment: $100\,GtC \equiv 0.23°C$ medium-term warming
δ_s	0.0004	Carbon exit from atmosphere	Pulse experiment: $100\,GtC \equiv 0.15°C$ long-term warming
ζ	0.027	Growth rate, Carbon Capture and Storage	Starting after 2100, Follows Krusell Smith (2022)
$\gamma^\oplus$	0.003406	Damage sensitivity	Nordhaus' DICE
$\gamma^\ominus$	$0.3\times\gamma^\oplus$	Damage sensitivity	Nordhaus' DICE & Rudik et al (2022)
α^T	0.5	Weight historical climate for optimal temp.	Marginal damage correlated with initial temp.
$T^\star$	14.5	Optimal yearly temperature	Average spring temperature / Developed economies

[22] It also aligns with my own estimation in Bourany (2022).

5.4 Energy markets

For the energy market, I match the energy mix of different countries, using the CES framework displayed above, as well as differences in costs of production. For the supply side, we use iso-elastic fossil extraction cost to replicate the oil-gas supply of fossil producers.

First, in this model, oil and gas are traded on international markets, with demand $\mathcal{P}_i e^f_{it_0}$ from the final good firm and supply $\mathcal{P}_i e^x_{it_0}$ from the fossil energy firm, extracting oil and gas from its own reserves. We use the extraction function ν^f_i to have the following isoelastic form of equation (13), which is homogeneous of degree one in $(e^x_i, \mathcal{R}_i)$. The inputs are paid in the price of the consumption good – normalized to one.[23] This implies the profit function in equation (15). I calibrate the three parameters $\mathcal{R}_i$, ν_i and $\bar{\nu}_i$ to match the three country-level variables $\mathcal{R}_i$, e^x_i and π^f_i. The reserves $\mathcal{R}_i$ are taken directly from the data on oil and gas reserves documented by Energy Institute (2024). I calibrate the slope of this cost function $\bar{\nu}_i$ to match exactly the production of oil and gas e^x_i, as informed by that same data source. This is displayed in Figure 4. I then calibrate the curvature of the cost function to match the share $\eta^\pi_i = \frac{\pi^f_i}{y_i \mathrm{p}_i + \pi^f_i}$ of fossil energy profit as share of GDP. I choose ν_i to minimize the distance – mean squared error – between the model share η^π_i and the data, successfully matching the share within $5-10$ percentage points. Differences in oil and gas energy rent across countries are not only determined by differences in cost and technology, but also in differences in trade costs and market power – by the existence of OPEC which control more than 28% of oil supply and around 15% of natural gas supply. This explains why it is difficult to match exactly the value η^π_i. However, to keep the simplicity and tractability of the model, I refrain from adding an additional Armington structure over energy sources, or oligopoly power over oil and gas as discussed in Bornstein et al. (2023) and Hassler et al. (2010).

Second, I match the energy mix of the different countries by relying on the two assumptions made in the model: (i) coal and renewable are only traded at the country level: $\bar{e}^c_i = e^c_i$ and $\bar{e}^r_i = e^r_i$ and (ii) the cost function is linear in goods, i.e. the production is Constant Returns to Scale, implying $q^c_i = \kappa^c_i$ and $q^r_i = \kappa^r_i$. This allows me to match the energy mix of each country by calibrating the energy costs parameters κ^c_i and κ^r_i for each country to match the data on coal share $\frac{e^c_i}{e^f_i + e^c_i + e^r_i}$ and non-carbon energy share $\frac{e^r_i}{e^f_i + e^c_i + e^r_i}$. Using the CES framework above, I match exactly the energy shares, successfully identifying countries that are more reliant on coal vs. oil and gas vs. non-carbon/renewable: for example, China and India are highly coal-dependent, and Russia, Middle-East and United-States/Canada are the biggest consumers of oil and gas.

5.5 Climate system

Finally, I calibrate the climate model described in Section 3.4 to match important features of the relationship between carbon emissions, temperatures and climate damages.

[23] I express the oil-gas extraction with a cost function $x^f_i = \nu^f_i(\cdot)$. We can also express analogously with the production function:

$$e^x_i = g(x^f_i) = \left(\tfrac{1+\nu_i}{\bar{\nu}_i}\right)^{\frac{1}{1+\nu}} \mathcal{R}_i^{\frac{\nu_i}{1+\nu_i}} (x^f_i)^{\frac{1}{1+\nu_i}}$$

with inputs x^f_i paid in the final good price. This production has constant returns to scale in $(x^f_i, \mathcal{R}_i)$.

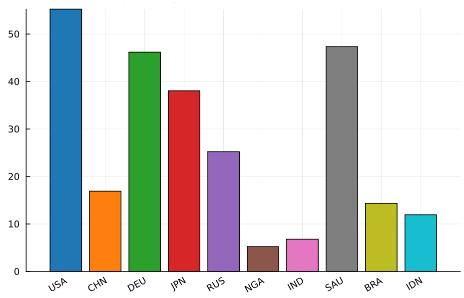


Figure 3: GDP per capita
Thsds 2011-USD PPP, avg. 2018-2023

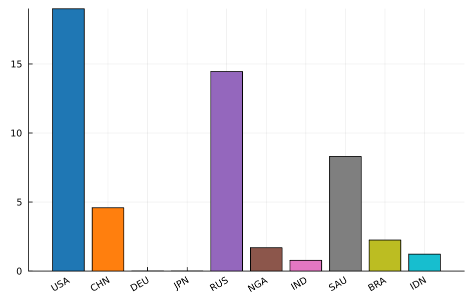


Figure 4: Oil and gas production
GTOE (gigatons oil equiv.), avg. 2018-2023

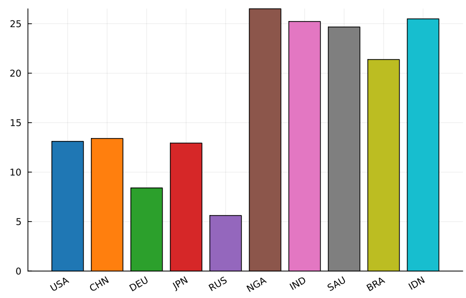


Figure 5: Temperatures
Avg., population-weighted, 2015

First, I calibrate two parameters of the global climate system: the climate sensitivity χ, i.e. the reaction of global temperature, $\mathcal{T}_t$, to the atmospheric concentration of CO_2, $\mathcal{S}_t$, and the carbon decay rate, δ_s, representing the exit of carbon of the atmosphere into carbon sinks – oceans, biosphere – and out of the higher atmosphere. To this end, as is standard in Integrated Assessment models, I match the pulse experiment dynamics of larger IAMs – CMIP5 in this case: for a "pulse" of $100GT$ of carbon released – corresponding to 10 years of emissions – the global temperature reaches its peak between $0.20°C$ and $0.25°C$ after 10 years and then decreases slightly to stabilize around $0.17°C$ after 200 years. I follow Dietz et al. (2021), and calibrate $\chi = 0.23$ and $\delta = 0.0004$ to match these two moments, as seen in Figure 2.

Moreover, this climate system is inherently unstable for emissions, $\mathcal{E}_t = \bar{\xi}_t \sum_{i \in \mathbb{I}} \epsilon_{it}$, with trends in population growth, n, and long-term TFP growth $\bar{g}$, where $n = 0.0035$ and $\bar{g} = 0.01$ are the long-term growth rates according to forecast by the UN and World-Bank. To counteract the non-stationarity of the climate system, I follow Krusell and Smith (2022) and assume that part of the emissions $\mathcal{E}_t$ are captured and stored, under the variable $\bar{\xi}_t$. I assume the exponential form, $\bar{\xi}_t = e^{-\xi t}$ and calibrate ζ to match the moment suggested in Krusell and Smith (2022): 50% is captured by 2125, and 100% by 2300 – which is $> 99.9\%$ in our model. This implies that in the Business-as-Usual scenario, global temperatures reach $\sim 4.5°$ by 2100 and are stabilized around $7°$ by 2400. More optimistic scenarios for Carbon Capture and Storage (CCS) could be imagined without affecting the main result since most of the damages are discounted heavily after 2100.

Second, I calibrate initial temperatures τ_{it_0} with data from Burke et al. (2015), and I display selected countries in Figure 5. Furthermore, I assume the linear pattern scaling $\dot{\tau}_{it} = \Delta_i \mathcal{T}_t$. I identify the scaling parameter in reduced-form by estimating this linear regression over the period $t = 1950{-}2015$ for each country and then aggregating by region i.[24] This procedure does not require extensive and granular data such at geographical characteristics, albedo, etc.

Third, to calibrate the damages, I use a quadratic function as in the DICE - IAM, and seen in equation (23), with the damage parameter $\gamma = 0.00340$. This value is intermediary between the value $\gamma = 0.00311$ in Krusell and Smith (2022), calibrated to match Nordhaus' DICE calibration of 6.6% of loss of global GDP when temperature anomaly $\mathcal{T}_t = 5$, and the updated calibration in

[24] To control for the fact that country j has an influence on world temperature $\mathcal{T}_t = \sum_i g_i \tau_{it}$, I estimate the jackknife linear equation with $\mathcal{T}_{t,\neq j} = \sum_{i \neq j} g_i \tau_{it}$ for each j, i.e. $\tau_{jt} = \Delta_j \mathcal{T}_{t,\neq j}$.

Barrage and Nordhaus (2024) which calibrate it at $\gamma = 0.003467$. For small values, I consider $\gamma^- = 0.3\gamma$, following the quantification in Rudik et al. (2021), who show that the negative productivity impact of cold temperatures is much weaker than for hot temperatures.

Finally, to calibrate $\tau_i^\star$, I use also an intermediary assumption between the following two cases: (i) the representative agent economy, like Barrage and Nordhaus (2024), would assume $\tau_i^\star = \tau_{it_0}$, which implies that $\tau_{it} - \tau_i^\star = \Delta_i \mathcal{T}_t$: differences in damages only comes from increases in aggregate temperature. The analysis by Bilal and Känzig (2024) shows that climate damage on GDP comes in large part from the increase in global temperature, causing extreme events. In contrast, (ii) a different view in heterogeneous countries economies would set $\tau_i^\star = \tau^\star$ the same for all regions, at an "ideal" temperature, as in Krusell and Smith (2022) and Kotlikoff, Kubler, Polbin, Sachs and Scheidegger (2021). In this case, differences in climate damages come essentially from differences in initial temperatures. I take the intermediary step and assume $\tau_i^\star = \alpha^T \tau^\star + (1-\alpha^T)\tau_{it_0}$, where $\alpha^\tau = 0.5$ and $\tau^\star = 14.5$ is the average spring temperature of developed economies – and around the yearly average of places like California or Spain.

5.6 Heterogeneity

In this section, I summarize the different dimensions of heterogeneity included in the model and aggregate the parameters of the calibration in Table 1.

Table 2: Heterogeneity across countries

Dimension of heterogeneity	Model parameter	Matched variable from the data	Source of the data
Population	Country size $\mathcal{P}_i$	Population $\mathcal{P}_i$	UN Population Prospect
TFP/technology/institutions	Firm productivity z_i	GDP per capita (2016-PPP) y_i	World Bank/Maddison project
Productivity in energy	Energy-augmenting productivity z_i^e	Energy cost share s_i^e	SRE Energy Institute (2024)
Cost of coal energy	Cost of coal production κ_i^c	Energy mix/coal share e_i^c/e_i	SRE Energy Institute (2024)
Cost of non-carbon energy	Cost of non-carbon production κ_i^r	Energy mix/coal share e_i^r/e_i	SRE Energy Institute (2024)
Local temperature	Initial temperature τ_{it_0}	Pop-weighted yearly temperature	Burke et al. (2015)
Pattern scaling	Pattern scaling Δ_i	Sensitivity of τ_{it} to world $\mathcal{T}_t$	Burke et al. (2015)
Oil-gas reserves	Reserves $\mathcal{R}_i$	Proved Oil-gas reserves $\mathcal{R}_i$	SRE Energy Institute (2024)
Cost of oil-gas extraction	Slope of extraction cost $\bar{\nu}_i$	Oil-gas extracted/produced e_i^x	SRE Energy Institute (2024)
Cost of oil-gas extraction	Curvature of extraction cost ν_i	Profit π_i^f / energy rent	World Bank / WDI

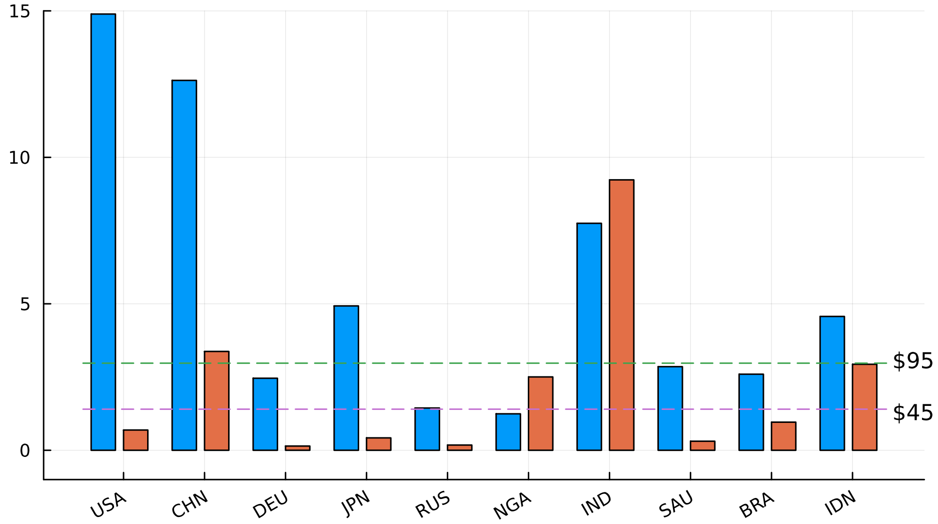
6 Quantitative model results

The results of this section are preliminary. I simulate the model with 68 countries. However, to display the quantitative results in the graphs, I show examples of ten large "example countries": the US, China, Germany, Japan, Russia, Nigeria, India, Saudi Arabia, Brazil, and Indonesia.

6.1 Local and Social Cost of Carbon

In the following graph, I first display the Social Cost of Carbon and Local Cost of Carbon derived above. Recall that in the Second-Best, we have $SCC = \sum_{\mathbb{I}} \widehat{\psi}_i^w \, LCC_i$.

In the next graph we plot the difference between $LCC_i = \frac{\psi_i^{\mathcal{S}}}{\lambda_i^w}$ (blue, left bars) and the carbon-tax relevant terms: $\widehat{\psi}_i^w LCC_i = \frac{\psi_i^{\mathcal{S}}}{\overline{\psi}^w}$ (red, right bars). We see that the US and China have the largest Local Costs of Carbon, since, as argued above, the LCC_i scales proportionally to population $\mathcal{P}_i$ and GDP per capita y_i. However, when we account for inequality and the social welfare weight – i.e. the Pareto weight times marginal utility of consumption – we observe that now India, Indonesia, and Nigeria are the countries with the higher "welfare/policy-relevant" Local Cost of Carbon.



In the competitive equilibrium – i.e. without mitigation policies implemented, the Social Cost of Carbon can be written in two ways: one without accounting for redistributive effects $\mathbb{E}^{\mathbb{I}}[\omega_i LCC_i]$ and one that does integrate inequality $\mathbb{E}^{\mathbb{I}}[\widehat{\psi}_i^w LCC_i]$. In our numerical exercise, we obtain:

$$\mathbb{E}^{\mathbb{I}}[\omega_i LCC_i] = \$95/tCO_2 \qquad \mathbb{E}^{\mathbb{I}}[\widehat{\psi}_i^w LCC_i] = \$45/tCO_2$$

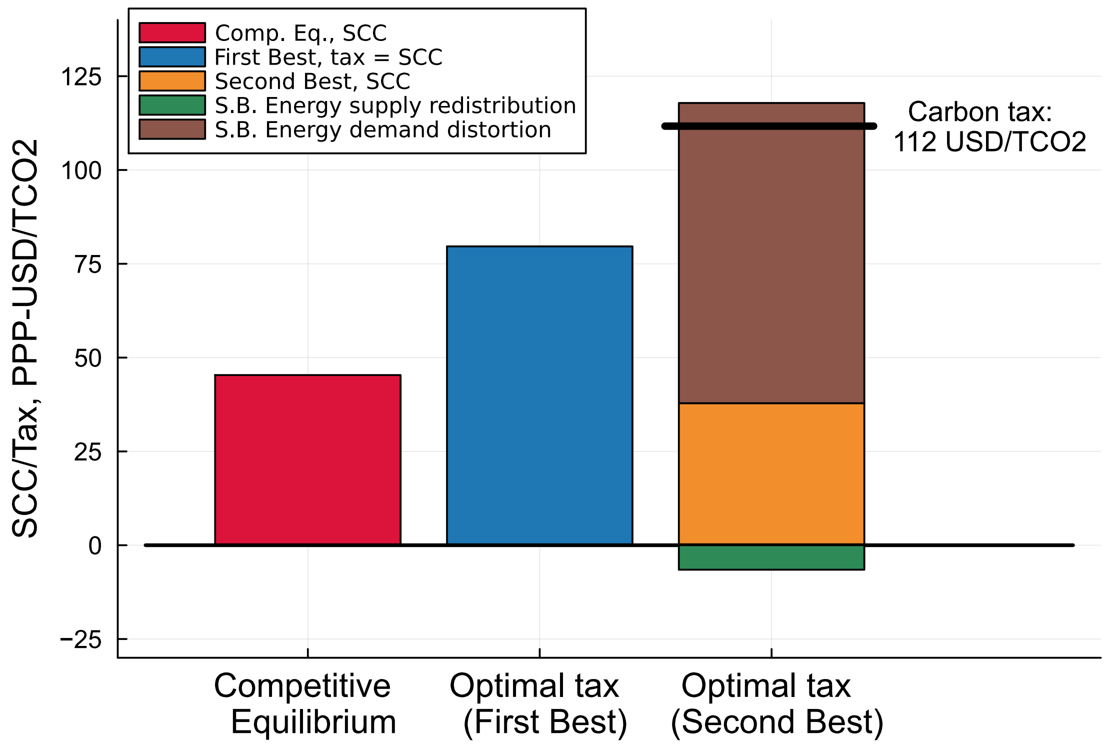
The result is displayed in Section 6.1.

The optimal carbon tax with heterogeneity and redistribution motives in the Second Best allocation is given as:

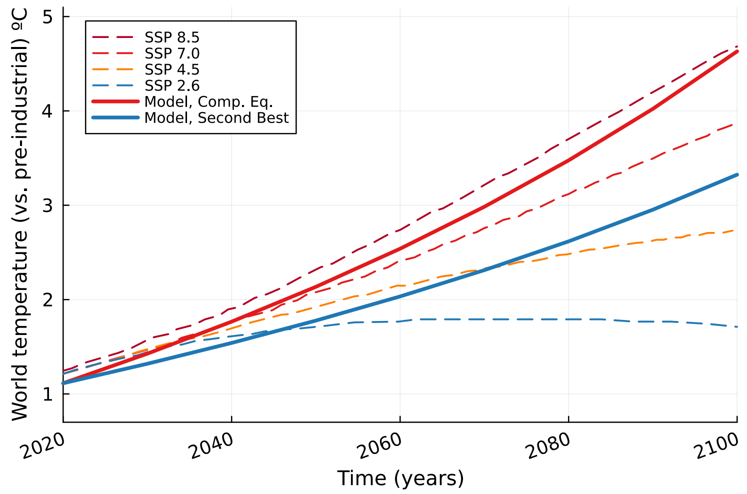
$$\mathrm{t}^{\varepsilon} = SCC + Supply\ Redistribution + Demand\ Distortion$$

Correcting for the two additional redistributive terms, the optimal carbon tax is 5% lower than the SCC, because the Supply Redistribution and Demand Distortion term largely offset each others. The results with utilitarian weights, i.e. $\omega_i = 1, \forall i$, are displayed in Section 6.1. The carbon tax much higher, as both the Supply Redistribution and Demand Distortion. The planner would use carbon tax as a tool for redistribution using energy price and terms-of-trade manipulation.

The dynamic setting allows us to compute the temperature path. In Section 6.1, I display the path of global temperature in the competitive equilibrium (i.e. the Business as Usual scenario)



vs. Second-Best with the optimal carbon tax. I show that the second-best allocation aligns closely with the Shared Socioeconomic Pathway (SSP) 4.5, i.e., the "Middle of the Road" Scenario. This is due to the fact that global emissions are only reduced by 35%, as the marginal costs of climate change equate the marginal cost of mitigation and the energy transition. As a result, it would not be optimal to reduce carbon emissions to net-zero in this class of Integrated Assessment Models.



7 Conclusion

In this paper, I examine how to design the optimal carbon policy in a world marked by multiple layers of inequality. Through both theoretical and quantitative analysis, I demonstrate that the traditional approach of setting a global carbon tax equal to the Social Cost of Carbon needs to be reconsidered when accounting for global inequalities in emissions, income, climate vulnerability, and policy impacts.

The key theoretical insights emerge from both a simplified model and a rich, dynamic framework. In the First-Best scenario with available redistributive instruments, the optimal policy follows the Pigouvian principle, where the carbon tax equals the Social Cost of Carbon. However, in the more realistic Second-Best scenario without cross-country transfers, the optimal policy must balance climate externalities with redistributive considerations. This leads to two main findings: First, the uniform global carbon tax needs to be adjusted for both supply redistribution and demand distortion effects. Second, when country-specific carbon taxes are possible, they should be inversely proportional to social welfare weights – the product of the planners' Pareto weights and the marginal utility of consumption – resulting in lower taxes for developing economies.

The quantitative analysis, based on a calibrated model covering 68 countries, reveals that accounting for inequality reduces the Social Cost of Carbon twofold (from $100 to $50). This reduction reflects the higher marginal value of wealth in poorer countries. The model also captures competing effects on the optimal carbon tax from energy markets equilibrium: downward pressure to protect fossil-fuel exporters versus upward pressure to minimize energy use distortions. These effects largely offset each other, resulting in an optimal carbon tax slightly above $110, aligned with existing estimates.

These findings have important implications for international climate policy. They suggest that a one-size-fits-all approach to carbon taxation could be suboptimal when considering global inequalities. Instead, policymakers should consider differentiated carbon pricing schemes that account for countries' economic development levels, heterogeneous climate damages, and energy market exposure. This research also highlights the importance of developing complementary redistributive mechanisms in international climate policy to achieve both environmental and equity objectives.

Future research would extend this analysis by examining the dynamic evolution of these effects as developing economies grow, climate impacts intensify, and fossil fuel reserves deplete. Moreover, it would assess the role of uncertainty and climate risk for the optimal policy as in Bourany (2023). Additionally, investigating the political economy constraints and implementation challenges of differentiated carbon pricing schemes matters considerably for practical policy design, as I analyzed in Bourany (2024*a*).

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A Toy model – Theoretical results

The first two sections are forthcoming. For a similar model – with multi-country trade – the proofs of the main theorems are analogous to the ones provided in the appendix of Bourany (2024*a*).

A.1 First Best

A.2 Second-Best

A.3 Cap and Trade

Competitive equilibrium

$$\max_{c_i, e_i, \varepsilon_i} U(c_i) \qquad s.t \qquad \begin{cases} c_i & = \mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e(e^x_i - e_i) - \mathcal{C}_i(e^x_i) + q^\varepsilon(\bar{\varepsilon}_i - \varepsilon_i) \quad [\lambda_i] \\ \xi e_i & \leq \varepsilon_i \qquad [\eta_i q^\varepsilon] \end{cases}$$

where η_i is the Lagrange multiplier/shadow for each dollar of carbon permits – hence the normalization by q^ε. The carbon intensity of energy is ξ: it requires ξ allowance to get enough carbon permit ε_i for one unit (e.g. ton of oil equivalent) of energy e_i. This yields the optimality condition of the firm for energy e_i and carbon permits ε_i:

$$MPe_i = q^e + \xi \eta_i q^\varepsilon \qquad [\varepsilon_i] \qquad \eta_i q^\varepsilon = q^\varepsilon$$

which implies that the implicit carbon tax is $MPe_i - q^e = \xi \tilde{\mathrm{t}}^\varepsilon = \xi q^\varepsilon$. Multiplier η_i for inequality constraint $e_i \geq \varepsilon_i$. Optimality of ε_i: $\eta_i = 1$.

The Lagrangian for the Second Best allocation in this context:

$$\begin{aligned} \mathcal{L}(c_i, e_i, \varepsilon_i, \bar{\varepsilon}_i, \overline{\mathcal{E}}, \boldsymbol{\lambda}) = & \sum_i \mathcal{P}_i \omega_i U(c_i) + \sum_i \omega_i \mathcal{P}_i \phi_i \Big(\mathcal{D}_i(\mathcal{S}) z_i F(e_i) + q^e(e^x_i - e_i) - \mathcal{C}_i(\mathcal{P}_i e^x_i)/\mathcal{P}_i + q^\varepsilon(\bar{\varepsilon}_i - \varepsilon_i) + \mathrm{t}^{ls}_i - c_i \Big) \\ & + \mu^e q^e \Big(\sum_j \mathcal{P}_i (e^x_j - e_j) \Big) + \mu^\varepsilon q^\varepsilon \Big(\overline{\mathcal{E}} + \sum_j \mathcal{P}_i (\bar{\varepsilon}_i - \varepsilon_i) \Big) + \mu^g \Big(\sum_i q^\varepsilon \mathcal{P}_i (\varepsilon_i - \bar{\varepsilon}_i) - \mathcal{P}_i \mathrm{t}^{ls}_i \Big) \\ & + \sum_i \mathcal{P}_i \omega_i \theta_i \big(q^e - \mathcal{C}'_i(\mathcal{P}_i e^x_i) \big) + \sum_i \mathcal{P}_i \omega_i \upsilon_i (MPe_i - q^e - \xi q^\varepsilon) \end{aligned}$$

Two options for rebate:
(i) Global rebate (government budget): $\sum_i \mathrm{t}^{ls}_i = \sum_i q^\varepsilon(\varepsilon_i - \bar{\varepsilon}_i)$ (multiplier μ^g), or:
(ii) Local lump-sum rebate $\mathrm{t}^{ls}_i = q^\varepsilon(\varepsilon_i - \bar{\varepsilon}_i)$ (in that case $\mu^g = 0$, we replace t^{ls}_i in budget constraint)

Planner's optimality conditions

- $[c_i]$

$$\phi_i = U'(c_i)$$

- $[e_i]$

$$\mathcal{P}_i \omega_i \phi_i [MPe_i - q^e] + \xi \sum_j \mathcal{P}_j \omega_j \phi_j \mathcal{D}'_i(\mathcal{S}) z_j F(e_j) - \mu^e + \mathcal{P}_i \omega_i \upsilon_i \mathcal{D}_i(\mathcal{S}) z_i F''(e_i) = 0$$

$$\mathcal{P}_i \omega_i \phi_i \xi q^\varepsilon = \xi \overline{\phi} SCC + \mu^e - \mathcal{P}_i \omega_i \upsilon_i \mathcal{D}_i(\mathcal{S}) z_i F''(e_i)$$

- $[e_i^x]$

$$\underbrace{\mathcal{P}_i\omega_i\phi_i[q^e - \mathcal{C}_i'(\mathcal{P}_i e_i^x)]}_{=0} + \mu^e\mathcal{P}_i - \mathcal{P}_i\omega_i\theta_i\mathcal{P}_i\mathcal{C}_i''(\mathcal{P}_i e_i^x)$$

$$\mathcal{P}_i\omega_i\theta_i = \mu^e/\big(\mathcal{C}_i''(\mathcal{P}_i e_i^x)\big)$$

- $[q^e]$

$$\sum_i \mathcal{P}_i\omega_i\phi_i(e_i^x - e_i) + \sum_i \mathcal{P}_i\omega_i\theta_i - \sum_i \mathcal{P}_i\omega_i\upsilon_i = 0$$

- $[\bar{\mathcal{E}}]$

$$\mu^\varepsilon q^\varepsilon = 0$$

Given that the planner controls directly the supply of carbon allowances, the market clearing is not a binding constraint for the optimal policy.

- $[\varepsilon_i]$ & $[\bar{\varepsilon}_i]$ & $[\mathrm{t}_i^{ls}]$ in the case of global rebate:

$$\begin{aligned}
[\varepsilon_i] \qquad & -\omega_i\mathcal{P}_i\phi_i q^\varepsilon - q^\varepsilon\mu^\varepsilon\mathcal{P}_i + \mu^g q^\varepsilon\mathcal{P}_i = 0 \\
& \mu^\varepsilon - \mu^g = -\omega_i\phi_i \\
[\bar{\varepsilon}_i] \qquad & \omega_i\mathcal{P}_i\phi_i q^\varepsilon + q^\varepsilon\mu^\varepsilon - \mu^g q^\varepsilon\mathcal{P}_i = 0 \\
& \mu^\varepsilon - \mu^g = -\omega_i\phi_i \\
[\mathrm{t}_i^{ls}] \qquad & \sum_i \mathcal{P}_i\omega_i\phi_i = \mu^g\sum_i \mathcal{P}_i
\end{aligned}$$

This implies that the planner implements full redistribution using the distribution of “free” carbon permits $\bar{\varepsilon}_i$ and lump-sum transfers t_i^{ls}. The exact mix $\{\mathrm{t}_i^{ls}, \bar{\varepsilon}_i\}$ is undetermined as long as the following condition holds:

$$\omega_i\phi_i = \omega_i U'(c_i) = \mu^g = \omega_j U'(c_j)$$

- $[\varepsilon_i]$ & $[\bar{\varepsilon}_i]$ Local rebate: no impact in the budget constraint

$$[\varepsilon_i] - q^\varepsilon\mu^\varepsilon\mathcal{P}_i = 0$$

$$[\bar{\varepsilon}_i] q^\varepsilon\mu^\varepsilon\mathcal{P}_i = 0$$

Given that the purchase of carbon permits, and hence the revenue of “free” carbon permits, are redistributed/taxed lump-sum, we obtain that $\bar{\varepsilon}_i$ is a redundant policy instrument. Moreover, the planner chooses the same policy as the agent, such that $\varepsilon_i = \xi e_i$.

- $[q^\varepsilon]$

$$\sum_i \mathcal{P}_i\omega_i\upsilon_i = 0$$

Again, as in the carbon taxation case, this implies no "aggregate distortion" at the global level.

B Quantitative model - Competitive equilibrium

Dynamics of the individual state variables $\underline{s}_{it} = (w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t)$:

$$\dot{w}_{it} = (r^\star_t - n_i - \bar{g}_i)w_{it} + \mathcal{D}(\tau_{it})F(k_{it}, e_{it}) - (r^\star_t + \delta)k_{it} + \pi^f_i(q^f_t, \mathcal{R}_{it}) - q^e_{it}e_{it} - c_{it}$$

$$\mathcal{E}_t = \bar{\xi}_t \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}(\xi^f e^f_{it} + \xi^c_{it})$$

$$\dot{\tau}_{it} = \zeta(\Delta_i\, \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})) \qquad \dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t$$

$$\dot{\mathcal{R}}_{it} = -e^f_{it} \qquad q^f_t = \bar{\nu}_t(e^x_{it}/\mathcal{R}_{it})^{\nu_i}$$

Household problem: Pontryagin Maximum Principle

$$\begin{aligned}
\mathcal{H}^{hh}(s, \{c,k,e^f,e^c,e^r\}, \{\lambda\}) = {} & e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}u(c_i, \tau_i) + e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\lambda^w_{it}\Big((r^\star_t-(n_i+\bar{g}_i))w_{it} + \mathcal{D}^y_i(\tau_{it})z_iF(k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) \\
& + \pi^f_i(q^f_t, \mathcal{R}_{it}) - (r^\star_t + \delta)k_{it} - q^f_t e^f_{it} - q^c_{it}e^c_{it} - q^r_{it}e^r_{it} - c_{it}\Big) \\
& + e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\lambda^S_{it}(\mathcal{E}_t - \delta_s \mathcal{S}_t) \\
& + e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\lambda^\tau_{it}(\zeta(\Delta_i\, \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})))
\end{aligned}$$

Choice of controls:

$$u_c(c_{it}, \tau_{it}) = \mathcal{D}^u(\tau_{it})u'(\mathcal{D}(\tau_{it})c_{it}) = \lambda^w_{it}$$

$$[k_t] \qquad MPk_{it} = r^\star_t + \delta$$

$$[x_t] \qquad MPe^x_{it} = \mathcal{D}^y_i(\tau_{it})z_i\ \partial_x F(k_{it}, e^f_{it}, e^c_{it}, e^r_{it}) = q^x_{it} \qquad \text{for } x \in \{f, c, r\}$$

$$[e^x_t] \qquad q^f_t = \nu^f_{i\,e^x}(e^x_{it}, \mathcal{R}_{it})$$

The Pontryagin maximum principle for the states $\{w_{it}, \tau_{it}, \mathcal{S}_t\}$

$$[w_t] \qquad \dot{\lambda}^w_{it} = \lambda^w_{it}(\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}_w(\cdot) = \lambda^w_{it}[(\rho - n_i - (1-\eta)\bar{g}_i) - (r^\star_t - n_i - \bar{g}_i)]$$

$$\Rightarrow \qquad \dot{\lambda}^w_{it} = \lambda^w_{it}(\rho + \eta\bar{g}_i - r^\star_t)$$

$$[\tau_{it}] \qquad \dot{\lambda}^\tau_{it} = \lambda^\tau_{it}(\rho-n_i-(1-\eta)\bar{g}_i + \zeta) + \underbrace{\gamma^y(\tau_{it}-\tau^\star_i)\mathcal{D}^y_i(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} z_iF(k_{it}, e_{it})\lambda^w_{it} + \underbrace{\gamma^u(\tau_{it}-\tau^\star_i)\mathcal{D}^u_i(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau_{it})c_{it})c_{it}$$

$$[\mathcal{S}_t] \qquad \dot{\lambda}^S_{it} = \lambda^S_{it}(\rho-n_i-(1-\eta)\bar{g}_i + \delta_s) - \zeta\,\chi\,\Delta_i\,\lambda^\tau_{it}$$

Recall if $\rho + \eta\bar{g}_i < r^\star_t$, then λ^w_t decreases (and consumption increases) over time.

Solving the ODE for the Local Cost of Carbon

$$\lambda^S_{it} = -\int_t^\infty e^{-(\rho-n_i-(1-\eta)\bar{g}_i+\delta_s)(s-t)}\zeta\chi\Delta_i\lambda^\tau_{is}ds$$

$$\text{with} \qquad \lambda^\tau_{it} = \int_t^\infty e^{-(\rho-n_i-(1-\eta)\bar{g}_i+\zeta)(s-t)}(\tau_{is}-\tau^\star_i)(1+(\alpha^\gamma-1)\mathbb{1}_{\{\tau_{is}<\tau^\star_i\}})\left[\gamma^y y_{is} + \gamma^u c_{is}\right]\lambda^w_{is}ds$$

$$\lambda^S_{it} \xrightarrow[\zeta\to\infty]{} -\int_t^\infty e^{-(\rho-n_i-(1-\eta)\bar{g}_i+\delta_s)(s-t)}\chi\Delta_i(\tau_{is}-\tau^\star_i)\left[\gamma^y y_{is} + \gamma^u c_{is}\right]\lambda^w_{is}ds\ ,$$

with output is $y_{it} = z_i \mathcal{D}_i^y(\tau_{it}) F(k_{it}, e_{it})$ and $\lambda_{it}^w = \mathcal{D}_i^u(\tau_{it}) u'\big(\mathcal{D}_i^u(\tau_{it}) c_{it}\big)$. Use the Euler equation, or costate dynamics:

$$\dot{\lambda}_{it}^w = \lambda_{it}^w(\rho + \eta \bar{g}_i - r_t^\star) \qquad \Rightarrow \qquad \lambda_{it}^w = \lambda_{is}^w e^{-\int_t^s (\rho+\eta\bar{g}_i - r_s^\star) du}$$

for $s > t$, which gives the Local Cost of Carbon:

$$\begin{aligned} LCC_{it} = -\frac{\lambda_{it}^S}{\lambda_{it}^w} \rightarrow \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \delta_s)(s-t)} \chi \Delta_i (\tau_{is} - \tau_i^\star)\big[\gamma^y y_{is} + \gamma^u c_{is}\big] e^{+\int_t^s (\rho + \eta \bar{g}_i - r_s^\star) du} ds \ , \\ LCC_{it} = \int_t^\infty e^{-\delta_s(s-t) - \int_t^s (r_u^\star - n_i - \bar{g}_i) du} \chi \Delta_i (\tau_{is} - \tau_i^\star)\big[\gamma^y y_{is} + \gamma^u c_{is}\big] ds \end{aligned}$$

This implies that the future damages are discounted faster if $r_{it}^\star > n_i + \bar{g}_i$. Conversely, if growth rate of population n_i and TFP $\bar{g}_i$ are high compared to the world interest rate – think of developing economies – then they would put more weights on future damages on output and consumption per capita.

C Quantitative model - First-Best

First-Best allocation results from the global welfare maximization of the planner, who has access to all the instruments:

$$\mathcal{W}_{t_0} = \max_{\{c, k, e^f, e^c e^r, e^x, \bar{e}^c, \bar{e}^r\}} \sum_{\mathbb{I}} \mathcal{P}_i \, \omega_i \int_{t_0}^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \, u\big(\mathcal{D}_i^u(\tau_{it}) \, c_{it}\big) \ dt$$

subject to the good and energy resource constraints and the climate system:

$$\sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\Big[c_{it} + (\dot{k}_{it} + (n_i + \bar{g}_i + \delta)k_{it}) + \nu_i^f(e_{it}^x, \mathcal{R}_{it}) + \kappa_i^c e_{it}^c + \kappa_{it}^r e_{it}^r\Big] = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) \qquad [\phi_t^w]$$

$$E_{it}^f = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \, e_{it}^f = \sum_{i\in\mathbb{I}} e^{(n_i+\bar{g}_i)t} e_{it}^x \qquad [\mu_t^f] \qquad \bar{e}_i^c = e_i^c \qquad [\mu_{it}^c] \qquad \bar{e}_i^r = e_i^r \qquad [\mu_{it}^r]$$

$$\dot{\mathcal{S}}_t = \mathcal{E}_t - \delta_s \mathcal{S}_t \qquad \mathcal{E}_t := \sum_{\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) \qquad [\phi_t^S]$$

$$\dot{\tau}_{it} = \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big) \qquad [\phi_{it}^\tau]$$

Let us define several objects: total population $\mathcal{P}_t = \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{n_i t}$, $\mathcal{P} = \mathcal{P}_{t_0} = \sum_i \mathcal{P}_i$ and global population growth rate:

$$n_t = \tfrac{1}{\mathcal{P}_t} \sum_{i\in\mathbb{I}} n_i \mathcal{P}_i e^{n_i t}$$

and the welfare-relevant discount rate:

$$\begin{aligned} \mathcal{P} e^{-\int_{t_0}^t \bar{\rho}_s ds} &= \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \\ \bar{\rho}_t &= \frac{1}{\mathcal{P}} \sum_{i\in\mathbb{I}} (\rho - n_i - (1-\eta)\bar{g}_i) \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds} = \frac{\sum_{i\in\mathbb{I}} (\rho - n_i - (1-\eta)\bar{g}_i) \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}}{\sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t}} \end{aligned}$$

Note, all the analysis can accomodate time-varying population growth rate n_{it} and time-varying TFP growth $\bar{g}_{it}$.

First-Best Optimal Control Problem – Pontryagin Principle

The Hamiltonian for the Social planner is:

$$
\begin{aligned}
\mathcal{H}^{fb}(\boldsymbol{s},\boldsymbol{c},\boldsymbol{\phi}) &= \sum_{\mathbb{I}} \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\, u(\mathcal{D}_i^u(\tau_{it})\, c_{it}) \\
&+ e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^w \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\Big(\mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) - (n_i+\bar{g}_i+\delta)k_{it} - e^{\nu_i(n_i+\bar{g}_i)t}\nu_i^f(e_{it}^x,\mathcal{R}_{it}) - \kappa_i^c e_{it}^c - \kappa_{it}^r e_{it}^r - c_{it}\Big) \\
&+ e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \sum_{i\in\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}\Big(e_{it}^x - e_{it}^f\Big) \\
&+ e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^S \Big\{ \sum_{\mathbb{I}} \mathcal{P}_i e^{(n_i+\bar{g}_i)t}(\xi^f e_{it}^f + \xi^c e_{it}^c) - \delta_s \mathcal{S}_t \Big\} \\
&+ \sum_{i\in\mathbb{I}} \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t} \phi_{it}^\tau\ \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big)
\end{aligned}
$$

Pontryagin maximum principle, optimality conditions, first for controls $\{c, e^f, e^c, e^r, e^x\}$

- Consumption $[c_{it}]$

$$
\begin{aligned}
\omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\, u_c\big(\mathcal{D}_i^u(\tau_{it})\, c_{it}\big) &= e^{-\int_{t_0}^t \bar{\rho}_s ds} \phi_t^w \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \\
\omega_i\, e^{-(\rho+\eta\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds}\, \mathcal{D}_i^u(\tau_{it}) u'\big(\mathcal{D}_i^u(\tau_{it})\, c_{it}\big) &= \phi_t^w
\end{aligned}
$$

- Energy sources $[e_{it}^f]$, $[e_{it}^c]$, $[e_{it}^r]$

$$
\begin{aligned}
\phi_t^w MPe_{it}^f &= \mathcal{D}_i(\tau_{it}) z_{it} F_{e^f}(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = \mu_t^f - \xi^f \phi_t^S \\
MPe_{it}^c &= \mathcal{D}_i(\tau_{it}) z_{it} F_{e^c}(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r) = \kappa_i^c - \xi^c \phi_t^S \qquad MPe_{it}^r = \kappa_i^r
\end{aligned}
$$

- Energy extraction $[e_{it}^x]$

$$
\begin{aligned}
\phi_t^w e^{\nu_i(n_i+\bar{g}_i)t} \nu_{i\,e^x}^f(e_{it}^x, \mathcal{R}_{it}) &= \mu_t^f \\
e^{\nu_i(n_i+\bar{g}_i)t} \bar{\nu}_i \Big(\frac{e_{it}^x}{\mathcal{R}_{it}}\Big)^{\nu_i} &= \frac{\mu_t^f}{\phi_t^w}
\end{aligned}
$$

  note that e^x is the extraction rate per effective capita: with population/TFP growth, the marginal cost become larger.
- Note that we simplify the problem by avoiding treating e_{it}^c and $\bar{e}_{it}^c$ and e_{it}^r and $\bar{e}_{it}^r$ as separate variables.

Pontryagin maximum principle, optimality conditions, second for states $\{k_{it}, \mathcal{S}_t, \tau_{it}\}_{it}$

- Capital $[k_i]$

$$
\begin{aligned}
\dot{\phi}_t^w &= \phi_t^w \bar{\rho}_t - \mathcal{H}_k^{fb}(\boldsymbol{s},\boldsymbol{c},\boldsymbol{\lambda}) = \phi_t^w \bar{\rho}_t + \phi_t^w(n_i+\bar{g}_i) - \phi_t^w(MPk_{it} - \delta) \\
\dot{\phi}_t^w &= \phi_t^w\big(\bar{\rho}_t + n_i + \bar{g}_i - (MPk_{it} - \delta)\big)
\end{aligned}
$$

Note if that there is only one country, we get $\bar{\rho}_t = \rho - n_i - (1-\eta)\bar{g}_i$ and then we obtain the standard Euler equation $\dot{\phi}^w_t = \phi^w_t(\rho + \eta\bar{g}_i - r^k_{it})$, with $r^k_{it} = MPk_{it} - \delta$

- Carbon concentration in atmosphere $[\mathcal{S}_t]$

$$\dot{\phi}^S_t = \phi^S_t(\bar{\rho}_t + \delta_s) - \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds} \zeta \Delta_i \chi \, \phi^\tau_{it}$$

- Temperature $[\tau_{it}]$, normalized by $e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}$ local discounting

$$\dot{\phi}^\tau_{it} = \phi^\tau_{it}(\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}^{fb}_\tau(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{\lambda})$$

$$\dot{\phi}^\tau_{it} = \phi^\tau_{it}(\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + \omega_i u'(\mathcal{D}^u_i(\tau_{it})\, c_{it})\mathcal{D}^u_i(\tau_{it})(\tau_{it} - \tau^\star_i)\gamma^u c_{it} + e^{-\int_{t_0}^t \bar{\rho}_s ds + (n_i+g_i)t + (\rho-n_i-(1-\eta)\bar{g}_i)t} \phi^w_t(\tau_{it} - \tau^\star_i)\gamma^y y_{it}$$

Proof of Proposition 6

Solving for the shadow value of temperature ϕ^τ_{it} and carbon ϕ^S_t

$$\dot{\phi}^\tau_{it} = \phi^\tau_{it}(\rho - n_i - (1-\eta)\bar{g}_i + \zeta) + e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho+\eta\bar{g}_i)t} \phi^w_t(\tau_{it} - \tau^\star_i)[\gamma^y y_{it} + \gamma^u c_{it}]$$

$$\phi^\tau_{it_0} = \int_{t_0}^\infty e^{-(\rho-n_i-(1-\eta)\bar{g}_i+\zeta)(t-t_0)} e^{-\int_{t_0}^t \bar{\rho}_s ds + (\rho+\eta\bar{g}_i)t} \phi^w_t(\tau_{it} - \tau^\star_i)[\gamma^y y_{it} + \gamma^u c_{it}]$$

$$\phi^\tau_{it_0} = \int_{t_0}^\infty e^{-\int_{t_0}^t \bar{\rho}_s ds - (\zeta - n_i - \bar{g}_i)(t-t_0)} \phi^w_t(\tau_{it} - \tau^\star_i)[\gamma^y y_{it} + \gamma^u c_{it}]$$

Solving for the SCC_i, we get:

$$\phi^S_{t_0} = -\int_{t_0}^\infty e^{-\int_{t_0}^t \bar{\rho}_s ds - \delta_s(t-t_0)} \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0)+\int_{t_0}^t \bar{\rho}_s ds} \zeta\Delta_i\chi \, \phi^\tau_{it} dt$$

$$\phi^S_{t_0} = -\int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0)} \zeta\Delta_i\chi \, \phi^\tau_{it} dt$$

$$\phi^S_{t_0} \xrightarrow[\zeta\to\infty]{} -\int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0)} \chi\Delta_i(\tau_{it} - \tau^\star_i)\big[\gamma^y y_{it} + \gamma^u c_{it}\big]\phi^w_t dt \ ,$$

Realizing that the marginal value of wealth:

$$\dot{\phi}^w_t = \phi^w_t(\bar{\rho}_t + n_i + \bar{g}_i - \underbrace{(MPk_{it} - \delta)}_{=r^k_{it}}) \qquad \Rightarrow \qquad \phi^w_{t_0} = \phi^w_t e^{-\int_{t_0}^t (\bar{\rho}_s + n_i + \bar{g}_i - r^k_{is})ds}$$

This implies that the Social Cost of Carbon defined as $SCC_{t_0} = -\frac{\phi^S_{t_0}}{\phi^w_{t_0}}$ can be rewritten as:

$$-\phi^S_{t_0} \xrightarrow[\zeta\to\infty]{} \int_{t_0}^\infty e^{-\delta_s(t-t_0)} \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0)} \chi\Delta_i(\tau_{it} - \tau^\star_i)\big[\gamma^y y_{it} + \gamma^u c_{it}\big]\phi^w_{t_0} e^{-\int_{t_0}^t (r^k_{is} - \bar{\rho}_s - n_i - \bar{g}_i)ds} dt \ ,$$

$$SCC_{t_0} \to \int_{t_0}^\infty \sum_{i\in\mathbb{I}} \omega_i \mathcal{P}_i e^{-[(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0) - \int_{t_0}^t \bar{\rho}_s ds]} \ e^{-\delta_s(t-t_0) - \int_{t_0}^t (r^k_{is} - n_i - \bar{g}_i)ds} \ \chi\Delta_i(\tau_{it} - \tau^\star_i)\big[\gamma^y y_{it} + \gamma^u c_{it}\big] dt \ ,$$

The aggregate discount factor is defined as $\sum_{i\in\mathbb{I}}\omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(t-t_0)} = \mathcal{P}e^{-\int_{t_0}^{t}\bar{\rho}_s ds}$, and given that c_{it}, y_{it} and τ_{it} are bounded, we can simplify the expression. Moreover, changing the order of the sum and integral by Fubini's theorem, we obtain:

$$
\begin{aligned}
SCC_{t_0} &\to \int_{t_0}^{\infty}\sum_{i\in\mathbb{I}}\omega_i\mathcal{P}_i e^{-\delta_s(t-t_0)-\int_{t_0}^{t}(r^k_{is}-n_i-\bar{g}_i)ds}\,\chi\Delta_i(\tau_{it}-\tau_i^\star)\big[\gamma^y y_{it}+\gamma^u c_{it}\big]dt\ ,\\
SCC_{t_0} &\to \sum_{i\in\mathbb{I}}\omega_i\mathcal{P}_i\int_{t_0}^{\infty} e^{-\delta_s(t-t_0)-\int_{t_0}^{t}(r^k_{is}-n_i-\bar{g}_i)ds}\,\chi\Delta_i(\tau_{it}-\tau_i^\star)\big[\gamma^y y_{it}+\gamma^u c_{it}\big]dt\ ,\\
SCC_{t_0} &\to \sum_{i\in\mathbb{I}}\omega_i\mathcal{P}_i\, LCC_{it}
\end{aligned}
$$

where the Local Cost of Carbon LCC_{it} is given in Appendix B. Given that, in the competitive equilibrium, we have free capital flows and frictionless borrowing, it implies $r^k_{it} = MPk_{it}-\delta = r^\star_t$, which gives the results of Proposition 6. □

D Quantitative model - Second-Best

Second-Best allocation results from the global welfare maximization of the planner, subject to choice of a global carbon tax, t^ε, and local lump-sum rebate: $\mathrm{t}^{ls}_{it} = \mathrm{t}^\varepsilon(\xi^f e^f_{it}+\xi^c e^c_{it})$.

$$
\mathcal{W}_{t_0} = \max_{\{c,k,e^f,e^c e^r,e^x,\bar{e}^c,\bar{e}^r\}}\sum_{\mathbb{I}}\mathcal{P}_i\,\omega_i\int_{t_0}^{\infty}e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\,u\big(\mathcal{D}^u_i(\tau_{it})\,c_{it}\big)\ dt
$$

subject to the good and energy resource market clearing and the climate system:

$$
\begin{aligned}
\dot{w}_{it} = (r^\star_t-(n_i{+}\bar{g}_i))w_{it} + \pi^f_i(q^f_t,\mathcal{R}_{it}) + \mathcal{D}^y_i(\tau_{it})z_iF(k_{it},e^f_{it},e^c_{it},e^r_{it}) - (r^\star_t+\delta)k_{it}&\\
-(q^f_t+\xi^f\mathrm{t}^\varepsilon_{it})e^f_{it} - (q^c_{it}+\xi^c\mathrm{t}^\varepsilon_{it})e^c_{it} - q^r_{it}e^r_{it} - c_{it} + \mathrm{t}^{ls}_{it}\ ,& \qquad [\psi^w_{it}]
\end{aligned}
$$

$$
E^f_{it} = \sum_{i\in\mathbb{I}}\mathcal{P}_i e^{(n_i+\bar{g}_i)t}\,e^f_{it} = \sum_{i\in\mathbb{I}}e^{(n_i+\bar{g}_i)t}e^x_{it} \quad [\mu^f_t] \qquad B_t = \sum_{i\in\mathbb{I}}\mathcal{P}_i e^{(n_i+\bar{g}_i)t}(w_{it}-k_{it}) = 0 \quad [\mu^b_t]
$$

$$
\bar{e}^c_i = e^c_i \quad [\mu^c_{it}] \qquad \bar{e}^r_i = e^r_i \quad [\mu^r_{it}]
$$

$$
\dot{\mathcal{S}}_t = \mathcal{E}_t-\delta_s\mathcal{S}_t \qquad \mathcal{E}_t := \sum_{\mathbb{I}}\mathcal{P}_i e^{(n_i+\bar{g}_i)t}(\xi^f e^f_{it}+\xi^c e^c_{it}) \qquad [\psi^S_t]
$$

$$
\dot{\tau}_{it} = \zeta\big(\Delta_i\chi\mathcal{S}_t-(\tau_{it}-\tau_{it_0})\big) \qquad [\psi^\tau_{it}]
$$

as well as the optimality conditions of the agents of the Competitive equilibrium

$$
\begin{aligned}
&[k_t] && MPk_{it} = r^\star_t+\delta && [\upsilon^k_{it}]\\
&[x_t] && MPe^x_{it} = \mathcal{D}^y_i(\tau_{it})z_i\ \partial_x F(k_{it},e^f_{it},e^c_{it},e^r_{it}) = q^x_{it} \quad \text{for } x\in\{f,c,r\} && [\upsilon^x_{it}]\\
&[e^x_t] && q^f_t = \nu^f_{i\,e^x}(e^x_{it},\mathcal{R}_{it}) && [\theta^x_{it}]
\end{aligned}
$$

Using the Primal approach, we can write the Hamiltonian, where the states are $\{w_{it},\mathcal{S}_t,\tau_{it}\}_{it}$, and the controls are $\{c_{it},b_{it},k_{it},e^f_{it},e^c_{it},e^r_{it},e^x_{it},\bar{e}^c_{it},\bar{e}^r_{it}\}_{it}$, and prices $\{r^\star_t,q^f_t,\mathrm{w}_{it},q^c_{it},q^r_{it}\}_{it}$ and where each country i variable is discounted by $\rho - n_i - (1-\eta)\bar{g}_i$:

$$\mathcal{H}^{sb}(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{\psi}) = \sum_{\mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \, u\big(\mathcal{D}_i^u(\tau_{it}) \, c_{it}\big)$$

$$+ \sum_{i \in \mathbb{I}} \psi_{it}^w \omega_i \, \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \Big(\big(r_t^\star - (n_i + \bar{g}_i)\big) w_{it} + \mathcal{D}_i(\tau_{it}) z_{it} F(k_{it}, e_{it}^f, e_{it}^c, e_{it}^r)$$

$$+ \pi_i^f(q_t^f, \mathcal{R}_{it}) - (r^\star + \delta) k_{it} - (q_t^f + \xi^f \mathrm{t}_{it}^\varepsilon) e_{it}^f - (q_{it}^c + \xi^c \mathrm{t}_{it}^\varepsilon) e_{it}^c - q_{it}^r e_{it}^r - c_{it} + \mathrm{t}_{it}^{ls} \Big)$$

$$+ \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [\mu_{it}^c (\bar{e}_{it}^c - e_{it}^c) + \mu_{it}^r (\bar{e}_{it}^r - e_{it}^r)]$$

$$+ e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \Big(e_{it}^x - e_{it}^f \Big) + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^b \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} \Big(w_{it} - k_{it} \Big)$$

$$+ e^{-\int_{t_0}^t \bar{\rho}_s ds} \psi_t^S \Big\{ \sum_{\mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t} (\xi^f e_{it}^f + \xi^c e_{it}^c) - \delta_s \mathcal{S}_t \Big\}$$

$$+ \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^\tau \; \zeta\big(\Delta_i \chi \mathcal{S}_t - (\tau_{it} - \tau_{it_0})\big)$$

$$+ \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \Big[\upsilon_{it}^f (q_t^f + \xi^f \mathrm{t}^\varepsilon - MPe_{it}^f) + \upsilon_{it}^c (q_{it}^c + \xi^c \mathrm{t}^\varepsilon - MPe_{it}^c) + \upsilon_{it}^r (q_{it}^r - MPe_{it}^r) + \upsilon_{it}^k (r_t^\star + \delta - MPk_{it})$$

$$+ \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \Big[\theta_{it}^x (\nu_{i\,e^x}^f (e_{it}^x, \mathcal{R}_{it}) - q_t^f) + \theta_{it}^c (\kappa_{it}^c - q_{it}^c) + \theta_{it}^r (\kappa_{it}^r - q_{it}^r) \Big]$$

PMP: Optimality conditions for the controls $\{c_{it}, b_{it}, k_{it}, e_{it}^f, e_{it}^c, e_{it}^r, e_{it}^x, \bar{e}_{it}^c, \bar{e}_{it}^r\}_{it}$ are:

- Consumption:

$$\omega_i \, \mathcal{P}_i u\big(\mathcal{D}_i^u(\tau_{it}) \, c_{it}\big) = \psi_{it}^w \omega_i \, \mathcal{P}_i$$

- Capital choice:

$$\omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [MPk_{it} - \delta - r_t^\star] - e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^b \sum_{i \in \mathbb{I}} \mathcal{P}_i e^{(n_i + \bar{g}_i)t}$$

$$- \omega_i \mathcal{P}_i e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} [\upsilon_{it}^f \partial_k MPe_{it}^f + \upsilon_{it}^c \partial_k MPe_{it}^c + \upsilon_{it}^r \partial_k MPe_{it}^r + \upsilon_{it}^k \partial_k MPk_{it}] = 0$$

$$\mu_t^b = -e^{-[(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds]} \omega_i \big[\upsilon_{it}^f \partial_k MPe_{it}^f + \upsilon_{it}^c \partial_k MPe_{it}^c + \upsilon_{it}^r \partial_k MPe_{it}^r + \upsilon_{it}^k \partial_k MPk_{it} \big]$$

  The multiplier μ_t^b represents the shadow value of liquidity of aggregate bonds. If we increased bond supply B_t, it would decrease the interest rate and improve the ability of firms to borrow and invest, decreasing the marginal value of capital. This redistributive effect has an impact on the firm inputs optimality conditions for input x, written with $\upsilon_{it}^x \partial_k MPx_{it}$. As a result, μ_t^b is the equilibrium value equalizing these different redistributive/distortive effects.

- Energy extraction – Oil-gas (Fossil) $[e_{it}^x]$

$$\omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \psi_{it}^w [q_t^f - \nu_{i\,e^x}^f (e_{it}^x, \mathcal{R}_{it})] + e^{-\int_{t_0}^t \bar{\rho}_s ds} \mu_t^f \mathcal{P}_i e^{(n_i + \bar{g}_i)t}$$

$$+ \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t} \theta_{it}^x \mathcal{P}_i \nu_{i\,e^x e^x}^f (e_{it}^x, \mathcal{R}_{it})$$

$$\mu_t^f = -e^{-[(\rho + \eta \bar{g}_i)t - \int_{t_0}^t \bar{\rho}_s ds]} \omega_i \, \theta_{it}^x \mathcal{P}_i e^{(n_i + \bar{g}_i)} \nu_{i\,e^x e^x}^f (e_{it}^x, \mathcal{R}_{it})$$

The multiplier μ^f_t is the shadow value of liquidity of aggregate oil-gas supply. If we increased supply E^f_t, it would decrease the oil-gas price rate q^f_t going down the supply curve, as denoted by the curvature $\nu^f_{i\,e^x e^x}$ and weighted by the shadow value of optimality of the fossil firm's extraction. Note also that we scaling the curvature of the cost by population $\mathcal{P}_i e^{(n_i+\bar{g}_i)t}$ – population growth push extraction along the supply curve.

- Energy production (Coal and renewable)

$$\begin{aligned} \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\mu^c_{it} &= 0 \\ \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\mu^r_{it} &= 0 \end{aligned}$$

There's no redistribution effects across countries through the market clearing, due to the fact that (i) the coal (and renewable) are traded locally, and (ii) there are no profits from coal and renewable production.

- Energy consumption – Oil-gas (Fossil)

$$\begin{aligned} &\omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[MPe^f_{it}-q^f_t] - e^{-\int_{t_0}^t \bar{\rho}_s ds}\mu^f_t\mathcal{P}_i e^{(n_i+\bar{g}_i)t} + \xi^f e^{-\int_{t_0}^t \bar{\rho}_s ds}\psi^S_t\mathcal{P}_i e^{(n_i+\bar{g}_i)t} + \\ &\quad - \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\left[\upsilon^f_{it}\partial_{e^f}MPe^f_{it} + \upsilon^c_{it}\partial_{e^f}MPe^c_{it} + \upsilon^r_{it}\partial_{e^f}MPe^r_{it} + \upsilon^k_{it}\partial_{e^f}MPk_{it}\right] = 0 \end{aligned}$$

$$\omega_i\,\mathcal{P}_i\,e^{-(\rho+\eta\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\psi^w_{it}\xi^f\mathrm{t}^\varepsilon_t = \mathcal{P}_i\mu^f_t - \mathcal{P}_i\xi^f\psi^S_t + \omega_i\mathcal{P}_i e^{-(\rho+\eta\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\left[\upsilon^f_{it}\partial_{e^f}MPe^f_{it} + \upsilon^c_{it}\partial_{e^f}MPe^c_{it} + \upsilon^r_{it}\partial_{e^f}MPe^r_{it} + \upsilon^k_{it}\partial_{e}\right.$$

- Energy consumption – Coal

$$\begin{aligned} &\omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[MPe^c_{it}-q^c_{it}] - \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\mu^c_{it} + \xi^c e^{-\int_{t_0}^t \bar{\rho}_s ds}\psi^S_t\mathcal{P}_i e^{(n_i+\bar{g}_i)t} + \\ &\quad - \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\left[\upsilon^f_{it}\partial_{e^c}MPe^f_{it} + \upsilon^c_{it}\partial_{e^c}MPe^c_{it} + \upsilon^r_{it}\partial_{e^c}MPe^r_{it} + \upsilon^k_{it}\partial_{e^f}MPk_{it}\right] = 0 \end{aligned}$$

$$\omega_i\,\mathcal{P}_i\,e^{-(\rho+\eta\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\psi^w_{it}\xi^c\mathrm{t}^\varepsilon_t = -\mathcal{P}_i\xi^c\psi^S_t + \omega_i\mathcal{P}_i e^{-(\rho+\eta\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\left[\upsilon^f_{it}\partial_{e^c}MPe^f_{it} + \upsilon^c_{it}\partial_{e^c}MPe^c_{it} + \upsilon^r_{it}\partial_{e^c}MPe^r_{it} + \upsilon^k_{it}\partial_{e^c}MPk_{i}\right.$$

- Energy consumption – Renewable

$$\begin{aligned} &\omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[MPe^c_{it}-q^r_{it}] - \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\mu^r_{it} + \\ &\quad - \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\left[\upsilon^f_{it}\partial_{e^r}MPe^f_{it} + \upsilon^c_{it}\partial_{e^r}MPe^c_{it} + \upsilon^r_{it}\partial_{e^r}MPe^r_{it} + \upsilon^k_{it}\partial_{e^f}MPk_{it}\right] = 0 \\ &\Rightarrow \quad \left[\upsilon^f_{it}\partial_{e^c}MPe^f_{it} + \upsilon^c_{it}\partial_{e^c}MPe^c_{it} + \upsilon^r_{it}\partial_{e^c}MPe^r_{it} + \upsilon^k_{it}\partial_{e^c}MPk_{it}\right] = 0 \end{aligned}$$

PMP: Optimality conditions for the controls over prices $\{r^\star_t, q^f_t, \mathrm{w}_{it}, q^c_{it}, q^r_{it}\}_{it}$

- Interest rate $[r^\star_t]$

$$\sum_i \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[w_{it}-k_{it}] + \sum_i \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\upsilon^k_{it} = 0$$

The redistributive effect on agents' budget, weighted by ψ^w_{it} should compensate for the distortionary effect on firms' optimality of capital, weighted by shadow value υ^k_{it}.

- Fossil energy price: $[q^f_t]$

$$\sum_i \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[e^x_{it}-e^f_{it}]+\sum_i \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}[\upsilon^f_{it}-\theta^x_{it}]=0$$

- Coal energy price: $[q^c_{it}]$

$$\omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[\bar{e}^c_i-e^c_{it}]+\omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}[\upsilon^c_{it}-\theta^c_{it}]=0$$
$$\Rightarrow \qquad \upsilon^c_{it}=\theta^c_{it}$$

- Renewable energy price: $[q^r_{it}]$, similarly:

$$\Rightarrow \qquad \upsilon^r_{it}=\theta^r_{it}$$

For coal and renewable, since the price is local, the "distortive/redistributive" effect on the supply equals the one of its demand.

- Wages w_{it} are determined directly by the Marginal Product of Labor $MP\ell_{it}$, since the labor supply is inelastic and normalized to 1.
- Carbon tax/Carbon price $[\mathrm{t}^\varepsilon]$

$$\sum_i \omega_i\,\mathcal{P}_i\,e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}[\xi^f\upsilon^f_{it}+\xi^c\upsilon^c_{it}]=0$$

The optimal carbon tax level balances out all the distortions for each country, for oil-gas and coal, according to shadow values υ^f_{it} and υ^c_{it}.

PMP: Optimality conditions for the states $\{w_{it},\mathcal{S}_t,\tau_{it}\}_{it}$.

- Wealth $[w_{it}]$

$$\dot{\psi}^w_{it}=\psi^w_{it}(\rho-n_i-(1-\eta)\bar{g}_i)-\mathcal{H}^{sb}_w(\boldsymbol{s},\boldsymbol{c},\boldsymbol{\psi})$$
$$\dot{\psi}^w_{it}=\psi^w_{it}(\rho+\eta\bar{g}_i-r^\star_t)+\frac{1}{\omega_i\mathcal{P}_i}e^{-\int_{t_0}^t\bar{\rho}_s ds+(\rho+\eta\bar{g}_i)t}\mu^b_t$$
$$\mu^b_t=e^{-[(\rho+\eta\bar{g}_i)t-\int_{t_0}^t\bar{\rho}_s ds]}\omega_i[\upsilon^f_{it}\partial_k MPe^f_{it}+\upsilon^c_{it}\partial_k MPe^c_{it}+\upsilon^r_{it}\partial_k MPe^r_{it}+\upsilon^k_{it}\partial_k MPk_{it}]$$
$$\dot{\psi}^w_{it}=\psi^w_{it}(\rho+\eta\bar{g}_i-r^\star_t)+e^{-\int_{t_0}^t\bar{\rho}_s ds+(\rho+\eta\bar{g}_i)t}e^{-[(\rho+\eta\bar{g}_i)t-\int_{t_0}^t\bar{\rho}_s ds]}\frac{1}{\mathcal{P}_i}[\dots]$$
$$\dot{\psi}^w_{it}=\psi^w_{it}(\rho+\eta\bar{g}_i-r^\star_t)+\frac{1}{\mathcal{P}_i}[\upsilon^f_{it}\partial_k MPe^f_{it}+\upsilon^c_{it}\partial_k MPe^c_{it}+\upsilon^r_{it}\partial_k MPe^r_{it}+\upsilon^k_{it}\partial_k MPk_{it}]$$

This implies time-varying liquidity motives for the marginal value of wealth. Abstracting from discounting $\rho-n_i-(1-\eta)\bar{g}_i$ and $\bar{\rho}_t$, if μ^b_t is positive (the planner would like to increase the supply of bond, decreasing return), it needs to be compensated for higher marginal value of wealth ψ^w_{it} in the future ($\dot{\psi}^w_{it}$ is higher if $\mu^b_t>0$) which implies higher consumption, today at time t.

- Temperature $[\tau_{it}]$

$$\dot{\psi}_{it}^{\tau} = \psi_{it}^{\tau}(\rho - n_i - (1-\eta)\bar{g}_i) - \mathcal{H}_{\tau}^{sb}(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{\psi})$$

$$\dot{\psi}_{it}^{\tau} = \psi_{it}^{\tau}(\rho - n_i - (1-\eta)\bar{g}_i \; + \zeta) + \underbrace{\gamma^y(\tau_{it} - \tau_i^\star)\mathcal{D}_i^y(\tau_{it})}_{-\partial_\tau \mathcal{D}^y} z_i F(k_{it}, e_{it})\psi_{it}^w + \underbrace{\gamma^u(\tau_{it} - \tau_i^\star)\mathcal{D}_i^u(\tau_{it})}_{-\partial_\tau \mathcal{D}^u} u'(\mathcal{D}^u(\tau_{it})c_{it})c_{it}$$

$$\dot{\psi}_{it}^{\tau} = \psi_{it}^{\tau}(\rho - n_i - (1-\eta)\bar{g}_i \; + \zeta) + (\tau_{it} - \tau_i^\star)[\gamma^y y_{it} + \gamma^u c_{it}]\psi_{it}^w$$

- Carbon concentration $[\mathcal{S}_t]$

$$\dot{\psi}_t^S = \psi_t^S \bar{\rho}_t - \mathcal{H}_{\tau}^{sb}(\boldsymbol{s}, \boldsymbol{c}, \boldsymbol{\psi})$$

$$\dot{\psi}_t^S = \psi_t^S(\bar{\rho}_t + \delta_s) - \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)t + \int_{t_0}^t \bar{\rho}_s ds} \zeta \Delta_i \chi \, \psi_{it}^{\tau}$$

Proof of Proposition 8

Solving for the differential equations for the marginal value of temperature ψ_{it}^{τ} and carbon ψ_{it}^{S}.

$$\psi_{it}^{\tau} = \int_t^\infty e^{-(\rho - n_i - (1-\eta)\bar{g}_i + \zeta)(s-t)} (\tau_{is} - \tau_i^\star)\left[\gamma^y y_{is} + \gamma^u c_{is}\right]\psi_{is}^w ds$$

$$\psi_t^S = -\int_t^\infty e^{-\delta_s(s-t) - \int_t^s \bar{\rho}_s ds} \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(s-t) + \int_t^s \bar{\rho}_s ds} \zeta \Delta_i \chi \, \psi_{is}^{\tau} ds$$

$$\psi_t^S = -\int_t^\infty e^{-\delta_s(s-t)} \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(s-t)} \zeta \Delta_i \chi \, \psi_{is}^{\tau} ds$$

$$\psi_t^S \xrightarrow[\zeta \to \infty]{} -\int_t^\infty e^{-\delta_s(s-t)} \sum_{i \in \mathbb{I}} \omega_i \, \mathcal{P}_i \, e^{-(\rho - n_i - (1-\eta)\bar{g}_i)(s-t)} \Delta_i \chi \left(\tau_{is} - \tau_i^\star\right)\left[\gamma^y y_{is} + \gamma^u c_{is}\right]\psi_{is}^w ds$$

Using the dynamics for the marginal value of wealth;

$$\psi_{it}^w = \int_t^T e^{-(\rho + \eta\bar{g}_i)(s-t) + \int_t^s r_u^\star du} \bar{\upsilon}_{is}^k ds + e^{-(\rho + \eta\bar{g}_i)(T-t) + \int_t^T r_u^\star du} \psi_{iT}^w$$

$$\bar{\upsilon}_{it}^k = \frac{1}{\mathcal{P}_i}[\upsilon_{it}^f \partial_k MPe_{it}^f + \upsilon_{it}^c \partial_k MPe_{it}^c + \upsilon_{it}^r \partial_k MPe_{it}^r + \upsilon_{it}^k \partial_k MPk_{it}]$$

First, let us assume that $\bar{\upsilon}_{it}^k \approx 0$ there are no liquidity effects, giving $\psi_{it}^w = e^{-(\rho + \eta\bar{g}_i)(T-t) + \int_t^T r_u^\star du} \psi_{iT}^w$.

I define the social welfare weights:

$$\overline{\psi}_t^w = \frac{1}{\mathcal{P}_t} \sum_{i \in \mathbb{I}} \omega_i \mathcal{P}_i e^{-[(\rho - n_i - (1-\eta)\bar{g}_i)t - \int_{t_0}^t \rho_s ds]} \psi_{it}^w$$

$$\widehat{\psi}_{it}^w = \frac{\omega_i \mathcal{P}_i \psi_{it}^w}{\overline{\psi}_t^w}$$

This allow to simplify the marginal value of carbon:

$$\psi_t^S \to -\int_t^{\infty} e^{-\delta_s(s-t)} \sum_{i\in\mathbb{I}} \omega_i \,\mathcal{P}_i \, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(s-t)} \Delta_i \chi\left(\tau_{is}-\tau_i^\star\right)\left[\gamma^y y_{is} + \gamma^u c_{is}\right]\psi_{is}^w ds$$
$$\psi_t^S \to -\int_t^{\infty} e^{-\delta_s(s-t)} \sum_{i\in\mathbb{I}} \omega_i \,\mathcal{P}_i \, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)(s-t)} \Delta_i \chi\left(\tau_{is}-\tau_i^\star\right)\left[\gamma^y y_{is} + \gamma^u c_{is}\right] e^{+(\rho+\eta\bar{g}_i)(s-t)-\int_t^s r_u^\star du}\psi_{it}^w ds$$

This implies the Social Cost of Carbon:

$$SCC_t = -\frac{\psi_t^S}{\overline{\psi}_t^w} = \int_t^{\infty} e^{-\delta_s(s-t)-\int_t^s r_u^\star du} \sum_{i\in\mathbb{I}} e^{(n_i+\bar{g}_i)(s-t)} \Delta_i \chi\left(\tau_{is}-\tau_i^\star\right)\left[\gamma^y y_{is} + \gamma^u c_{is}\right] \frac{\omega_i \,\mathcal{P}_i \, \psi_{it}^w}{\overline{\psi}_t^w} ds$$

$$SCC_t = \sum_{i\in\mathbb{I}} \widehat{\psi}_{it}^w LCC_{it} \qquad \widehat{\psi}_{it}^w = \frac{\omega_i \,\mathcal{P}_i \, \psi_{it}^w}{\overline{\psi}_t^w}$$

$$LCC_{it} = \int_t^{\infty} e^{-\delta_s(s-t)-\int_t^s (r_u^\star - n_i - \bar{g}_i)du} \, \Delta_i \chi\left(\tau_{is}-\tau_i^\star\right)\left[\gamma^y y_{is} + \gamma^u c_{is}\right] ds$$

which gives the results of Proposition 8. □

Proof of Proposition 9

Solving for the optimal carbon tax involves for solving for the objects in the optimality condition for energy choices. Take the energy choice for fossil-fuels:

$$\omega_i \,\mathcal{P}_i \, e^{-(\rho+\eta\bar{g}_i)t}\psi_{it}^w \xi^f \mathrm{t}_t^\varepsilon = e^{-\int_{t_0}^t \bar{\rho}_s ds}\mathcal{P}_i \mu_t^f - e^{-\int_{t_0}^t \bar{\rho}_s ds}\mathcal{P}_i \xi^f \psi_t^S$$
$$+\omega_i \mathcal{P}_i e^{-(\rho+\eta\bar{g}_i)t}\left[\upsilon_{it}^f \partial_{e^f} MPe_{it}^f + \upsilon_{it}^c \partial_{e^f} MPe_{it}^c + \upsilon_{it}^r \partial_{e^f} MPe_{it}^r + \upsilon_{it}^k \partial_{e^f} MPk_{it}\right]$$

We need to solve each of the objects in turn: (i) the marginal value of carbon, ψ_t^S, related to the Social Cost of Carbon, as seen in the previous proposition, (ii) the marginal value of oil supply μ_t^f, related to the energy supply redistribution, (iii) the marginal value of firms' inputs optimality conditions $\upsilon_{it}^f, \upsilon_{it}^c, \upsilon_{it}^r, \upsilon_{it}^k$.

<u>*Supply redistribution*</u>

We want to solve for μ_t^f. First, take the optimality for $[q_t^f]$.

$$\sum_i \omega_i \,\mathcal{P}_i \, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\theta_{it}^x = \sum_i \omega_i \,\mathcal{P}_i \, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi_{it}^w [e_{it}^x - e_{it}^f] + \sum_i \omega_i \,\mathcal{P}_i \, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\upsilon_{it}^f$$

The redistribution effect on oil-gas quantity between exporter and importers $e_{it}^x - e_{it}^f$ exactly compensate – for the planner – the equilibrium effect on demand υ_{it}^f and supply θ_{it}^x.

Second, using the optimality of $[e_{it}^x]$:

$$\mu_t^f = -e^{-[(\rho+\eta\bar{g}_i)t-\int_{t_0}^t \bar{\rho}_s ds]}\omega_i \, \theta_{it}^x \; \mathcal{P}_i e^{(n_i+\bar{g}_i)t} \; \nu_{i\,e^x e^x}^f(e_{it}^x, \mathcal{R}_{it})$$
$$\omega_i \,\mathcal{P}_i \theta_{it}^x = -\frac{\mathcal{P}_i e^{-(n_i+\bar{g}_i)t}}{\nu_{i\,e^x e^x}^f(e_{it}^x, \mathcal{R}_{it})} e^{-[(\rho+\eta\bar{g}_i)t-\int_{t_0}^t \bar{\rho}_s ds]}\mathcal{P}_i \mu_t^f$$

As a result, we obtain:

$$\sum_i \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\theta^x_{it} = -\mu^f_t \sum_i \frac{\mathcal{P}_i e^{(n_i+\bar{g}_i)t}}{\nu^f_{i\,e^x e^x}(e^x_{it},\mathcal{R}_{it})} e^{(\rho+\eta\bar{g}_i)t-\int_{t_0}^t \bar{\rho}_s ds} e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}$$

$$\sum_i \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\theta^x_{it} = -e^{-\int_{t_0}^t \bar{\rho}_s ds}\mu^f_t \sum_i \frac{1}{\nu^f_{i\,e^x e^x}(e^x_{it},\mathcal{R}_{it})}$$

$$\Rightarrow \qquad e^{-\int_{t_0}^t \bar{\rho}_s ds}\mu^f_t = -\Big[\sum_i \nu^f_{i\,e^x e^x}(e^x_{it},\mathcal{R}_{it})^{-1}\Big]^{-1}\sum_i \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\theta^x_{it}$$

$$\Rightarrow \qquad e^{-\int_{t_0}^t \bar{\rho}_s ds}\mu^f_t = \Big[\sum_i \nu^f_{i\,e^x e^x}(e^x_{it},\mathcal{R}_{it})^{-1}\Big]^{-1}\Big\{\sum_i \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}[e^f_{it}-e^x_{it}] - \sum_i \omega_i\, \mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\upsilon^f_{it}\Big\}$$

<u>*Demand distortion*</u>

We use a Nested CES framework for production. I express the formula without time indices for simplicity.

$$\text{Energy} \qquad e_i = \Big(\sum_\ell (\omega^\ell)^{\frac{1}{\sigma_e}}(e^\ell_i)^{\frac{\sigma_e-1}{\sigma_e}}\Big)^{\frac{\sigma_e}{\sigma_e-1}} \qquad\qquad \text{Output} \qquad y_i = z_i\Big((1-\varepsilon)^{\frac{1}{\sigma}}(z^e_i\, e_i)^{\frac{\sigma-1}{\sigma}} + \varepsilon^{\frac{1}{\sigma}}(k^\alpha_i \ell^{1-\alpha}_i)^{\frac{\sigma-1}{\sigma}}\Big)^{\frac{\sigma}{\sigma-1}}$$

Optimality for fossil energy demand features this complicated term, which we can simplify using the CES structure. The curvature of production, represented by terms like $\partial_{e^f} MPe^f_i$ are related the the elasticity of energy use.

$$\bar{\upsilon}^f_i = [\upsilon^f_i \partial_{e^f} MPe^f_i + \upsilon^c_i \partial_{e^f} MPe^c_i + \upsilon^r_i \partial_{e^f} MPe^r_i + \upsilon^k_i \partial_{e^f} MPk_i]$$

$$= \frac{1}{e^f_i}\Big[-\upsilon^f_i(q^f+\xi^f \mathrm{t}^\varepsilon)[\frac{1-s^f}{\sigma^e} + s^f\frac{1-s^e}{\sigma^y}] + \upsilon^c_i(q^c_i+\xi^f \mathrm{t}^\varepsilon)s^f_i[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y}] + \upsilon^r_i q^r_i s^f_i[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y}] + \upsilon^k_i(r^\star+\bar{\delta})\frac{s^{e^r/y}_i}{\sigma^y}\Big]$$

with Energy share in production: $s^e_i = \frac{e_i q^e_i}{y_i}$, Fossil share in energy mix $s^f_i = \frac{e^f_i q^f}{e_i q^e_i}$ and similarly $s^c_i = \frac{e^c_i q^c_i}{e_i q^e_i}$ and $s^r_i = \frac{e^r_i q^r_i}{e_i q^e_i}$.

When we normalize by $\overline{\psi}$, we obtain:

$$\widehat{\bar{\upsilon}}^f_i = \frac{1}{e^f_i}\Big[-\widehat{\upsilon}^f_i(q^f+\xi^f \mathrm{t}^\varepsilon)[\frac{1-s^f}{\sigma^e} + s^f\frac{1-s^e}{\sigma^y}] + \widehat{\upsilon}^c_i(q^c_i+\xi^f \mathrm{t}^\varepsilon)s^f_i[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y}] + \widehat{\upsilon}^r_i q^r_i s^f_i[\frac{1}{\sigma^e} - \frac{1-s^e}{\sigma^y}] + \widehat{\upsilon}^k_i(r^\star+\delta)\frac{s^{e^r/y}_i}{\sigma^y}\Big]$$

We can obtain similar formulas for $\widehat{\bar{\upsilon}}^f_{it}$, $\widehat{\bar{\upsilon}}^c_{it}$, $\widehat{\bar{\upsilon}}^r_{it}$, and $\widehat{\bar{\upsilon}}^k_{it}$.

Rewriting the optimal carbon tax

Take the optimality condition for energy choice, where I replaced $\bar{\upsilon}^f_{it}$. Sum over countries i and normalize by world population $\mathcal{P}_t$

$$\omega_i \,\mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it}\xi^f \mathrm{t}^\varepsilon_t = e^{-\int_{t_0}^t \bar{\rho}_s ds}\mathcal{P}_i e^{n_i+\bar{g}_i}\mu^f_t - e^{-\int_{t_0}^t \bar{\rho}_s ds}\mathcal{P}_i\xi^f\psi^S_t + \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\,\bar{\upsilon}^f_{it}$$

$$\xi^f \mathrm{t}^\varepsilon_t \sum_{i\in\mathbb{I}} \omega_i\,\mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\psi^w_{it} = e^{-\int_{t_0}^t \bar{\rho}_s ds}\underbrace{\sum_i \mathcal{P}_i e^{(n_i+\bar{g}_i)t}}_{=\mathcal{P}_t}\left[\mu^f_t - \xi^f\psi^S_t\right] + \sum_i \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t}\,\bar{\upsilon}^f_{it}$$

$$\xi^f \mathrm{t}^\varepsilon_t \overline{\psi}^w_t = \left[\mu^f_t - \xi^f\psi^S_t\right] + \sum_i \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\,\bar{\upsilon}^f_{it}$$

We define:

$$\begin{aligned}
\textit{Supply Redistribution}_t = \frac{\mu^f_t}{\overline{\psi}^w_t} &= \Big[\sum_i \nu^f_{i\,e^xe^x}(e^x_{it},\mathcal{R}_{it})^{-1}\Big]^{-1}\Big\{\sum_i \omega_i\,\mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\frac{\psi^w_{it}}{\overline{\psi}^w_t}[e^x_{it}-e^f_{it}] \\
&\quad + \sum_i \omega_i\,\mathcal{P}_i\, e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\frac{\upsilon^f_{it}}{\overline{\psi}^w_t}\Big\} \\
&= \Big[\sum_i (\nu^f_{i\,e^xe^x})^{-1}\Big]^{-1}\Big\{\sum_i \widehat{\psi}^w_{it}[e^x_{it}-e^f_{it}] + \sum_i \widehat{\upsilon}^f_{it}\Big\} \\
\textit{Demand Distortion}_t &= \frac{1}{\overline{\psi}^w_t}\sum_i \omega_i\mathcal{P}_i e^{-(\rho-n_i-(1-\eta)\bar{g}_i)t+\int_{t_0}^t \bar{\rho}_s ds}\,\bar{\upsilon}^f_{it} = \sum_i \widehat{\bar{\upsilon}}^f_{it}
\end{aligned}$$

We obtain:

$$\begin{aligned}
\xi^f \mathrm{t}^\varepsilon_t &= \xi^f SCC_t + \Big[\sum_i (\nu^f_{i\,e^xe^x})^{-1}\Big]^{-1}\Big\{\sum_i \widehat{\psi}^w_{it}[e^x_{it}-e^f_{it}] + \sum_i \widehat{\upsilon}^f_{it}\Big\} + \sum_i \widehat{\bar{\upsilon}}^f_{it} \\
\xi^f \mathrm{t}^\varepsilon_t &= \xi^f SCC_t + \textit{Supply Redistribution}_t + \textit{Demand Distortion}_t
\end{aligned}$$

This implies the formula in Proposition 9 $\square$.