The Inequality of Climate Change and Optimal Energy Policy

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Introduction

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  (iii) reallocation of activity through trade, the leakage effect
  (+) higher income countries not exposed as much as developing economies.

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  - (ii) energy markets, between importer and exporters
  - (iii) reallocation of activity through trade, the leakage effect
  - (+) higher income countries not exposed as much as developing economies.

- As a result, different countries are affected differently by carbon taxation,

  $\Rightarrow$ What is the optimal carbon policy in the presence of climate externality and inequality?

  - Optimal taxation design depends crucially on redistribution instruments i.e. lump-sum transfers across countries

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- Study an Integrated Assessment Model (IAM) with heterogeneous countries to:
  - Evaluate the welfare costs of global warming (Social Cost of Carbon)
  - Solve for the optimal Ramsey policy for carbon taxation
  - Analyze the strategic implications of joining/designing climate agreements
  - Provide a numerical methodology for this Het. Agents model

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  - Provide a numerical methodology for this Het. Agents model
- Preview of the results:
  - Social Cost of Carbon need to be adjusted for inequality level
  - Taxation of energy also accounts for supply and demand elasticity
  - Country-specific taxes: poorer countries will pay relatively lower taxes

Literature

- Climate change & optimal carbon taxation
  - RA model: Nordhaus DICE (1996-), Weitzman (2014), Golosov et al (2014)
  - HA model: Krusell, Smith (2022), Kotlikoff, Kubler, Polbin, Scheidegger (2021)
  - Spatial models: Cruz, Rossi-Hansberg (2022, 2023) among others
  - Climate policy w/ inequality (within country): Belfiori, Hur, Carroll (2024), Le Grand, Oswald, Ragot, Saussay (2024), etc.

  ⇒ *Optimal and constrained policy with heterogeneous countries & trade*
- Optimal policy in heterogeneous agents models
  - Policy with limited instruments: Diamond (1973), Davila, Walther (2022)
  - Bhandari et al (2021), Le Grand, Ragot (2022), Davila, Schaab (2022) …

  ⇒ *Application to climate and carbon taxation policy*
- Unilateral vs. climate club policies:
  - Climate clubs: Nordhaus (2015), Non-cooperative taxation: Chari, Kehoe (1990), Suboptimal policy: Hassler, Krusell, Olovsson (2019)
  - Trade policy: Kortum, Weisbach (2023), Farrokhi, Lashkaripour (2024), Hsiao (2022), Costinot, Donalson, Vogel, Werning (2015), and many others

  ⇒ *Climate cooperation and socially optimal climate policy*

Roadmap

- Toy model
- Optimal taxation of carbon
  - First-best: all instruments available
  - Constrained efficient: Without lump-sum transfers
  - Constrained efficient: Heterogeneous carbon tax
- Quantitative model

Toy Model – Household & Firms

Static deterministic Neoclassical economy

- countries $i \in \mathbb{I}$, heterogeneous in productivity z_i, temperature T_i, energy extraction cost $\mathcal{C}_i$
- In each country, five agents:

(i) Rep. household, (ii) final good firm, (iii-v) oil-gas (fossil), coal and renewable energy producers

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1. Representative household problem in each country i (passive):

$$\mathcal{V}_i = u(c_i) \qquad\qquad c_i = \underbrace{w_i \ell_i}_{\text{labor income}} + \underbrace{\pi_i^f}_{\text{fossil firm profit}} + \underbrace{t_i^{ls}}_{\text{lump-sum transfers}}$$

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2. Competitive final good producer in country i

$$\max_{\ell_i, e_i^f, e_i^c, e_i^r} \underbrace{\mathcal{D}(T_i)}_{\text{climate damage}} \underbrace{z_i}_{TFP} f(\ell_i, e_i^f, e_i^c, e_i^r) - w_i \ell_i - \underbrace{(q^f + \mathrm{t}_i^\varepsilon) e_i^f}_{\text{fossil (oil-gas)}} - \underbrace{(q_i^c + \mathrm{t}_i^\varepsilon) e_i^c}_{\text{coal}} - \underbrace{q_i^r e_i^r}_{\text{non-carbon energy}}$$

- Climate policy: Carbon tax t_i^f

Model – Energy markets & Emissions

3. Competitive fossil fuels energy producer, selling on international fossil market:
   - Supply of fossil energy e_i^x by extraction at cost $\mathcal{C}_i^f$

$$\pi_i^f = \max_{e_i^x} q^f e_i^x - \mathcal{C}_i^f(e_i^x)$$

$$E^f = \sum_{\mathbb{I}} e_i^f = \sum_{\mathbb{I}} e_i^x$$

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4. Coal energy firm: elastic supply e_i^c at price $q_i^c = z_i^c$
5. Renewable energy firm: elastic supply e_i^r at price $q_i^r = z_i^r$

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▶ Climate system:
- GHGs from oil-gas and coal affect temperatures:

$$T_i = \bar{T}_{i0} + \Delta_i \mathcal{E} = \bar{T}_{i0} + \underbrace{\Delta_i}_{\substack{\text{pattern}\\ \text{scaling}}} \sum_{\mathbb{I}} \underbrace{(e_i^f + e_i^c)}_{\text{GHG from energy}}$$

Model – Equilibrium

- Given policies $\{\mathrm{t}_i^f, \mathrm{t}_i^{ls}\}_i$, a **competitive equilibrium** is a set of decisions $\{c_i, e_i^f, e_i^r, e_i^x\}_i$, states $\{T_i\}_i$ and prices $\{q^f, q_i^c, q_i^r, w_i\}_i$ such that:
  - Households choose $\{c_i\}_i$ to max. utility s.t. budget constraint
  - Firm choose inputs $\{e_i^f, e_i^c, e_i^r\}_i$ to max. profit
  - Oil-gas firms extract/produce $\{e_i^x\}_i$ to max. profit. + Elastic renewable, coal supplies $\{e_i^c, e_i^r\}$
  - Emissions $\mathcal{E}_t$ affects climate $\{T_i\}_i$.
  - Government budget clear $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^{\varepsilon}(e_i^f + e_i^c)$
  - Prices $\{q^f, q_i^c, q_i^r\}$ adjust to clear the markets for energy $\sum_{\mathbb{I}} e_i^x = \sum_{\mathbb{I}} e_i^f$, and e_i^c, e_i^r
    The good market clearing holds by Walras law.

Optimal world policy – Summary of results

- Competitive equilibrium Details eq 0
  - Passive policies $t^f = 0$, and large cost of climate change

- First-Best, with unlimited instruments Details eq 1
  - Welfare: $\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{c}, \boldsymbol{e}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$
  - Social Planner redistribute across countries with lump-sum transfers t_i^{ls}
  - Set the optimal Pigouvian carbon tax to $t^f = SCC$

- Second-best Ramsey policy, with limited instruments Details eq 2
  - Optimal carbon tax accounts for (i) inequality and local climate damage, (ii) energy supply elasticities, (iv) energy demand distortions

- Endogenous Participation? If countries can exit climate agreements Details eq 3
  - All formulas corrected for participation constraints (multipliers affect distribution weights)
  - Optimal design of climate agreements / climate clubs ⇒ JMP

Quantification

- Quantification and calibration More details
  - Quadratic damage as in Nordhaus DICE

$$y_i = \mathcal{D}_i(T)\bar{y} \qquad \text{with} \quad \mathcal{D}_i(T) = e^{-\gamma_i (T - T_i^\star)^2}$$

  - Energy parameters to match production/reserves, Isoelastic cost function

$$\mathcal{C}_i(e_i^x) = \bar{\nu}_i \left(e_i^x / \mathcal{R}_i\right)^{1+\nu} \mathcal{R}_i$$

    - Cost $\bar{\nu}_i$ and Reserves $\mathcal{R}_i$ to match data for production and reserve
  - Production $\bar{y} = zf(\ell_i, k_i, e_i^f, e_i^r)$, labor, capital, fossil, renewable
    - Nested CES energy vs. labor-capital Cobb-Douglas bundle (elasticity $\sigma_y < 1$), and fossil/renewable $\sigma_e > 1$.
    - TFP, and DTC, z_i, z_i^e, calibrated to match GDP / energy shares data.
  - Population, from WDI data

Competitive equilibrium

- Key objects:
  - Marginal value of wealth $\lambda_i^w = u'(c_i)$
  - Marginal value of carbon ψ_i^ε for country i
  - "Local social cost of carbon" (LCC) for region i:

$$LCC_i := -\frac{\partial \mathcal{V}_i / \partial \mathcal{S}}{\partial \mathcal{V}_i / \partial c_i} = \frac{\psi_i^\varepsilon}{\lambda_i^w} = \Delta_i \gamma_i (T_i - T_i^\star) y_i > 0$$

    – Stationary equilibrium closed-form formula, analogous to GHKT (2014) Closed Form Solution Here

First-Best, Optimal policy with transfers

- First-Best, Maximizing welfare of the Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t},\boldsymbol{x},\boldsymbol{c},\boldsymbol{q}\}_i} \sum_{\mathbb{I}} \omega_i \, u(c_i) = \sum\nolimits_{\mathbb{I}} \mathcal{W}_i$$

  - Full set of instruments $\mathbf{t} = \{\mathrm{t}_i^{\varepsilon}, \mathrm{t}_i^{ls}\}$, including transfers *across countries*

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  - Full set of instruments $\mathbf{t} = \{\mathrm{t}_i^{\varepsilon}, \mathrm{t}_i^{ls}\}$, including transfers *across countries*
  - Key objects: Local vs. Global Social Cost of Carbon,

$$SCC^{fb} := -\frac{\partial \mathcal{W} / \partial \mathcal{S}}{\partial \mathcal{W} / \partial \bar{c}} = \frac{\psi^{\varepsilon}}{\lambda^{w}} = \frac{\sum_{\mathbb{I}} \psi_i^{\varepsilon}}{\frac{1}{I} \sum_{\mathbb{I}} \lambda_i^{w}} \qquad LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{S}}{\partial \mathcal{W}_i / \partial c_i} = \frac{\psi_i^{\varepsilon}}{\lambda_i^{w}}$$

First-Best, Optimal policy with transfers

- *Proposition 1:* Optimal carbon tax:

$$\mathrm{t}^{\varepsilon} = SCC^{fb}$$

  - Result as in Representative Agent economy, c.f. Nordhaus DICE (1996), GHKT (2014)

$$SCC^{fb} = \frac{\psi^{\varepsilon}}{\lambda^{w}} = -\sum_{\mathbb{I}} \frac{\psi_i^{\varepsilon}}{\lambda_i^{w}} = \textstyle\sum_{\mathbb{I}} LCC_i$$

- Lump-sum transfers redistribute across countries, s.t.

$$\omega_i u'(c_i) = \lambda_i^w = \bar{\lambda}^w = \lambda_j^w = \omega_j u'(c_j) \quad \forall i,j \in \mathbb{I}$$

  - Imply cross-countries lump-sum transfers $\exists\, i \;\; s.t.\; \mathrm{t}_i^{ls} \geq 0$ or $\exists\, j \;\; s.t.\; \mathrm{t}_j^{ls} \leq 0$

Ramsey policy with limited transfers

- Second best without access to lump-sum transfers: choice of a carbon tax $\{\mathfrak{t}^f, \mathfrak{t}^r\}$
  - Tax receipts redistributed lump-sum: $\mathfrak{t}_i^{ls} = \mathfrak{t}^f e_i^f$
  - Inequality across regions:

$$\widehat{\lambda}_i^w = \frac{\omega_i \lambda_i^w}{\bar{\lambda}^w} = \frac{\omega_i u'(c_i)}{\frac{1}{I}\sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1$$

$\Rightarrow$ ceteris paribus, poorer countries have higher $\widehat{\lambda}_i^w$

  - Social Cost of Carbon integrates these inequalities:

$$SCC^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$$

$$SCC^{sb} = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i^w, LCC_i\right)$$

Ramsey Problem – Optimal Carbon and Energy Policy

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy

$$\text{Supply Redistribution}^{sb} + \text{Demand Distortion}^{sb} = \underbrace{\mathcal{C}^{f}_{EE}}_{\substack{\text{agg. supply}\\ \text{distortion}}} \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right)}_{\substack{\text{terms-of-trade}\\ \text{redistribution}}} - \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\upsilon}_i, \frac{q^f(1-s^f_i)}{\sigma_i e_i}\right)}_{\substack{\text{demand}\\ \text{distortion}}}$$

- ○ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^e_i energy cost share and σ_i energy demand elasticity Details

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∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^e_i energy cost share and σ_i energy demand elasticity Details

- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \qquad t^f = SCC^{sb} + \text{Supply Redistribution}^{sb} + \text{Demand Distortion}^{sb}$$

  – Reexpressing demand terms:

$$t^{\varepsilon} = \left(1 + \mathbb{C}\text{ov}_i\left(\widehat{\lambda}^w_i, \widehat{\frac{\sigma_i e_i}{1-s^e_i}}\right)\right)^{-1} \left[\sum_{\mathbb{I}} LCC_i + \mathbb{C}\text{ov}_i\left(\widehat{\lambda}^w_i, LCC_i\right) + \mathcal{C}^f_{EE}\, \mathbb{C}\text{ov}_i\left(\widehat{\lambda}^w_i, e^f_i - e^x_i\right)\right]$$

Step 2: Ramsey Problem – Country-specific energy tax

- Suppose the planner has access to a *distribution* of carbon price.
- *Proposition 3:* Optimal country-specific fossil energy tax:

$$\Rightarrow \qquad t_i^{\varepsilon} = \frac{1}{\widehat{\lambda}_i^w}\left[SCC^{sb} + \text{Supply Redistribution}^{sb}\right]$$

  – Social cost of carbon: $SCC^{sb} = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$

⇒ Reduce the tax burden for poorer/more “valuable” countries

Ramsey Problem – Theoretical extensions (in the paper)

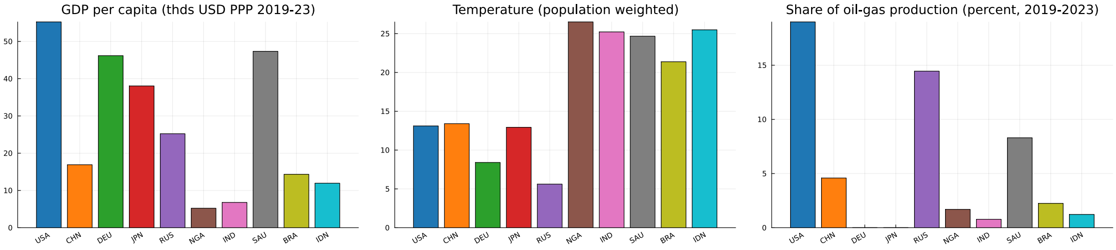
- Trade: redistributive effect through leakage effect
  - Trade model à la Armington Details trade
  - Additional trade-off: account for distortions on goods central in trade networks
- Uncertainty:
  - Stochastic climate impact, e.g. climate sensitivity Δ_i or damage curvature γ_i, Details uncertainty
  - Interaction in SCC between uncertainty and inequalities: covariance between cost of climate change and social welfare weights
- Dynamic considerations
  - Valuation of reserves / Hotelling rent: carbon tax serves as an instrument for intertemporal substitution of fossil production. c.f. Heal, Schlenker (2019), Cruz, Rossi-Hansberg (2022)
  - Curb capital demand + distort consumption/saving decision, c.f. H.A. model, BEGS/LG-R
- Methodological contribution:
  - solving IAM with arbitrary heterogeneity $\Leftrightarrow$ solving forward-backward ODEs

Model – Dynamics & extensions

1. Climate system with inertia / closer to standard IAMs
2. Firms
   - Include capital to produce $f(\ell, k, e^f, e^c, e^r)$
   - Separate coal energy e^c from oil and gas: no energy rent π^f, higher emissions $\mathcal{E}$
   - Match the energy mix of each country (WIP)
3. Households
   - Consumption / saving in bonds / in capital $\Rightarrow$ Keynes-Ramsey rule
   - International markets to borrow bonds (in zero net supply)
4. Energy markets
   - Fossil energy extraction/depleting reserves $\Rightarrow$ Hotelling problem (WIP)
   - Price of clean energy trending down
5. Population $\mathcal{P}_i$ and growth dynamics (for each country (WIP))
6. (Exogenous) growth: TFP change and Energy-augmenting Directed TC (WIP)

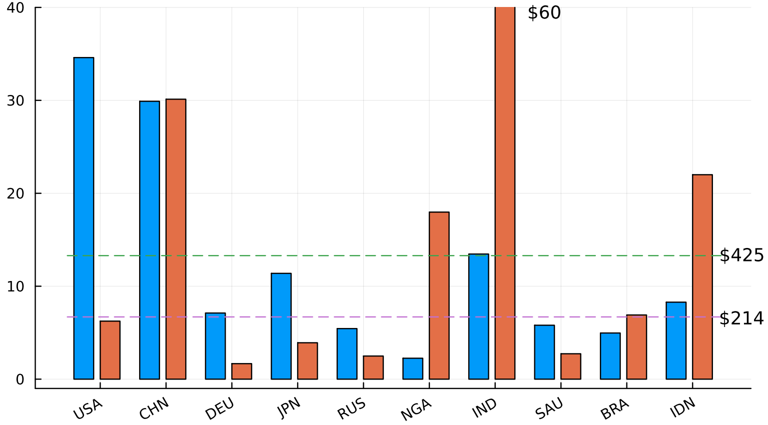
Quantitative application

- Sample of 60 countries, Average over years 2019-2023
- Matching country data on macro variables (GDP per capita, energy use), energy markets (production, reserves)



Local Cost of Carbon & Social Cost of Carbon

- Recall: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i^w \, LCC_i$
- Difference: $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$ (blue, left bars) vs. $\widehat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\bar{\lambda}^w}$ (red, right bars)



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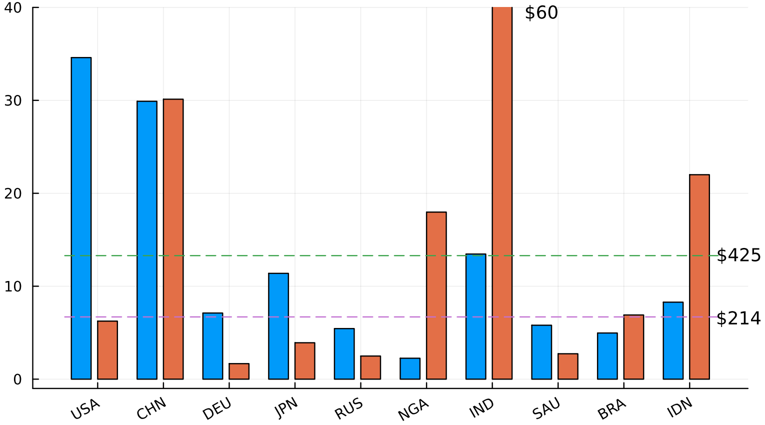


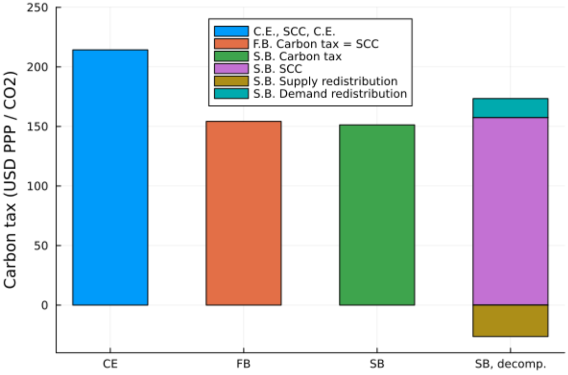
The Carbon Tax is not (only) the Social Cost of Carbon!

- Optimal carbon tax with heterogeneity:

$$t^{\varepsilon} = SCC + \text{Supply Redistrib.} + \text{Demand Distort}^{\circ}$$

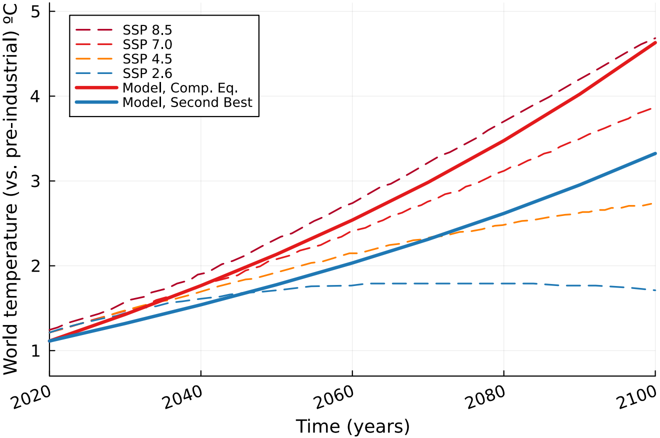
⇒ Correct for additional redistributive terms

- The optimal energy tax is 5% lower than the SCC (here with Negishi Pareto weights)
- With utilitarian weights $\omega_i = 1, \forall i$, tax much higher: use carbon tax as a tool to do redistribution Details



Climate dynamics

- Temperature path:
  Competitive equilibrium (Business as Usual) vs. Second-Best Carbon tax



Conclusion

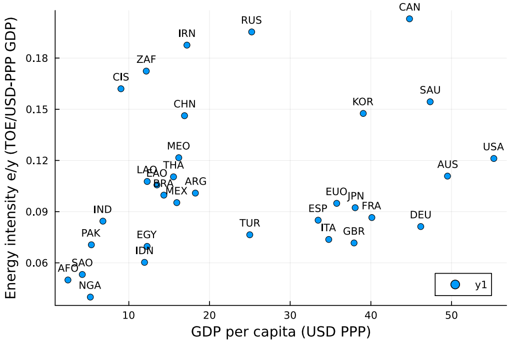
- In this project, I solve for the optimal climate policy
  - Accounting for inequality as it depends on the availability of transfer mechanisms
  - Redistributing through GE effects on energy and good markets ⇒ terms-of-trade effects
  - Additional trade-related and dynamic motives co-funded in energy taxation
- Incentives and implementability
  - What if some countries deviate from apply the appropriate energy tax?
  - Game theoretical consideration due to participation constraints
  - Implementation of a “climate club”:
    Penalty tariffs or Carbon-Adjustment Mechanisms for non-participants crucial for enforcing carbon policy

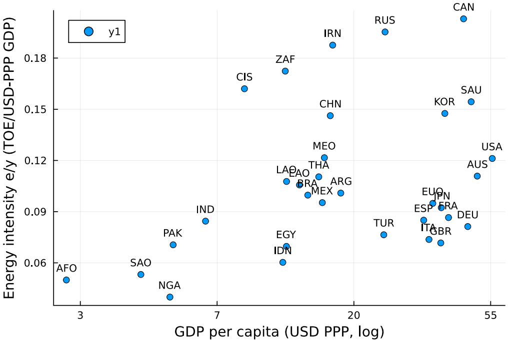
  ⇒ Job Market Paper: “The Optimal Design of Climate Agreements”

Appendices

Energy intensity

- Energy intensity e_i/y_i vs. GDP per capita y_i





Step 0: Competitive equilibrium & Trade

- Each household in country i maximize utility and firms maximize profit
- Standard trade model results:
  - Consumption and trade:

$$s_{ij} = \frac{c_{ij}p_{ij}}{c_i\mathbb{P}_i} = a_{ij}\frac{(d_{ij}(1+\mathrm{t}_{ij}^b)\mathrm{p}_j)^{1-\theta}}{\sum_k a_{ik}(d_{ik}(1+\mathrm{t}_{ik}^b)\mathrm{p}_k)^{1-\theta}} \qquad \& \qquad \mathbb{P}_i = \Big(\sum_j a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta}\Big)^{\frac{1}{1-\theta}}$$

  - Energy consumption doesn't internalize climate damage:

$$\mathrm{p}_i MPe_i = q^e$$

  - Inequality, as measured in local welfare units:

$$\lambda_i = u'(c_i)$$

  - "Local Social Cost of Carbon", for region i

$$LCC_i = \frac{\partial \mathcal{W}_i/\partial \mathcal{E}}{\partial \mathcal{W}_i/\partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f)\frac{\mathrm{p}_i}{\mathbb{P}_i} \qquad (> 0 \text{ if heat causes losses})$$

back

Step 1: World First-best policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - Full array of instruments: cross-countries lump-sum transfers t_i^{ls}, individual carbon taxes t_i^f on energy e_i^f, bilateral tariffs t_{ij}^b
  - Budget constraint: $\sum_i \mathrm{t}_i^{ls} = \sum_i \mathrm{t}_i^f e_i^f + \sum_{i,j} \mathrm{t}_{ij}^b c_{ij} d_{ij} \mathrm{p}_j$

- Maximize welfare subject to
  - Market clearing for good $[\mu_i]$, market clearing for energy μ^e

back

Step 1: World First-best policy

- Social planner results:
  - Consumption:

$$\omega_i u'(c_i) = \Big[\sum_j a_{ij}(d_{ij}\omega_j\mu_j)^{1-\theta}\Big]^{\frac{1}{1-\theta}}$$

  - Energy use:

$$\omega_i\mu_i MPe_i = \mu^e + SCC$$

  - Social cost of carbon:

$$SCC = -\frac{\sum_j \Delta_j\omega_j\mu_j\mathcal{D}'_j(T_j)\bar{y}_j}{\frac{1}{I}\sum_j \omega_j\mu_j}$$

back

Step 2: World optimal Ramsey policy

- Maximizing welfare of the world Social Planner:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{i \in \mathbb{I}} \omega_i \, u(c_i) = \sum_{\mathbb{I}} \mathcal{W}_i$$

  - One single instrument: uniform carbon tax t^f on energy e_i^f
  - Rebate tax lump-sum to HHs $\mathrm{t}_i^{ls} = \mathrm{t}^f e_i^f$

- Ramsey policy: Primal approach, maximize welfare subject to
  - Budget constraint $[\lambda_i]$, Market clearing for good $[\mu_i]$, market clearing for energy
  - Optimality (FOC) conditions for good demands $[\eta_{ij}]$, energy demand & supply, etc.
  - Trade-off faced by the planner:
    - (i) Correcting externality, (ii) Redistributive effect, (iii) Distort energy demand and supply

back

Step 2: World optimal Ramsey policy

- The planner takes into account
  (i) the marginal value of wealth λ_i
  (ii) the shadow value of good i, from market clearing, μ_i:

$$\text{w/o trade} \qquad \omega_i u'(c_i) = \omega_i \lambda_i$$

$$\text{vs. w/ trade in goods:} \qquad \omega_i u'(c_i) = \left(\sum_{j \in \mathbb{I}} a_{ij} (d_{ij} \mathrm{p}_j)^{1-\theta} \left[\omega_i \lambda_i + \omega_j \mu_j + \eta_{ij} (1 - s_{ij}) \right]^{1-\theta} \right)^{\frac{1}{1-\theta}}$$

- Relative welfare weights, representing inequality

$$\text{w/o trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i \lambda_i}{\bar{\lambda}} = \frac{\omega_i u'(c_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_j u'(c_j)} \lessgtr 1 \qquad \Rightarrow$$

ceteris paribus, poorer countries have higher $\widehat{\lambda}_i$

$$\text{vs. w/ trade:} \qquad \widehat{\lambda}_i = \frac{\omega_i (\lambda_i + \mu_i)}{\frac{1}{I} \sum_{\mathbb{I}} \omega_i (\lambda_i + \mu_i)} \lessgtr 1$$

Step 2: Optimal policy – Social Cost of Carbon

- Key objects: Local vs. Global Social Cost of Carbon:
  - Marginal cost of carbon $\psi_i^{\mathcal{E}}$ for country i
  - "Local social cost of carbon" (LCC) for region i:

$$LCC_i := \frac{\partial \mathcal{W}_i / \partial \mathcal{E}}{\partial \mathcal{W}_i / \partial w_i} = \frac{\psi_i^{\mathcal{E}}}{\lambda_i} = -\Delta_i \mathcal{D}'(T_i) z_i f(e_i^f) \mathrm{p}_i \qquad (> 0 \text{ if heat causes losses})$$

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  - Social Cost of Carbon for the planner:

$$SCC := \frac{\partial \mathcal{W}/\partial \mathcal{E}}{\partial \mathcal{W}/\partial w} = \frac{\sum_{\mathbb{I}} \omega_i \psi_i^{\mathcal{E}}}{\frac{1}{I}\sum_{\mathbb{I}} \omega_i(\lambda_i + \mu_i)}$$

  - Social Cost of Carbon integrates these inequalities:

$$SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i LCC_i = \sum_{\mathbb{I}} LCC_i + \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, LCC_i\right)$$

Step 2: Optimal policy – Other motives

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers
  2. Distort energy demand, of countries that need more or less energy
- New measure: Social Value of Fossil (SVF)

$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \mathcal{C}^f_{EE}\, \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right) \; - \; \mathbb{C}\mathrm{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)$$

∘ Params: $\mathcal{C}^f_{EE}$ agg. fossil supply elasticity, s^f_i energy cost share and σ energy demand elasticity

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$$SVF := \frac{\partial \mathcal{W}/\partial E}{\partial \mathcal{W}/\partial w} = \underbrace{\mathcal{C}^f_{EE}}_{\text{agg. supply distortion}} \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, e^f_i - e^x_i\right)}_{\text{terms-of-trade redistribution}} - \underbrace{\mathbb{C}\text{ov}_i\left(\widehat{\lambda}_i, \frac{q^f(1-s^f_i)}{\sigma}\right)}_{\text{demand distortion}}$$

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- *Proposition 2:* Optimal fossil energy tax:

$$\Rightarrow \qquad t^f = SCC + SVF$$

  – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i\, LCC_i$

back

Step 2: Optimal policy – Details

- Taxing fossil energy has additional redistributive effects:
  1. Through energy markets: distort supply, lowers eq. fossil price, benefit net importers

$$\text{Supply Redistribution} = \Big(\sum_i \frac{1}{C^i_{ee}(e^x_i)}\Big)^{-1} \sum_i \widehat{\lambda}_i (e^f_i - e^x_i)$$
$$= q^f \Big(\sum_i \frac{e^x_i}{\nu_i}\Big)^{-1} \sum_i \widehat{\lambda}_i (e^f_i - e^x_i)$$

  2. Distort energy demand, of countries that need more or less energy:
     FOC $MPe^f_i = q^f$, multiplier $\propto \widehat{\upsilon}^f_i$

$$\text{Demand Distortion} = \frac{1}{I}\sum_i \widehat{\upsilon}^f_i \partial_{e^f} MPe^f_i + \widehat{\upsilon}^r_i \partial_{e^f} MPe^f_i$$
$$= -(q^f + \mathrm{t}^f)\frac{1}{I}\sum_i \widehat{\upsilon}^f_i \frac{1}{e^f_i}\Big[\frac{1-s^f_i}{\sigma^e} + s^f_i \frac{1-s^e_i}{\sigma^y}\Big] + \frac{1}{I}\sum_i \widehat{\upsilon}^r_i \frac{q^r_i}{e^f_i} s^f_i \Big[\frac{1}{\sigma^e} - \frac{1-s^e_i}{\sigma^y}\Big]$$

Step 2: Optimal policy – Details

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- *Proposition 2:* Optimal fossil energy tax: back

$$\Rightarrow \qquad \mathfrak{t}^f = SCC + \text{Supply Redistrib.} + \text{Demand Distortion}$$

  – Social cost of carbon: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i LCC_i$

Step 3: Ramsey Problem with participation constraints

- Consider that countries can "exit" climate agreement.
- For a climate "club" of $\mathbb{J} \subset \mathbb{I}$ countries:
  - Countries $i \in \mathbb{J}$ are subject to a carbon tax t^f
  - Countries $i \in \mathbb{J}$ can unilaterally leave, subject to retaliation tariff $\mathrm{t}^{b,r}$ on goods and get consumption $\tilde{c}_i$
  - Countries $i \notin \mathbb{J}$ trade in goods subject to tariff t^b with club members and countries outside the club. They still trade with the club members in energy at price q^f

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- Participation constraints:

$$u(c_i) \geq u(\tilde{c}_i) \qquad [\nu_i]$$

- Welfare:

$$\mathcal{W} = \max_{\{\mathbf{t}, \boldsymbol{e}, \boldsymbol{q}\}_i} \sum_{\mathbb{J}} \omega_i \, u(c_i) + \sum_{\mathbb{J}^c} \alpha \omega_i \, u(c_i)$$

Step 3: Ramsey Problem with participation constraints

- Participation constraints

$$u(c_i) \geq u(\tilde{c}_i) \quad [\nu_i]$$

- *Proposition 3.1:* Second-Best social valuation with participation constraints
  - Participation incentives change our measure of inequality

$$\text{w/ trade:} \qquad \omega_i(1+\nu_i)u'(c_i) = \Big(\sum_{j \in \mathbb{I}} a_{ij}(d_{ij}\mathrm{p}_j)^{1-\theta} \big[\omega_i \widetilde{\lambda}_i + \omega_j \widetilde{\mu}_j + \widetilde{\eta}_{ij}(1-s_{ij}) \big]^{1-\theta} \Big)^{\frac{1}{1-\theta}}$$

$$\Rightarrow \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(\widetilde{\lambda}_i + \widetilde{\mu}_i)}{\frac{1}{J}\sum_{\mathbb{J}} \omega_i(\widetilde{\lambda}_i + \widetilde{\mu}_i)} \neq \widehat{\lambda}_i$$

$$\text{vs. w/o trade} \qquad \widehat{\widetilde{\lambda}}_i = \frac{\omega_i(1+\nu_i)u'(c_i)}{\frac{1}{J}\sum_{\mathbb{J}} \omega_j(1+\nu_j)u'(c_j)} \neq \widehat{\lambda}_i$$

  - Result: $\omega_i(1+\nu_i)$ are the "endogenous Pareto weights"

Step 3: Participation constraints & Optimal policy

- *Proposition 3.2:* Second-Best taxes:
  - Taxation with imperfect instruments:
    - Climate change & general equilibrium effects on fossil market affects all countries $i \in \mathbb{I}$
    - Need to adjust for the "outside" countries $i \notin \mathbb{J}$ not subject to the tax, which weight on the energy market as $\vartheta_{\mathbb{J}^c} \approx \frac{E_{\mathbb{J}^c}}{E_{\mathbb{I}}} \frac{\nu\sigma}{q^f(1-s^f)}$
      with ν fossil supply elasticity, σ energy demand elasticity and s^f energy cost share.
  - Optimal fossil energy tax $t^f(\mathbb{J})$:

$$\Rightarrow \quad t^f(\mathbb{J}) = SCC + SVF$$

$$= \frac{1}{1-\vartheta_{\mathbb{J}^c}} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i LCC_i + \frac{1}{1-\vartheta_{\mathbb{J}^c}} \mathcal{C}^f_{EE} \sum_{i\in\mathbb{I}} \widetilde{\lambda}_i (e^f_i - e^x_i) - \sum_{i\in\mathbb{J}} \widetilde{\lambda}_i \frac{q^f(1-s^f_i)}{\sigma}$$

  - Optimal tariffs/export taxes $t^{b,r}(\mathbb{J})$ and $t^b(\mathbb{J})$: In search for a closed-form expression
    As of now, only opaque system of equations (fixed point w/ demand/multipliers)

back

Climate uncertainty and the Cost of Carbon:

- Stochastics: for any shock ϵ with distribution $\epsilon \sim \varphi(\epsilon)$
- New measure for inequalities:

$$\widehat{\lambda}^w_{it}(\epsilon) = \frac{\lambda^k_{it}(\epsilon)}{\mathbb{E}_{i,\epsilon}[\lambda^w_{it}(\epsilon)]} = \frac{\omega_i u'\big(c_{it}(\epsilon)\big)}{\int_\epsilon \int_j \omega_j u'\big(c_{j,t}(\epsilon)\big)\, dj\, d\varphi(\epsilon)}$$

- Uncertainty-adjusted SCC writes:

$$\mathbb{E}_\epsilon[SCC] = \int_{\mathcal{E}} \int_{\mathbb{I}} \widehat{\lambda}^w_{it}(\epsilon) LCC_{it}(\epsilon) d\varphi(\epsilon)$$

$$= \underbrace{\mathbb{E}_j\Big[\mathbb{C}\text{ov}_\epsilon\Big(\widehat{\lambda}^w_{it}(\epsilon), LCC_{jt}(\epsilon)\Big)\Big]}_{=\text{effect of aggregate risk } \epsilon} + \underbrace{\mathbb{C}\text{ov}_j\Big[\mathbb{E}_\epsilon\big(\widehat{\lambda}^w_{it}(\epsilon)\big), \mathbb{E}_\epsilon\big(LCC_{jt}(\epsilon)\big)\Big]}_{=\text{effect of heterogeneity across } j} + \underbrace{\mathbb{E}_{j,\epsilon}[LCC_{jt}(\epsilon)]}_{=\text{average exp. damage}}$$

$$> \mathbb{E}_\epsilon[\overline{SCC}(\epsilon)] \qquad \& \qquad > SCC_t$$

$\Rightarrow$ Climate uncertainty reinforces the unequal costs of climate change! back

Sequential solution method

- Summary of the dynamic model:
  - ODEs for states $\{\boldsymbol{x}\} = \{w_{it}, \tau_{it}, \mathcal{R}_{it}, \mathcal{S}_t\}_{it}$
  - Backward ODE for the costates $\{\boldsymbol{\lambda}\} = \{\lambda^w_{it}, \lambda^\tau_{it}, \lambda^{\mathcal{S}}_t, \lambda^{\mathcal{R}}_{it}\}_{it}$
  - Non-linear equations (FOCs) for household controls $\{\boldsymbol{c_1}\} = \{c_{it}, b_{it}, k_{it}\}_{it}$ and static demands for energy/capital $\{\boldsymbol{c_2}\} = \{e^f_{it}, e^r_{it}, k_{it}\}_{it}$ and static supplies $\{\boldsymbol{c_3}\} = \{e^x_{it}, \bar{e}^r_{it}\}_{it}$.
  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)

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  - Market clearing as equation for prices $\{\boldsymbol{q}\} = \{q^f_t, r^\star_t\}_t$
  - Existence and Uniqueness, c.f. Mean Field Game theory (Carmona-Delarue)
- Global Numerical solution:
  - Discretize agents (countries) space $i \in \mathbb{I}$ with M and time-space $t \in [t_0, t_T]$ with T periods
  - Express as a large vector $\boldsymbol{y} = \{\boldsymbol{x}, \boldsymbol{\lambda}, \boldsymbol{c}, \boldsymbol{q}\}$ in a large non-linear function

$$F(\boldsymbol{y}) = \boldsymbol{0}$$

  - Solve for the large system with $N = (N_{ind,vars} \times M + N_{agg,vars}) \times T$ unknowns and N equations with gradient-descent – Newton-Raphson methods.

Impact of increase in temperature

- Marginal values of the climate variables: λ_{it}^{s} and λ_{it}^{τ}

$$\dot{\lambda}_{it}^{\tau} = \lambda_{it}^{\tau}(\rho+\zeta) + \overbrace{\gamma_i(\tau_{it}-\tau_i^{\star})\mathcal{D}^y(\tau_{it})}^{-\partial_\tau \mathcal{D}^y(\tau_{it})} f(k_{it},e_{it})\lambda_{it}^{k} + \overbrace{\phi_i(\tau_{it}-\tau_i^{\star})\mathcal{D}^u(\tau_{it})^{1-\eta} c_{it}^{1-\eta}}^{\partial_\tau u(c,\tau)}$$

$$\dot{\lambda}_{it}^{S} = \lambda_{it}^{S}(\rho+\delta^s) - \zeta\,\chi\,\Delta_i\,\lambda_{it}^{\tau}$$

- Costate λ_{it}^{S}: marg. cost of $1Mt$ carbon in atmosphere, for country i. Increases with:
  - Temperature gaps $\tau_{it}-\tau_i^{\star}$ & damage sensitivity of TFP γ_i^y and utility γ_i^u
  - Development level $f(k_{it},e_{it})$ and c_{it}
  - Climate params: χ climate sensitivity, Δ_i “catching up” of τ_i and ζ reaction speed
  - back

Cost of carbon / Marginal value of temperature

- Solving for the cost of carbon and temperature ⇔ solving ODE

$$\dot{\lambda}_{it}^{\mathcal{T}} = \lambda_t^{\mathcal{T}}(\widetilde{\rho} + \Delta\zeta) + \gamma(\tau - \tau^\star)\mathcal{D}^y(\tau)f(k,e)\lambda_t^k + \phi(\tau - \tau^\star)\mathcal{D}^u(\tau)u(c)$$

$$\dot{\lambda}_t^S = \lambda^S{}_t(\widetilde{\rho} + \delta^s) - \int_{\mathbb{I}} \Delta_i \zeta \chi \lambda_{it}^{\mathcal{T}}$$

- Solving for $\lambda_t^{\mathcal{T}}$ and $\lambda_t^{\mathcal{S}}$, in stationary equilibrium $\dot{\lambda}_t^S = \dot{\lambda}_t^{\mathcal{T}} = 0$

$$\lambda_{it}^{\mathcal{T}} = -\int_t^\infty e^{-(\widetilde{\rho}+\zeta)u}(\tau_u - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_u)y_{\mathcal{T}}\lambda_u^k + \phi\mathcal{D}^u(\tau_u)u(c_u)\Big)du$$

$$\lambda_{it}^{\mathcal{T}} = -\frac{1}{\widetilde{\rho} + \Delta\zeta}(\tau_\infty - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_\infty)y_\infty\lambda_\infty^k + \phi\mathcal{D}^u(\tau_\infty)u(c_\infty)\Big)$$

$$\begin{aligned}
\lambda_t^S &= -\int_t^\infty e^{-(\widetilde{\rho}+\delta^s)u}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,u}^{\mathcal{T}}dj\,du \\
&= \frac{1}{\widetilde{\rho}+\delta^s}\zeta\chi\int_{\mathbb{I}}\Delta_j\lambda_{j,\infty}^{\mathcal{T}} \\
&= -\frac{\chi}{\widetilde{\rho}+\delta^s}\frac{\zeta}{\widetilde{\rho}+\zeta}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_\infty\lambda_{j,\infty}^k + \phi\mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj \\
\lambda_t^S &\xrightarrow[\zeta\to\infty]{} -\frac{\chi}{\widetilde{\rho}+\delta^s}\int_{\mathbb{I}}\Delta_j(\tau_{j,\infty} - \tau^\star)\Big(\gamma\mathcal{D}^y(\tau_{j,\infty})y_{j,\infty}\lambda_{j,\infty}^k + \mathcal{D}^u(\tau_{j,\infty})u(c_{j,\infty})\Big)dj
\end{aligned}$$

Cost of carbon / Marginal value of temperature

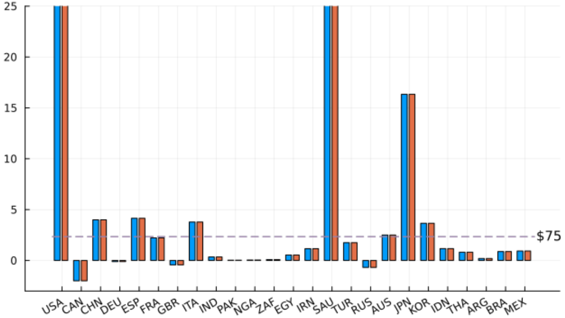
- ***Proposition (Stationary LCC)***:
  When $t \to \infty$ and for a BGP with $\mathcal{E}_t = \delta_s \mathcal{S}_t$ and $\tau_t \to \tau_\infty$, the LCC is *proportional* to climate sensitivity χ, marg. damage γ_i^y, γ_i^u, temperature, and output, consumption.

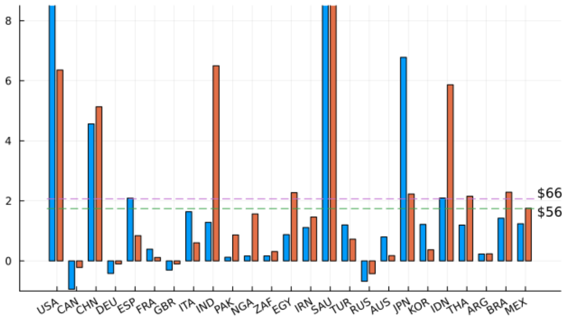
$$LSCC_{it} \equiv \frac{\Delta_i \chi}{\rho - n + \bar{g}(\eta - 1) + \delta^s} (\tau_\infty - \tau^\star) \Big(\gamma \mathcal{D}^y(\tau_\infty) y_\infty + \phi \mathcal{D}^u(\tau_\infty) c_\infty \Big)$$

  - Stationary equilibrium: $\dot{\lambda}_t^S = \dot{\lambda}_t^T = 0$
  - Fast temperature adjustment $\zeta \to \infty$
  - Back

Local Cost of Carbon & Carbon Tax – First and Second Best

- Recall: $SCC = \sum_{\mathbb{I}} \widehat{\lambda}_i^w LCC_i$
- Difference: $LCC_i = \frac{\psi_i^S}{\lambda_i^w}$ vs. $\widehat{\lambda}_i^w LCC_i = \frac{\psi_i^S}{\bar{\lambda}^w}$





Carbon tax and SCC differ!

- $SCC = \frac{\psi^S}{\bar{\lambda}^w}$ + Decomposition   Back
- t^f = SCC + Supply Redistrib. + Demand Distort°